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Design of Anti-Backlash Transmissions for Precision Position Control Systems

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Position control systems typical of machine tools and robots usually operate at relatively high torque and low speed. While servo motors having these characteristics are now available, their cost, size and weight generally exceed those of the traditional servo motor and gear reducer. Critical applications require transmissions with zero backlash and high stiffness, for which there are several solutions. This paper reviews various anti-backlash techniques, discusses their subtleties and applications, develops models, and presents design methodologies. Although presented in the context of geared transmissions, the ideas apply to other machine elements and situations requiring precision and robustness. Two case studies, a robot revolute joint and a machine tool axis of rotation, help to reinforce the theory and raise practical issues.

Keywords: *anti-backlash gears; anti-backlash transmission; position control; preload*

Introduction

Position control systems typical of machine tools and robots, operate at relatively low speeds while being subject to changing, often reversing load conditions. Electric servo motors usually are the actuators of choice, but these operate most efficiently at much higher speeds. Furthermore, the motor's weight, cost and heat generation increase in proportion to its output torque. The proposed situation favors a relatively large transmission ratio over a directly coupled system. In some cases a transmission is almost unavoidable, for example on a long machine axis.

An important characteristic of a transmission ratio R is the R^2 effect on inertia and stiffness reflected through the transmission. An ideal transmission, one without inertia, compliance, friction or backlash, reduces the load inertia reflected to the motor by R^2 and increases the servo stiffness reflected to the load by R^2. Compared to a directly coupled system, the closed loop natural frequency increases by R, and the damping ratio increases by R^2 assuming that all other system parameters remain constant, and that the reflected inertia of the load remains much larger than the motor inertia. This makes the system easier to control with off-the-shelf components, e.g., PID controllers.

Mechanical transmissions such as gear trains obviously have inertia, compliance and friction, but not necessarily backlash. These disadvantages together with added cost and complexities make direct drive systems very attractive for applications requiring high precision, rapid traverse speeds or minimal particle generation. Therefore, careful study of each application is important before committing to a strategy. The study may compare total system cost, size, weight, stiffness, resolution, accuracy and reliability. This paper assumes that the outcome of the study favors a large transmission ratio, and then the goal becomes, *design a nearly ideal transmission.*

Backlash frequently is the most serious problem associated with geared transmissions. Although required for proper tooth action, too much backlash may lead to: limit cycling for systems with output position feedback, unacceptable position errors for systems with motor position feedback, or chatter for systems excited by time varying loads, for example in milling. Fortunately, several techniques exist for eliminating backlash in geared transmissions. While the main focus of this paper is on geared anti-backlash transmissions, the concepts and design issues apply to other machine elements; for example, ball screws, spindle bearings and linear bearings.

References on anti-backlash transmissions are very few, but a number of patents exist for applications primarily in robotics and machine tools.[1] Most of the patents use a technique that is applicable when loads are light or primarily in one direction. One patent for a large tracking antenna uses a better technique to achieve higher stiffness and equal bi-directional load capacity.

This paper presents as background several anti-backlash techniques and discuses their applications, advantages and disadvantages. Mathematical models, developed later, provide deeper understanding and important relations for design. Finally there is enough foundation established to design anti-backlash transmissions. Two case studies, a robot revolute joint and a machine tool axis of rotation illustrate real world problems and interesting solutions.

Background

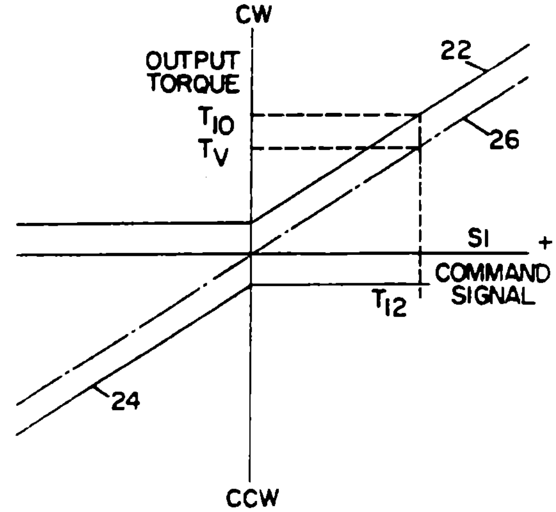
Several techniques for reducing or eliminating backlash in transmissions are available, and the designer must decide which one provides the best compromise on competing issues for a particular application. The designer should first consider some simple techniques that may well satisfy the application for the lowest cost.

The designer frequently can use gravity or another conservative force to oppose the load on the transmission so that it never experiences a reversing load. This requires a conventional transmission between the motor and the load, and a preferably constant force applied to the load from another source. This method has the disadvantage that it requires a larger servo motor to work against the constant force.

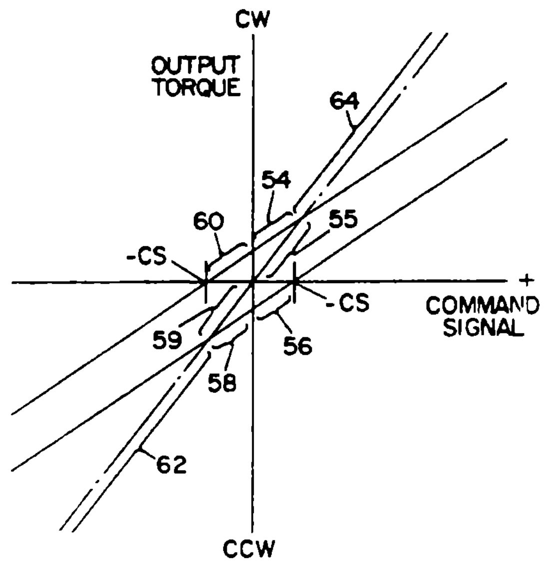
Another simple technique called crowded centers, provides backlash control by adjustment of the gear center distance. The adjustment may be either rigid or compliant, akin to a belt tension adjustment, but with far less margin for error. With a rigid adjustment, it is possible to remove virtually all the backlash from constant sources, i.e., size tolerances. Removing the rest with this method requires extremely precise components, resulting in exceedingly high manufacturing cost. In addition, any wear of components degrades the effectiveness of the system.

