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Selection Intensity in Genetic Algorithms with Generation Gaps

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Abstract

This paper presents calculations of the selection intensity of common selection and replacement methods used in genetic algorithms (GAs) with generation gaps. The selection intensity measures the increase of the average fitness of the population after selection, and it can be used to predict the average fitness of the population at each iteration as well as the number of steps until the population converges to a unique solution. In addition, the theory explains the fast convergence of some algorithms with small generation gaps. The accuracy of the calculations was verified experimentally with a simple test function. The results of this study facilitate comparisons between different algorithms, and provide a tool to adjust the selection pressure, which is indispensable to obtain robust algorithms.

1 INTRODUCTION

To maintain a constant number of individuals in their populations, genetic algorithms have a mechanism that deletes unwanted individuals to make room for the newly-created ones. Most frequently, the entire population is replaced every generation. In this case, the algorithm is called a "generational GA", and it represents an extreme case of replacement methods. In the other extreme, there are "steady-state" GAs that replace a single individual in every iteration. The fraction of the population that is replaced is controlled by a parameter called generation gap (denoted by $G \in [\frac{1}{n}, 1]$, where n is the size of the population).

Although there are numerous observations that the generation gap affects the selection pressure, this effect has not been quantified accurately. The purpose of this paper is to present calculations of the selection intensity in GAs with arbitrary generation gaps. The selection intensity is the normalized increase of the average fitness of the population after selection. It can be used to predict the average fitness of the population at each iteration as well as the number of steps until the population converges to a unique solution. The calculations consider the selection algorithm used to choose the parents and the mechanism used to replace existing members of the population with the offspring.

The paper is organized as follows. The next section contains a brief review of previous work on analysis of overlapping populations. Section 3 defines the selection intensity and reviews previous work on characterizing it in serial and parallel generational GAs. Section 4 has the calculations for the selection intensity of GAs with generation gaps. Experiments that verify the accuracy of the calculations are presented in section 5. Finally, section 6 presents a summary and the conclusions of this study.

2 GENERATION GAPS

De Jong (1975) was the first to evaluate empirically the performance of GAs with overlapping populations. He introduced the generation gap G as a parameter to the GA, and found that at low values of G the algorithm had a severe loss of alleles, which resulted in poor search performance. In De Jong's algorithm, the newly created individuals replaced random members of the population. He hypothesized that the poor performance was caused by the high variance in the individuals' lifetime and the number of offspring produced. Later, De Jong and Sarma (1993) presented additional empirical evidence, and suggested alternative deletion methods to reduce the variance.

Whitley (1989) introduced GENITOR, a "steady state" GA in which the worst individual was deterministically replaced every iteration. Goldberg and Deb (1991) analyzed GENITOR, and they observed that it has a high selective pressure even when the parents were selected randomly. This suggests that the deletion of worst individuals was the major factor in the selection intensity.

The following deletion methods are common:

- Insert offspring at random (uniformly).
- Replace the worst individuals.
- Choose using any selection algorithm normally used to select the parents (e.g., fitness-proportional, exponential or linear ranking, tournaments, etc.).
- Delete the oldest (FIFO).
- Combinations or elitist variants of the above.

There has been considerable research on the effect of these deletion methods on the convergence of GAs. For example, Syswerda (1991) compared generational and steady state GAs with fitness-proportional selection of parents and several replacement methods. Assuming an infinite population (so that effects due to small populations do not appear) and using random deletion of individuals, Syswerda showed that the generation gap had no effect on the allocation of copies to strings. However, changing the deletion strategy to least-fit, exponential ranking, or fitness-proportionate deletion caused the steady state algorithm to converge much faster than the generational GA. Calculations presented later in this paper will confirm and quantify these observations.

Chakraborty, Deb, and Chakraborty (1996) used Markov chains to obtain the probability that a specific class of individuals takes over the population at each iteration. They only considered random, worst-fit, and exponential ranking deletion, but their framework can be extended to other replacement strategies. Smith and Vavak (1999) did just that, and observed that replacing the oldest member or replacing randomly may result in loss of the optimal value. De Jong and Sarma (1993) observed similar losses of the optimal value even when the initial population had 10% of the optimal individuals. Smith and Vavak noted that the loss can be corrected simply by using an elitist replacement strategy that ensures that the best individual in the current generation survives to the next. The simple correction suggests that variability in the number of offspring or the individuals' lifetime is not the cause of failure.

Interestingly, De Jong and Sarma (1993) end their paper noting that "...the important behavioral changes [between generational and steady state GAs] are due to the changes in the exploration/exploitation balance resulting from the different selection and deletion strategies used. This is where we should continue our analysis efforts." That is precisely the purpose of this paper: to quantify accurately the selection intensity (the exploitation part). De Jong and Sarma also question whether an algorithm that selects a block of the best individuals and replaces a block of the worst would reduce the variance without changing the selection pressure. Section 4 shows that the answer is negative, and that indeed the selection pressure changes significantly as a function of G (the size of the blocks).

3 SELECTION INTENSITY

This section briefly reviews previous work on quantifying the intensity of selection methods. In addition, this section reviews recent work that characterizes the selection intensity caused by migration of individuals between populations in parallel GAs. The next section builds on the models presented here.

3.1 SELECTION METHODS

Some common selection methods are proportionate selection (Holland, 1975), linear ranking (Baker, 1985), tournament selection (Brindle, 1981), $(\mu \dagger \lambda)$ selection (Schwefel, 1981), and truncation selection (Mühlenbein & Schlierkamp-Voosen, 1993). In linear ranking selection, individuals are selected with a probability that is linearly proportional to the rank of the individuals in the population. The desired expected number of copies of the best (n^+) and worst ($n^- = 2 - n^+$) individuals are supplied as parameters to the algorithm. In tournament selection, s individuals are randomly sampled from the population (with or without replacement), and the best individual in the sample is selected. The process is repeated until the mating pool is filled. In $(\mu + \lambda)$ selection, λ offspring are created from μ parents, and the μ best individuals out of the union of parents and offspring are selected. In (μ, λ) selection ($\lambda \geq \mu$) the μ best offspring are selected to survive. Truncation selection selects the top $1/\tau$ of the population and creates τ copies of each individual. It is equivalent to (μ, λ) selection with $\mu = \lambda/\tau$.

Mühlenbein and Schlierkamp-Voosen (1993) introduced the use of the selection intensity to study the convergence of selection schemes. The selection intensity is defined as

Selection	Parameters	I
Tournament	s	$\mu_{s:s}$
(μ, λ)	μ, λ	$\frac{1}{\mu}\sum_{i=\lambda-\mu+1}^{\lambda}\mu_{i:\lambda}$
Linear Ranking	n^+	$(n^+-1)\frac{1}{\sqrt{\pi}}$
Proportional	σ^t, μ_t	σ^t/μ_t

Table 1: Selection intensity for common selection schemes.

$$I = \frac{\bar{f}_s^t - \bar{f}^t}{\sigma^t}, \tag{1}$$

where $\bar{f}_s^t$ is the mean fitness of the selected individuals, and $\bar{f}^t = \frac{1}{n}\sum_{i=1}^{n} F_i^t$ is the mean fitness of the population, σ^t is the standard deviation of the population, and the superscript t denotes the generation number. The numerator is called the selection differential, and is usually denoted as s^t.

The challenge to calculate the intensity of a selection method is to compute the mean fitness of the selected individuals, $\bar{f}_s^t$. This has been accomplished analytically for some common selection schemes of generational GAs. In particular, Bäck (1995) and Miller and Goldberg (1995) independently derived the selection intensity for tournament selection, and Bäck (1995) also derived I for (μ, λ) selection. Blickle and Thiele (1996) calculated the intensity of linear ranking, and Mühlenbein and Schlierkamp-Voosen (1993) calculated I for proportional selection. Table 1 contains the known selection intensities (adapted from (Miller & Goldberg, 1996)). Note that I is independent of the distribution of the current population, except for proportional selection.

3.2 MULTI-POPULATION GAs

Regardless of their implementation on uni- or multiprocessor computers, GAs with multiple populations exhibit a different behavior than GAs with a single population. Much has been written about this, but one of the main causes of the disparity seems to be the additional selection intensity caused by choosing migrants and replacements according to their fitness (Cantú-Paz, ress).

The selection intensity caused by migration is

$$I_{mig} = I_e + I_r, \tag{2}$$

where I_e is the selection intensity caused by selecting the emigrants, and I_r is the intensity caused by selecting replacements in the receiving deme. Using δ to denote the number of neighbors of a deme (the degree of the connectivity graph) and ρ to denote the migration rate (i.e., the fraction of the population that migrates every generation), $I_e \approx \delta\phi(\Phi^{-1}(1-\rho))$ if the best individuals are selected to migrate, and $I_e = 0$ if the migrants are chosen randomly. $\phi(z) = \exp(-z^2/2)/\sqrt{2\pi}$ and $\Phi(z) = \int_{-\infty}^{z}\phi(x)dx$ are the PDF and CDF respectively of a standard Gaussian distribution with mean 0 and standard deviation of 1.