As an alternative, a compliant adjustment mechanism has freedom to follow variable error sources and can achieve zero backlash. Some gear checking machines use this principle by meshing the tested gear against a master gear under a light preload and by measuring variations in center distance. The crowded center approach becomes cumbersome for a gear train having several stages of gear reduction. In addition, gears manufactured with backlash allowance may operate with tip-to-root interference at a crowded center distance. Gear details, therefore, should specify zero backlash allowance.

A very effective and flexible method for eliminating backlash uses two identical motors and transmissions connected to the output. Figure 1 from U.S. Patent 3,833,847, shows two methods for controlling a dual motor drive system.[2] The patent claims the electric drive system as original, but earlier versions of the same idea used hydraulic drive systems, e.g., large construction cranes. The first control method uses unidirectional torque at each motor so that neither transmission experiences a reversing load. This method does not take full advantage of both motors, but it is suitable for position feedback located at either motor. The second control method simply applies an opposite bias torque to each motor. This method has regions where both motors work together, but since their torques reverse, the location of position feedback should be at the output. Limit cycling is not a problem if at least one motor drives through a stiff path.



(a)



(b)

Figure 1: The coordination of two drive motors can be very simple. Motor torques (solid lines) add to give a net output torque (center line). In (a), each motor applies an opposite unidirectional torque. In (b), the motors have a constant bias torque between them.

The idea of using preload between rolling element bearings of back-to-back or face-to-face configuration is quite common and effective for increasing stiffness and removing lost motion in a spindle. The same idea applies to other machine elements such as ball screws and gear trains. Figure 2 illustrates this concept with three configurations that are functionally equivalent. Parts (a) and (b) come from U.S. Patent 4,953,417, and show two distinct transmission paths from the input (Item 10) to the output (Item 11).[3] By means of a preload adjustment device (Item 13), one path can oppose the other to eliminate all backlash in the transmission. A compliance (Item 12) added to one path serves to reduce variations in the preload that arise due to component errors.

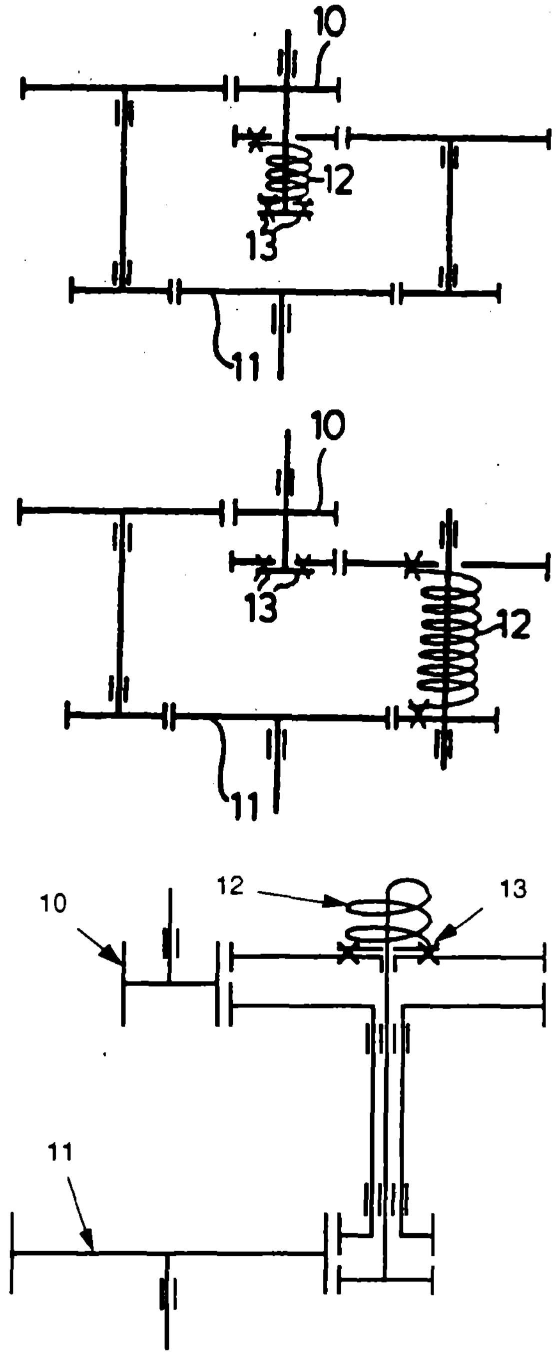


Figure 2: A Type 1 anti-backlash transmission has an added compliance in one path. Part (c) has concentric paths and requires less space.

Since the configurations in Parts (a) and (b) have two gears in mesh with the output gear, people tend to name them dual-pinion. This is unfortunate because they are functionally no different from the configuration in Figure 2c, which people tend to name split-pinion. In this paper, we give the name Type 1 to designs that add compliance to one path, effectively eliminating the stiffness of that path. Type 1 designs have one stiff path.

Figure 3 shows a dual-pinion design from U.S. Patent 3,665,482.[4] It has two stiff paths connecting the output gear to the motor (Item 10) through a differential (Item 11). A torque applied by a compliant spring (Item 23) causes the bevel gears (Items 20) to counter rotate, thus removing backlash in each path and generating a preload. This particular design is overly complicated, but it illustrates what we call Type 2. A Type 2 design has two stiff paths and does not sacrifice drive stiffness to achieve an even preload. Presentations of simpler Type 2 designs come later in the paper.

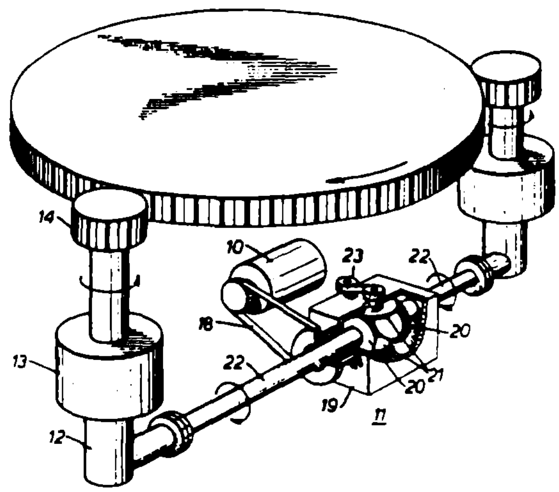


Figure 3: Both paths of this large tracking antenna contribute their full stiffness to the drive, making it a Type 2 design.