Similarly, $I_r \approx \phi(\Phi^{-1}(1-\delta\rho))$ if the worst individuals in the receiving deme are replaced by the migrants, and $I_r = 0$ if replacements are chosen randomly. The total selection intensity is the sum of the intensity of the method used to select parents (see table 1) and I_{mig}. We shall see in the next section that the equations for the selection intensity in GAs with overlapping populations are very similar to those above.

4 GENERATION GAPS AND SELECTION INTENSITY

To calculate the average fitness of the population in the next iteration, we take the weighted average of the individuals selected to reproduce and the individuals that were not replaced (which we call survivors):

$$\bar{f}^{t+1} = G\bar{f}_s^t + (1-G)\bar{f}_{surv}^t, \tag{3}$$

where $\bar{f}_s^t$ is the expected fitness of the selected individuals, $\bar{f}_{surv}^t$ is the expected fitness of the survivors, and t is the iteration number. To simplify things, we may also write the average fitness of the population as a weighted sum:

$$\bar{f}^t = G\bar{f}^t + (1-G)\bar{f}^t. \tag{4}$$

Collecting similar terms, we can write the selection differential as:

$$\begin{aligned} s^t &= \bar{f}^{t+1} - \bar{f}^t \\ &= G(\bar{f}_s^t - \bar{f}^t) + (1-G)(\bar{f}_{surv}^t - \bar{f}^t) \\ &= s_s^t + s_{surv}^t. \end{aligned} \tag{5}$$

This equation clearly shows that the selection pressure has two independent causes, namely the selection of the parents and the selection of replacements (or survivors). First, we consider the selection of parents because it is usually done with the same methods of generational GAs (i.e., proportional selection, linear ranking, tournaments, etc.). Thus, we can calculate $\bar{f}_s^t$ as $\sigma^t I_g + \bar{f}^t$, where I_g is the intensity of the selection method (see table 1). Now we can rewrite the first term of the equation above as

$$G(\bar{f}_s^t - \bar{f}^t) = G(\sigma^t I_g), \tag{6}$$

and using the definition of selection intensity $I_s = \frac{s_s^t}{\sigma^t}$ we obtain:

$$I_s = G I_g. \tag{7}$$

Essentially, this means that selection algorithms retain their pressure, but the overall intensity is lower because fewer individuals are generated in each iteration.

The remainder of this section examines the intensity caused by the selection of replacements (or the survivors). First, note that when the survivors are chosen randomly, their expected fitness is equal to the average of the population before selection, and therefore $s_{surv}^t = 0$, and there is no selection intensity.

The interesting case is when the replacements are selected according to their fitness. We examine in detail the bounding case where the worst individuals in the population are replaced. Other replacement methods induce a lower pressure than deleting the least fit. Actually, if the selection intensity of the replacement method is known, it can be shown that the intensity is GI_g.

The major assumption that we make is that the fitness values $f_i^t, i \in [1, n]$ can be interpreted as samples of random variables F_i^t with a common distribution $N(\bar{f}^t, \sigma^t)$. We may arrange the variables in increasing order as

$$F_{1:n}^t \leq F_{2:n}^t \leq \dots \leq F_{n:n}^t.$$

These are the order statistics of the F_i^t variables, and we can use them to calculate the average fitness of the survivors. Without loss of generality, we assume a maximization problem. The mean fitness of the $surv = (1 - G)n$ best individuals that survive (i.e., are not replaced) is

$$\bar{f}_{surv}^t = G \cdot \sum_{i=n-surv+1}^{n} E(F_{i:n}^t). \tag{8}$$

The random variables can be normalized as

$$Z_{i:n} = \frac{F_{i:n}^t - \bar{f}^t}{\sigma^t} \sim N(0,1),$$

and the average fitness of the survivors may be rewritten in terms of the normalized variables

$$\begin{aligned} \bar{f}_{surv}^t &= \frac{1}{surv} \sum_{i=n-surv+1}^{n} (E(Z_{i:n})\sigma^t + \bar{f}^t) \\ &= \sigma^t \cdot \frac{1}{surv} \sum_{i=n-surv+1}^{n} E(Z_{i:n}) + \bar{f}^t. \end{aligned} \tag{9}$$