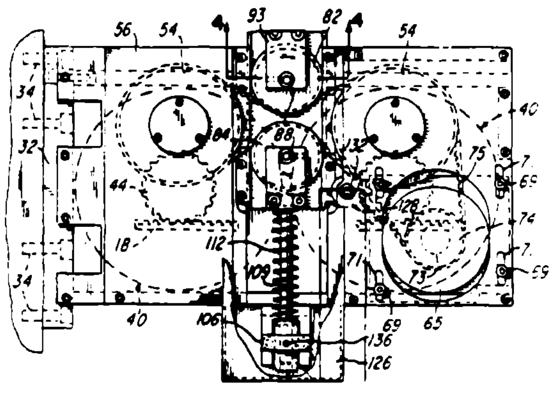


Figure 4: The left path of this slide drive has the added compliance of the preload spring making the design Type 1.

Figure 4 comes from U.S. Patent 4,714,388 and shows a rather complicated dual-pinion design that functions as a Type 1.[5] Two identical planetary transmissions (Items 40) engage a rack (Item 18) with their output gears (Items 44), and have their inputs (Items 54) driven by a timing belt. A spring loaded slide with two idler pulleys causes the input pulleys to counter rotate in a manner like the previous example. However with the motor input located at Item 73, the left path has the added compliance of the spring loaded slide. Had the motor input been at Item 88, the design would be Type 2, and the inventor would have achieved nearly twice as much stiffness.

In many cases, the available space for the transmission is a limiting factor. Then the designer should consider compact, high ratio transmissions but be well aware of their fundamental limitations. Large gear ratios in a single stage are typical of worm gears and differential drives; for example, epicyclic gear trains, Harmonic™ drives and cycloidal drives.[6] Typical backlash control is by the crowded center method. Worm gears have either rigid or spring loaded center adjustments; whereas, differential drives usually control backlash by tolerance or by selective fit.

The problem is that compact transmissions tend to have significant losses. Worm gears depend on fully developed hydrodynamic lubrication to have good efficiency, which is difficult to achieve for position control. Figure 5 comes from U.S. Patent 3,494,215 and shows a unique anti-friction worm gear.[7] Fordson Drive of Dearborn Michigan owns the patent, but we do not know if it is commercially available.

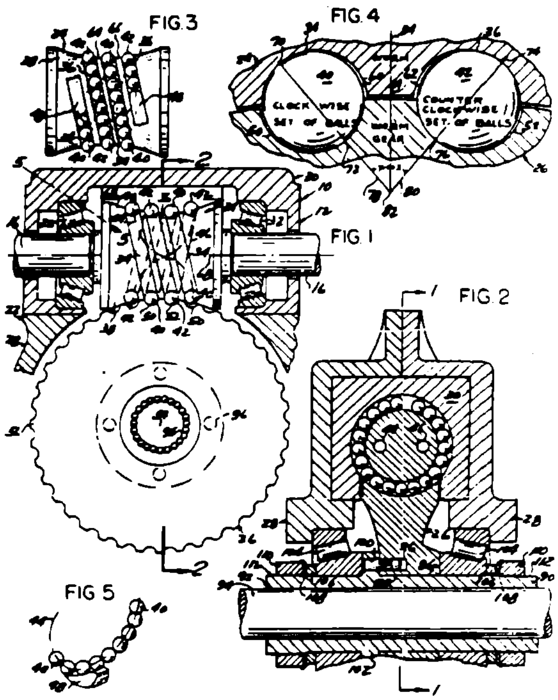


Figure 5: An anti-friction, anti-backlash worm drive is analogous to a ball screw.

Differential drives achieve very large reduction ratios by outputting the difference between two almost identical high speed meshes. Since the high speed meshes see output sized loads, losses can be overwhelming. The differential screw is a familiar, linear example. The so called *perpetual wedge* uses involute gears arranged differentially to achieve gear ratios in excess of 50:1; however in this configuration, the typical efficiency is below 50% making it impossible to back drive.[8] Successful designs eliminate much of their frictional losses but are still prone to fatigue failures. The cycloidal drive, for example, uses roller bearings to combat friction. The designer should be somewhat conservative when sizing commercial differential drives, and should expect instability if they are difficult to back drive.

Modeling Anti-backlash Gear Trains

The idea of applying preload between two parallel transmission paths was central to the methods presented previously. Furthermore, we distinguished between designs that function either as Type 1 or Type 2. Simple linear spring models are sufficient to fully explain the characteristics of each type and to make design decisions. The force-deflection curve for a real transmission path may have some nonlinear trend due to Hertzian contact in ball bearings, but often as one path hardens, the other path softens giving a total stiffness that is nearly constant.

The details of calculating path stiffnesses are not essential to the development of the models; however, four facts are important to keep in mind:

- Stiffnesses in parallel add.
- Compliances (reciprocal of stiffness) in series add.
- A stiffness reflected through a transmission ratio R increases by a factor R^2.
- Similarly, a linear stiffness acting at a lever arm r multiplied by r^2 gives the equivalent torsional stiffness.

Finding the total stiffness of a path becomes an assembly of component stiffnesses. Since most components are in series, it is often easier to work with compliance reflected to a common location and type, e.g., torsional compliance at the output. Most bearing manufacturers will supply force-deflection data for their bearings; the derivative of which is the stiffness. Beam theory gives the stiffness of most other components. Particularly valuable is the tangential stiffness of a typical gear mesh given by Equation 1, where E is the elastic modulus and w is the face width.[9]

$$k_{mesh} = \frac{E \cdot w}{11} \qquad (1)$$

The preload diagram is a good visual aid for understanding what happens to forces and deflections in each path of an anti-backlash transmission or other such preloaded device. Figure 6a shows the force-deflection curve for each path, represented by k_1 and k_2. Both are linear except for their backlash at zero force represented by b in the figure. Preloading one path against the other to a force F_L requires a total deflection δ_L around the loop formed by the two paths, hence the subscript L. Thus the curves overlap as shown and have at the point of intersection, zero net drive force F_D. By convention, the drive has zero displacement δ_D at the intersection.

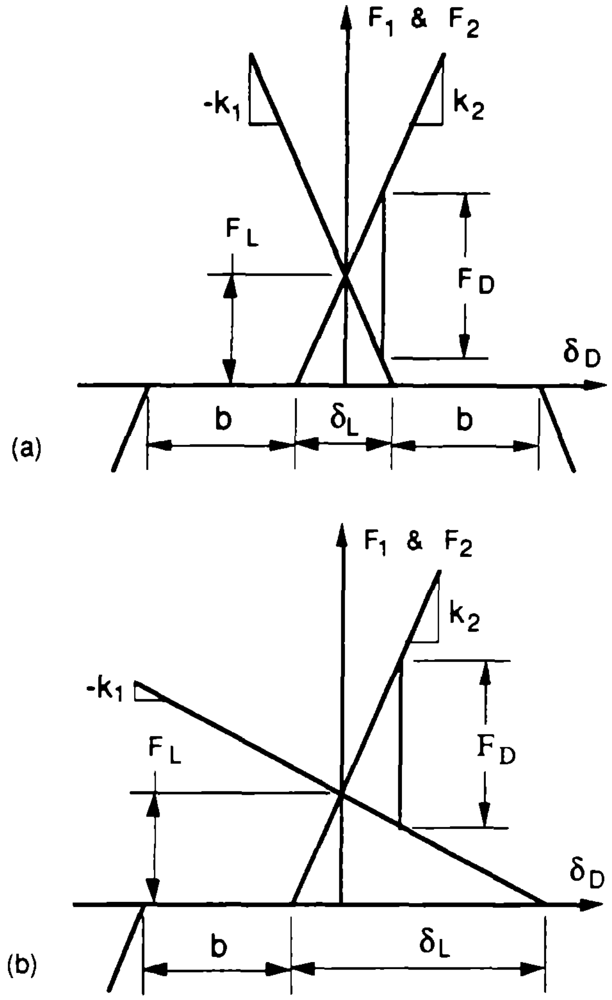


Figure 6: Preload diagrams show cases of equal path stiffnesses (a), and significantly different path stiffnesses (b). The case represented in (b) is more tolerant of transmission errors within the loop.

The vertical line of length F_D in Figure 6a represents the drive load applied to the transmission and its location along the δ_D axis represents the deflection of the transmission. Equation 2 follows directly from the figure and gives the drive stiffness k_D as simply the parallel sum of path stiffnesses. Should one path enter a region of backlash, then the drive stiffness reduces to the stiffness of the other path. This may be an acceptable condition in some applications, but most designs always operate with preload.

$$k_D = \frac{F_D}{\delta_D} = k_1 + k_2 \tag{2}$$

Each transmission path will have errors from such sources as tooth form errors, pitch line runouts and bearing runouts. The errors cause δ_L to vary during operation and as a result, F_L varies in proportion. Equation 3 gives the loop stiffness k_L as the series sum of path stiffnesses.

$$\frac{1}{k_L} = \frac{\delta_L}{F_L} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 \cdot k_2} \tag{3}$$

Figure 6b shows a situation where one path stiffness is considerably less than the other as would be the case for the Type 1 designs presented earlier. Typically the compromise is to achieve maximum drive stiffness for an allowable variation in preload, perhaps 10%. An appropriate indicator is the ratio of drive stiffness to loop stiffness, given by Equation 4. The ratio is minimal when $k_1 = k_2$, and it increases as k_1 and k_2 become different.

$$\frac{k_D}{k_L} = \frac{(k_1 + k_2)^2}{k_1 \cdot k_2} \tag{4}$$

The minimal condition may be appropriate if the application requires equal bi-directional stiffness outside the preload region, and control of transmission errors is very good as compared to δ_L. Otherwise a Type I design commonly has extra compliance in one path, making it many times more accommodating to transmission errors.

A Type 2 design is different because the added compliance is sensitive to variations in preload but is insensitive to variations in drive load. To accomplish this, a mechanism like the differential (Item 11) in Figure 3, divides the drive force evenly between the two transmission paths. Then it becomes a simple matter to add a spring to the mechanism, which applies a preloading force from path to path. The benefit is lower loop stiffness without sacrificing drive stiffness.

Figure 7 shows the linear spring model for a Type 2 design. The spring stiffnesses k_1 and k_2 represent the two transmission paths, and k_3 is the preload spring. In Part (a), the path forces F_1 and F_2 add to equal the drive force F_D as a result of the drive displacement δ_D. In an actual design, F_D would be the torque applied to the input shaft. In addition, F_1, F_2 and F_3 balance to give a second equation. By representing forces in terms of spring deflections and eliminating the deflection of k_3, the two equations combine to give Equation 5 for the drive stiffness.

$$k_D = \frac{F_D}{\delta_D} = (k_1 + k_2) - \frac{(k_1 - k_2)^2}{(k_1 + k_2 + k_3)} \tag{5}$$

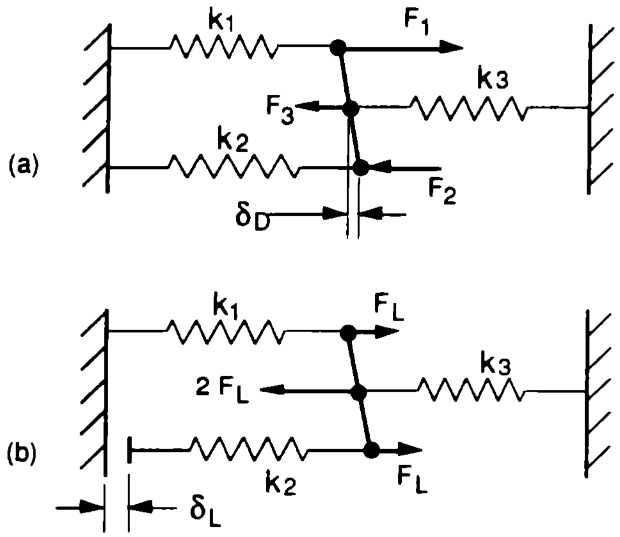


Figure 7: A displacement of the drive results in forces given in (a). A displacement in the loop due to component errors results in forces given in (b).

In Figure 7b, there is only loop force as a result of loop displacement. Proceeding as before, Equation 6 gives the loop stiffness k_L as the series sum of stiffnesses.

$$\frac{1}{k_L} = \frac{\delta_L}{F_L} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{4}{k_3} \tag{6}$$

Usually the transmission paths are identical and thus have the same stiffness, $k_1 = k_2$. Equations 5 and 6 simplify somewhat and one can see that a small k_3 can significantly reduce k_L without effecting k_D. If on the other hand k_3 was infinitely large (zero compliance), then Equations 5 and 6 would reduce, respectively, to Equations 2 and 3 for the Type 1 model.