Now, we can calculate the selection differential caused by deleting the worst individuals as

$$\begin{aligned} s_{surv}^t &= (1 - G)(\bar{f}_{surv}^t - \bar{f}^t) \\ &= \frac{1}{n} \cdot \sigma^t \cdot \sum_{i=n-surv+1}^{n} E(Z_{i:n}). \end{aligned} \tag{10}$$

Since the selection differential is also $s^t = I \cdot \sigma^t$, the selection intensity in this case is

$$I_{surv} = \frac{1}{n} \cdot \sum_{i=n-surv+1}^{n} E(Z_{i:n}). \tag{11}$$

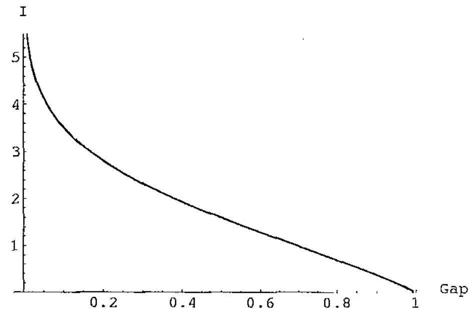
The expected value of the i-th order statistic of a sample of size n is defined as

$$\begin{aligned} \mu_{i:n} &= E(Z_{i:n}) \\ &= n\binom{n-1}{i-1} \int_{-\infty}^{\infty} z\phi(z)\Phi^{i-1}(z)[1 - \Phi(z)]^{n-i} dz, \end{aligned} \tag{12}$$

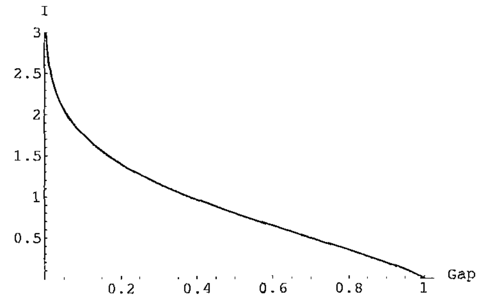
where $\phi(z)$ and $\Phi(z)$ are the PDF and CDF respectively of the fitness distribution (in our case a standard Gaussian distribution with mean 0 and standard deviation of 1). The values of $\mu_{i:n}$ are computationally expensive to calculate, but for a Gaussian distribution they are tabulated for $n \leq 400$ (Harter, 1970). Nevertheless, computing the sum in equation 11 can be cumbersome, but the following approximation exists[1] (Burrows, 1972; Bäck, 1995):

$$\sum_{i=n-surv+1}^{n} \mu_{i:n} \approx n\phi(\Phi^{-1}(1 - G)), \tag{13}$$

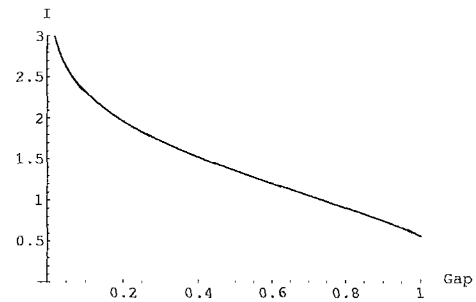
[1] Bäck shows that for $n > 50$ the approximation is indistinguishable from the real values.



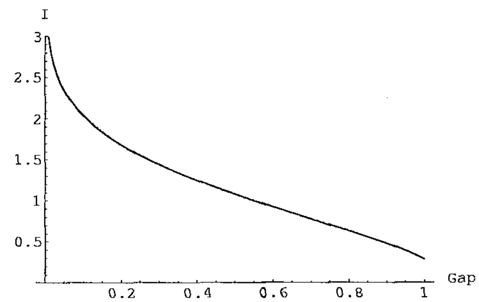
(a) Select best, delete worst



(b) Select best, delete randomly (or select randomly and delete worst, or (μ, λ))



(c) Tournaments ($s = 2$), delete worst



(d) Ranking ($n^+ = 1.5$), delete worst

Figure 1: Selection intensity of different selection and replacement strategies varying the generation gap.

and therefore equation 11 can be approximated as

$$I_{surv} \approx \phi(\Phi^{-1}(G)), \tag{14}$$

and the total intensity is simply

$$I = I_s + I_{surv}. \tag{15}$$

It is important to realize that the selection intensity is an adimensional quantity that does not depend on the fitness function or on the generation t. The only assumption made to calculate the intensity is that the fitness values have a normal distribution, but any other distribution may be used as long as $E(F_{i:n})$ may be computed (by substituting the appropriate PDF and CDF in equation 12).