Designing Anti-backlash Gear Trains

A task sometimes more difficult than design is specifying the capability required of the design. Ideally, one would determine the specification based on physical understanding of the process involved and the necessary quality of the output. Then one could design to meet the specification subject to constraints such as cost, space and time. One or more iterations would result if the constraints were too tight. Other times your customer or the market place defines the specification. No such specification existed for the two case studies described in the following sections. Instead each design was a *best effort* subject to a well-defined space constraint and *reasonable cost*.

The vagueness of a typical problem statement makes this section something other than a procedure for design. Design is seldom like that. The goal of this section is to develop an understanding of important issues, and to share some valuable techniques that illustrate practical implementation of the theory presented.

Consider a gear train with several stages of reduction, and determine conceptually the distribution of inertia, compliance, friction and backlash. To easier visualize, assume that each stage increases the transmission ratio R_i relative to the motor by the same factor, i.e., a geometric progression.

Speed decreases and force increases at each stage in proportion to R_i so that the transmitted power is the same at each stage, neglecting losses. Although friction force increases in proportion to R_i, its sliding velocity decreases to give approximately the same power loss per stage. Using fewer stages decreases losses, but this generally requires more space.

The designer will frequently size each stage to achieve nearly uniform stresses throughout the gear train. Assume for the moment that both inertia and stiffness of a stage increase in proportion to R_i. In that case the reflected inertia and stiffness of a stage would decrease in proportion to R_i due to the R^2 effect. Many gear trains show this trend, and as a result, stages nearest the motor contribute most of the inertia, while most of the compliance lies in stages nearest the output. In other words the high speed gears have more kinetic energy while the low speed gears store more elastic energy.

Backlash is a nonlinearity that can adversely affect the performance of a position control system, causing it either to limit cycle or to be less accurate and repeatable. A stage contributes to the total effect its backlash multiplied by R_i. Thus backlash nearest the output is most detrimental.

Removing all backlash may not be necessary or optimal, depending on the application. Each preloaded mesh experiences increased friction, wear and fatigue, in addition to costing more and requiring extra space. Usually, only a few stages nearest the output need to be anti-backlash. The other stages may have minimum backlash achieved by tolerance or by adjustment. This strategy eliminates almost all backlash, adds negligible inertia and nearly doubles the stiffness, assuming a Type 2 design. The two case studies use this strategy.

Type 1 Design

A Type 1 design can be simpler, more compact and less expensive than a Type 2, thus it is the preferred design for applications requiring moderate stiffness, and where the loads are light or predominately in one direction. The Type 1 design in Figure 4 offers none of the advantages mentioned, and the application seems better suited for a Type 2 design. This example indicates that these advantages are not inherent, and occur only by design.

Figure 8 is a hypothetical Type 1 design that shows all aspects important for the discussion. The paths are concentric to reduce space. Fewer anti-friction bearings are necessary, but they require preload since the direction of load may change. The inner path is naturally compliant with its shaft size determined to accommodate anticipated loop errors. The maximum load carried by the inner path is considerably less than that of the outer path, accordingly the gear face widths differ. A frictional interface in the inner path is one very good way to adjust preload, and a pair of set screws or a special tool creates the torque. The amount of preload set should be slightly greater than the maximum expected load tending to unload the stiff path. This recommendation provides some degree of conservatism as one can see from Figure 6b.

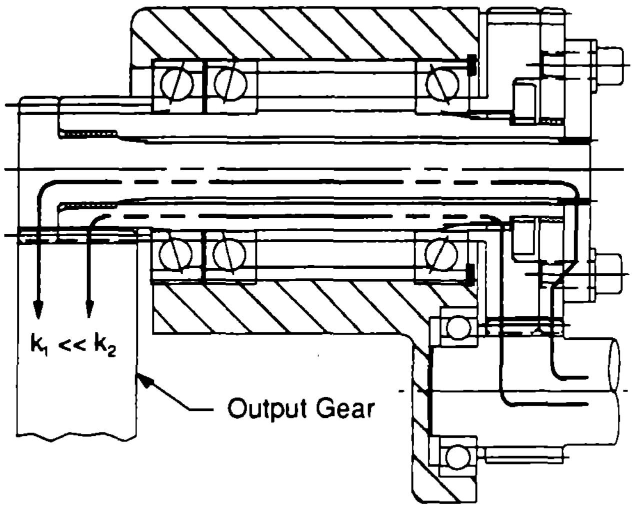


Figure 8: This concentric Type 1 design (split-pinion) has a frictional interface on the compliant, inner shaft to allow for preload adjustment.

Type 2 Design

Applications requiring high stiffness and equal bi-directional load capability favor the Type 2 design. Although a split-pinion Type 2 design is possible, the dual-pinion configuration is most natural. As noted before, a dual-pinion Type 1 is a waste of hardware. The distinguishing feature for the Type 2 design is a preload device usually located at the common input to the two transmission paths. This device must apply preload independently from the drive torque. In other words, it must decouple preload and drive torque. It must be free to move with low stiffness compared to the path stiffnesses and have minimal friction. As always, it must have a means to set or adjust the preload.

The following are three ways that the device could move one or more gears to generate preload in each path independently from the drive motion:

- Radial motion of a shared gear in a direction having equal components along each line of action, as shown in Figure 9.
- Axial motion of a pair of right and left handed helical gears, or of one herringbone gear.
- Opposite angular motion of two gears related by a differential as shown in Figure 3.

Radial motion is simple and compact. Figure 9 shows the optimum configuration where the two lines of action are parallel and opposite to the externally applied preload force F_e. This configuration minimizes the lateral force on the linear motion bearing. The optimum is fairly flat within several degrees of the operating pressure angle. Since both case studies use this configuration, further discussions will follow.

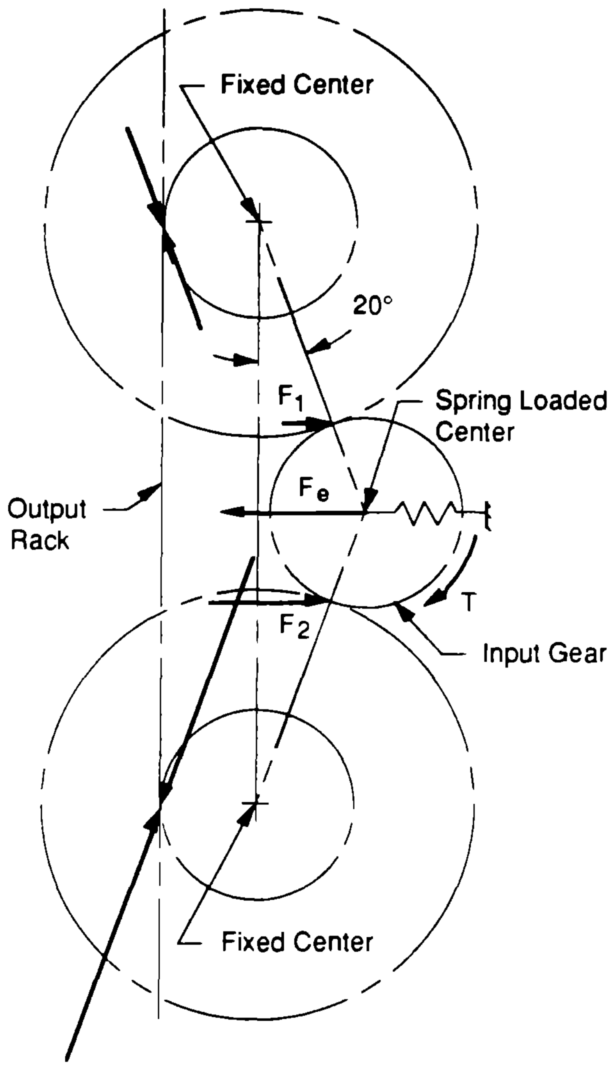


Figure 9: The sum of contact forces $F_1 + F_2$ balances the external preload force F_e. The difference between contact forces balances the drive torque T.

The diagram for axial motion preload looks just like Figure 9 except that the external preload force acts along the axis of the input shaft. Axial motion produces counter rotation of the left and right handed helical gears in mesh with the input shaft. The easiest way to provide axial motion is by letting the bearings for the input shaft slide in their housing. Clearance in the bearings is usually not a problem, but the ratio of friction to preload can become large for too small of a helix angle.

People who understand the automobile differential should instantly understand the angular motion preload device. Referring to Figure 3, a torque applied to Item 23 by a spring causes counter rotation of Items 20, thus removing backlash and generating preload in the loop. The input to the anti-backlash transmission is Item 19. The differential makes this method by far the most costly.

Usually a spring deflected through a measured distance generates the external force for preload. Its stiffness should be several times smaller than one path stiffness. Then the loop stiffness is essentially all due to the preload spring, making it easy to calculate preload variation. The preferred method for setting the spring is to have a frictional interface for adjustment somewhere in the loop. This also solves the *timing* problem of getting the last assembled gear to go into mesh. The problem is particularly significant to the radial preload method because it controls the center location of the input gear. The first case study required an alternative solution to the timing problem.

The preload in the loop should be at least one half of the maximum expected load to prevent one path from becoming unloaded. If that happens, the drive stiffness reduces to the series sum of the loaded path and the preload spring. This relatively high preload seems to be a disadvantage because it accentuates wear and increases friction; however, the Type 1 design requires a preload equal to the maximum expected load.

Case Study: A Robot Revolute Joint

The robot in this study has four degrees of freedom. Two identical links house the wrist and elbow joints with both having vertical axes of rotation. A third revolute joint at the shoulder gives the robot full planer motion. The shoulder mounts to a vertical slide to give the robot volumetric range. Only the joint located in the link was the subject of a re-design effort to decrease backlash and to double the gear ratio. The envelop of the link did not change.

Figure 10 shows the joint end of the robot link as it would appear from above with covers removed. The bevel gear mesh has minimal backlash when properly set. The other gears are in the optimal configuration for radial motion preload. In this design, the servo motor, bevel gear assembly and input gear move as a unit on three parallel blade flexures. Two compression springs provide the external preloading force.

A difficulty arose because the envelope did not allow room for an angular adjustment interface. Without one, the input gear would not mesh properly with the two cluster gears, except by accident or by design. The correct angular relationship between gears would have to be built into the assembly. There are two ways to proceed but only one makes sense.

Start by manufacturing each cluster gear with the same angular orientation between its large and small gears. One method is to lightly press and bond the large gear to the shaft of the small gear using a jig for angular alignment. Then it becomes primarily a mathematics problem to find the location of the gear centers to make the teeth mesh properly.