The maximum of equation 14 occurs at $G = 0.5$ and is $\phi(0) = 1/\sqrt{2\pi} = 0.3989$. However, if we convert I_r to its equivalent "generational" intensity[2] $I_g = I_{surv}/G$, the highest value of I_g is at $G = 1/n$, and it can be of considerable magnitude. For example, for $n = 256$, $I_g = 2.96$, and for $n = 1000$, $I_g = 3.36$. So, even if the parents are selected randomly, replacing the worst individuals may cause a considerable selection pressure. This is consistent (at least qualitatively) with Goldberg and Deb's (1991) observations of GENITOR.

[2]We may think of this as the normalized increase of the mean fitness of the population after n individuals have been selected.

Figure 1 has plots of the (generational) selection intensity of algorithms with different methods to select the parents and the replacements. To make the graphs, $\Phi^{-1}(x)$ was calculated numerically using Mathematica 3.0 as $\sqrt{2}$ `InverseErf[0,2x-1]`.

The plots show that the combination of selecting the best individuals as parents and deleting the worst individuals has the higher selection intensity. Note that when the best individuals are selected, at $G = 1$ there is no selection pressure, because the algorithm simply copies the entire population. In addition, the selection intensity when the best are selected and replacement is random is identical to (μ, λ) selection with $G = \mu/\lambda$. In the case of tournaments and linear ranking with random deletion, the graphs would be horizontal lines at 0.5642 and 0.2820, respectively. This is consistent with Syswerda's (1991) observations on random deletion, although he was considering proportional selection.

We must be cautious when comparing algorithms with the same selection intensity, because they are not equivalent algorithms. The selection intensity only considers the change of the population's mean fitness over time, and ignores the higher moments of the distribution. Blickle and Thiele (1996) made an analysis of the variance of several (generational) selection methods, and Rogers and Prügel-Bennett (1999) have a detailed analysis of the first four moments of a roulette-wheel algorithm that uses Boltzmann weights. Different selection algorithms impact the higher moments in different ways and may affect the quality of the solutions found. A reasonable heuristic is that given a choice between algorithms with the same selection intensity, we should prefer the one that produces the highest variance of fitness (Bäck, 1995).

Another aspect that we must take into consideration when designing GAs is that the convergence time of a GA is inversely proportional to the selection intensity. Rogers and Prügel-Bennett (1999) observed that they could replicate the dynamics of a generational GA with a steady state GA using half the function evaluations. However, they were using selection to choose both the parents and the replacements, effectively doubling the selection intensity. Although it may be tempting to use higher selection intensities, this may cause the algorithm to converge too fast (this is sometimes called premature convergence). Perhaps this contributed to the poor performance of the GAs in De Jong's empirical studies with small generation gaps.

5 EXPERIMENTS

This section presents experimental evidence that verifies the accuracy of the calculations of the previous section. The experiments use a $l = 500$ bit One-Max function, $F = \sum_{i=1}^{l} x_i$, where $x_i \in \{0, 1\}$ are the individual bits in the chromosome. Mühlenbein and Schlierkamp-Voosen (1993) showed that with an initial random population, the number of generations until convergence is given by $G = \frac{\pi}{2}\frac{\sqrt{l}}{I}$. We use this result to test the accuracy of equation 15. Again, we consider that a generation is when n individuals have been processed, so we convert the number of iterations until convergence to generations by multiplying by G.

The population size is $n = 500$ individuals, which is sufficient to ensure convergence to the optimum in all cases. The GA uses uniform crossover with probability 1.0, and no mutation. The results shown are the average of 20 independent runs for each parameter setting.

Figure 2 compares the theoretical predictions with experimental results. The graphs show the number of generations until convergence using best-fit selection, pairwise tournaments, and linear ranking with $n^+ = 1.5$. Both random and worst-fit deletion were used. Additional experiments with $n^+ = 2$ yielded the same results as pairwise tournaments, as was expected because the two algorithms have the same selection intensity.