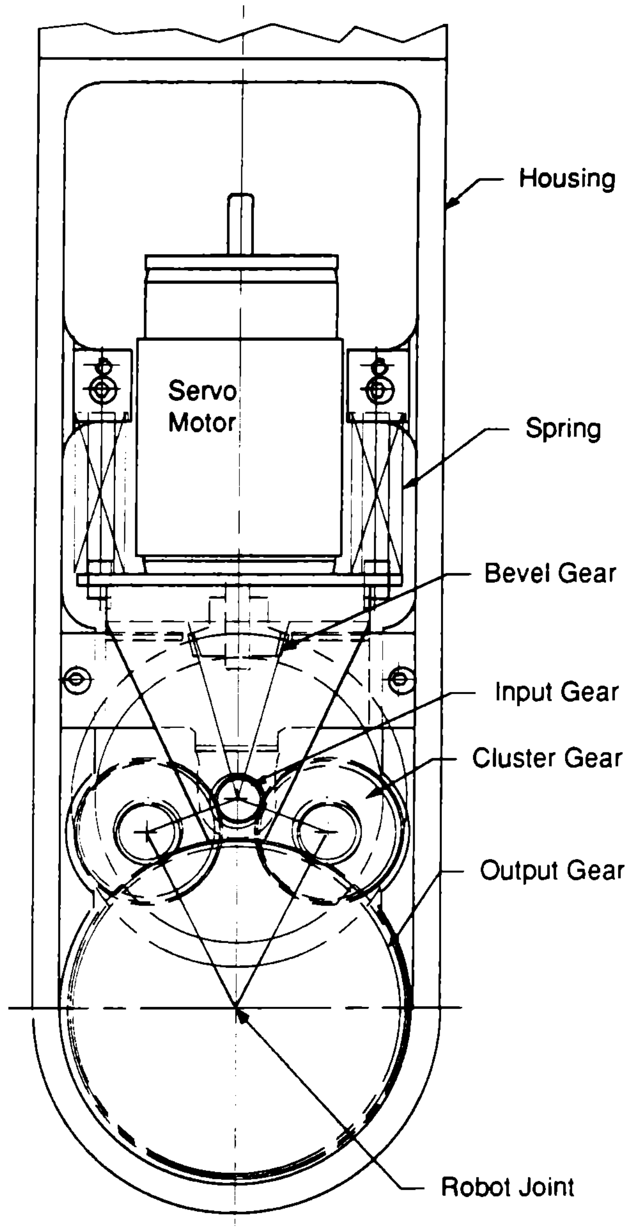


Figure 10: A dual-pinion configuration can be compact. The servo motor and bevel gear assembly move on three parallel blade flexures to provide radial motion preload. The scale is half size.

Figure 11 shows the construction used to set up the mathematical model. Place the gears in a straight line with their centers crowded to remove all backlash. Since both cluster gears are identical, this configuration guarantees that all gears mesh together. Rolling the cluster gear up the output gear as shown in (b) causes the input gear to rotate in a clockwise direction through an angle determined by the angle A_2, the gear ratios R_1, R_2, and by the angular backlash B_2 present at the operating center C_2. In similar fashion, rolling the input gear back to the original centerline causes an additional clockwise rotation determined by A_1, R_1 and B_1 at C_1. Any half-tooth increment i of the input gear is also an acceptable configuration. The mirror image path contributes exactly the opposite rotation making half increments whole. This construction gives one equation for two unknowns A_1 and A_2. The second equation comes directly from the geometry.

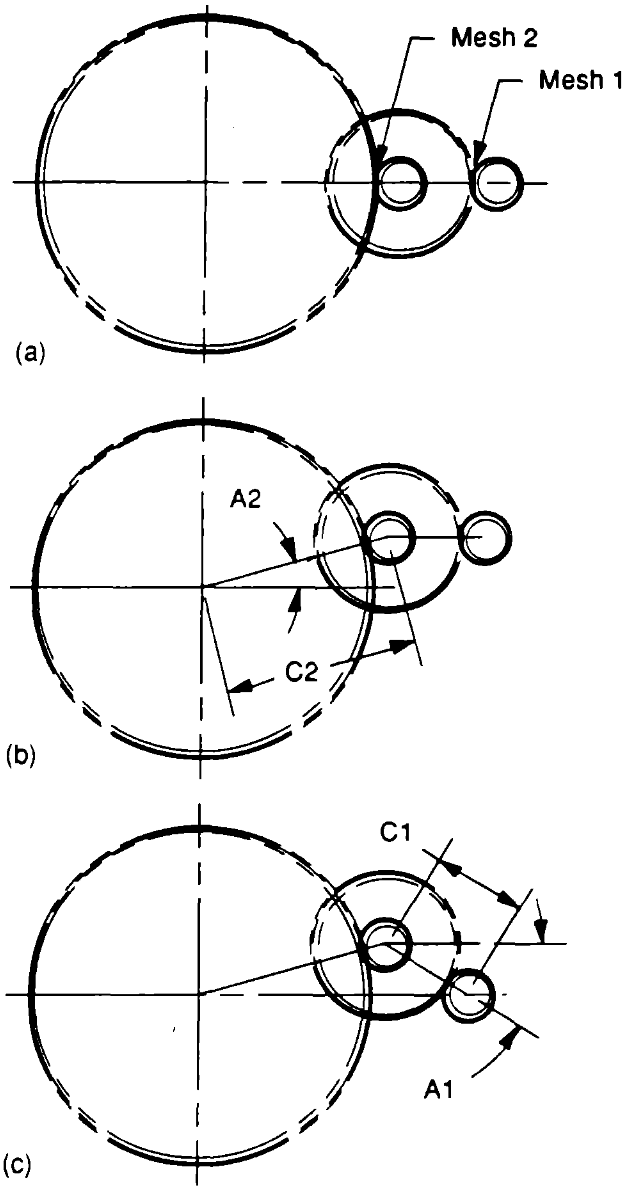


Figure 11: Construction of the mathematical model begins with a configuration (a) where both paths are identical. Other configurations (c) are possible where the mirror image path would also mesh with the input.

$$\left[A_2 \cdot (R_2+1) - \frac{B_2}{2}\right] \cdot R_1 + A_1 \cdot (R_1+1) - \frac{B_1}{2} = \frac{180}{N} i \qquad (7)$$

$$C_1 \sin A_1 = C_2 \sin A_2 \qquad (8)$$

Initially, the number of half-tooth increments i is unknown. Choose a nominal configuration such as $A_1 = 70^\circ$, solve for A_2 from Equation 8, and then solve for i from Equation 7. After rounding i to the nearest integer, an iterative numerical solution of Equations 7 and 8 gives the proper geometry represented by A_1 and A_2. A spreadsheet with a built in solver is very convenient for manipulating various errors and tolerances that factor into the equations. See the Appendix for the spreadsheet used in this study.

From estimates of manufacturing tolerances, the maximum calculated range of the input gear is 0.012 inches, which corresponds to a change in center distance of 0.004 inches. The center distance should be greater than standard to ensure that each mesh operates with backlash despite variations and tolerances.

Case Study: A Machine Tool C-Axis

A common option for a CNC turning machine (lathe) is the capability to use rotating tools in addition to the normal turning tools. Such a machine may operate either in a turning mode or in a milling mode to suit a particular application. The machine could conceivably switch modes several times during a single piece part. While in the milling mode, the lathe spindle becomes the C-axis, which is equivalent to a rotary table on a milling machine. The C-axis must have very stiff and accurate, angular position control. The lathe spindle requires only velocity control. The maximum speed required for each mode is vastly different, 1/4 rpm verses 4000 rpm.

The C-axis in this study was to be an add-on option to the standard, two-range headstock. As a result, the primary constraint was the envelop. The machine controller constrained the gear ratio to be either 180:1 or 360:1 from the encoder to the spindle. Time and money constraints dictated the use of proven techniques and off-the-shelf equipment.

A few things were evident from the start. The C-axis would have to disengage from the spindle during turning mode. The spindle bull gear, used for low range operation, was the best connection to the spindle. The company had good success with dual-pinion designs for similar applications, such as rotary tables. However, no one in the company had made one that completely disengaged from mesh.

Previous dual-pinion designs used axial motion preload. This was not compatible with axial motion of the dual pinions, which was the best way to disengage from the bull gear. After considerable contemplation, a new preload method emerged. Recognize from Figure 12 the optimal configuration for radial motion preload. The conceptual step from axial motion to radial motion was not smooth or deliberate. Rather it was a sudden inspiration that now seems obvious.

Two particularly interesting features of the design are worth explaining in detail. Referring to Figure 12, the extra length of the input gear provides the added compliance necessary to limit variations in preload. It is a simply supported beam designed to have zero slope at the mesh. Not shown in the figure are frictional interfaces in each cluster gear. With the C-axis engaged by hydraulic pressure, an adjustment to the interface causes the spring to deflect, and this gives a measure of preload.

Now the problem becomes how to engage and disengage a preloaded gear train. Fortunately the standard headstock used helical gears. Relative axial motion between the dual pinions also causes relative rotation due to the helix angle. When sliding into or out of engagement, one pinion leads the other to provide backlash until both pinions are hard against travel stops. The amount of lead is easy to calculate from estimates of backlash and other geometry. The method is very simple and works perfectly.

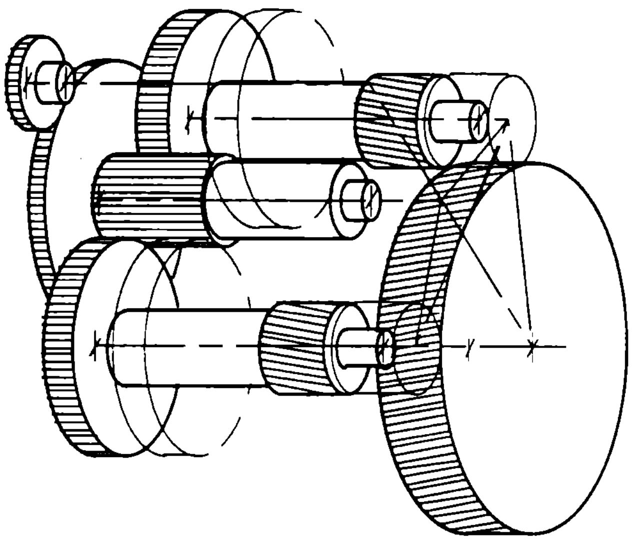


Figure 12: This isometric sketch shows the dual pinions disengaged from the spindle gear. The long pinion serves as a beam spring for radial motion preload.

Summary

The advantages and disadvantages of transmissions in position control systems were the subjects of the Introduction. In particular, the designer should consider the application and the alternatives carefully before committing to a strategy. Assuming that a transmission ratio is necessary, several techniques presented in the Background are available to control backlash. Dual path preloaded gear trains became the focus of the paper as two types emerged from patented designs.

The split-pinion configuration usually has one stiff path and one compliant path to accommodate component errors and tolerance. A dual-pinion configuration having one stiff path and one compliant path is functionally equivalent to the split-pinion and is all too common. A better design incorporates compliance at the input to the dual paths, thereby taking full advantage of the stiffness of each path.

Important to the design of anti-backlash transmissions are models for their stiffness from input to output, referred to as drive stiffness, and for their internal stiffness indicating sensitivity to component errors, referred to as loop stiffness. The modeling section developed simple, accurate relations for both quantities on both types of drives, and hopefully gave the future designer a better understanding. The design section discussed equally important qualitative issues and presented three methods to achieve preload. Two case studies presented particular problems and solutions.

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Appendix

Error Analysis and Center Location Program						
Input numbers in bold.						
Description	**Symbol/Unit**	**Gear 1**	**Gear 2**	**Gear 3**	**Gear 4**	
General Information						
No. of Teeth	N	**16**	**48**	**21**	**112**	
Diametral Pitch	P	**32**	**32**	**32**	**32**	
Pressure Angle	Ø (deg)	**20**	**20**	**20**	**20**	
Pitch Circle Dia.	D (in)	0.5	1.5	0.65625	3.5	
Error Information						
Bearing Eccentr.	(in t.i.r.)	**0.0002**	**0.0002**	**0.0002**	**0.0005**	
Pitch Line Eccen	(in t.i.r.)	**0.0008**	**0.0008**	**0.0016**	**0.0012**	
Tooth To Tooth	(in t.i.r.)	**0.0005**	**0.0005**	**0.0005**	**0.0005**	
Total Error	(in t.i.r.)	0.0015	0.0015	0.0023	0.0022	
Variation in x1 to accommodate errors.						**Total**
Δx1	(in)	0.0015	0.0015	0.0053	0.0039	
% of Sum		12	12	43	32	100
% of RSS		22	22	76	57	177
		Δx1	**ΔC1**			
Sum	(in)	0.0122	0.0042			
RSS	(in)	0.0069	0.0024			
(Sum + RSS)/2	(in)	0.0095	0.0033			
Description	**Symbol/Unit**	**Mesh 1**	**Mesh 2**			
Mesh Information						
Center Distance	Cnom (in)	**1.0040**	**2.0781**			
Backlash	Bmax (in)	**0.0050**	**0.0042**			
	Bmin (in)	**0.0040**	**0.0008**			
Ang. Backlash	Ba.max (rad)	0.0200	0.0128			
	Ba.min (rad)	0.0160	0.0024			
Mesh Ratio	R	3.0000	5.3333			
See Synthesis of Gear Centers Procedure						
Calculate A2 and N for maximum backlash given A1 and center distances.						
A1	(rad)	-1.2217				
A2	(rad)	0.4712				
N		35.1701				
Use Solver to obtain A1 and A2 given N rounded to a smaller integer.						
A1	(rad)	-1.2129	-69.4939			
A2	(rad)	0.4696	26.9051			
error		0.0000				
Center distance components						
x1	(in)	0.3517				
y1	(in)	-0.9404				
x2	(in)	1.8532				
y2	(in)	0.9404				
Allow x1 to change to accommodate minimum backlash.						
A1	(rad)	-1.2085	-69.2426			
C1	(in)	1.0057				
x1	(in)	0.3564				
Δx1	(in)	0.0047				