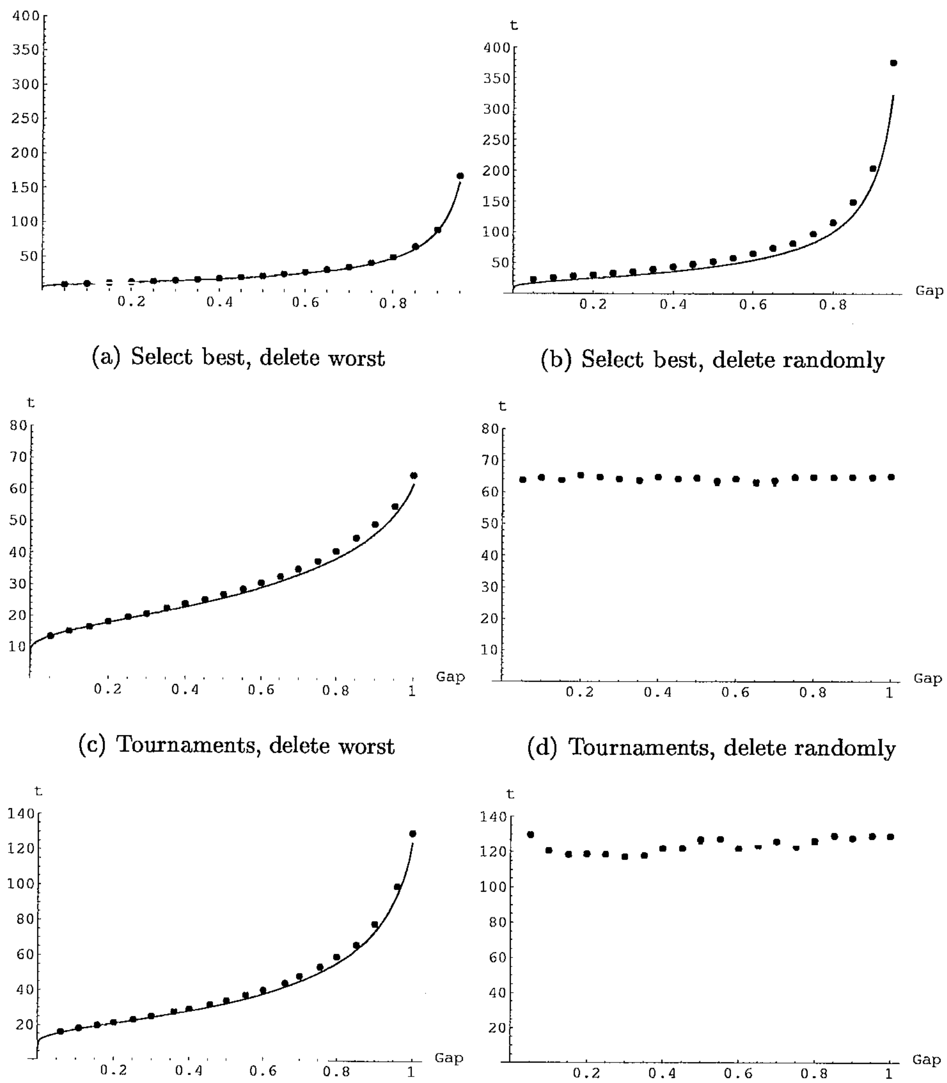
6 CONCLUSIONS

This paper presented calculations of the selection intensity of genetic algorithms with arbitrary generation gaps. The accuracy of the theory was verified experimentally, and it was used as a possible explanation for previous observations reported by others. The resulting equations are similar to those that model the selection intensity of migration in multi-population GAs. This suggests the possibility of exchanging ideas and analysis techniques to further advance our understanding of the two types of algorithms.

Future work should consider the effect of selection on the higher moments of the distribution of fitness. This is important because algorithms with the same selection intensity may reduce the variance (diversity) of the population in different ways and may also change the shape of the distribution. Studying these effects may help to design recombination or mutation operators that balance the effects of selection.

It is well known that algorithms with higher selection pressure need larger populations to succeed (Mühlenbein & Schlierkamp-Voosen, 1993; Harik, Cantú-Paz, Goldberg, & Miller, 1997). This introduces a tradeoff because higher selection pressures result in faster convergence, but larger populations require more computations. The tradeoff suggests that there is an optimal population size and selection pressure that minimize the total computational work. Future work along these lines may produce a framework that relates selection intensity, population size, and solution quality.

Such a framework would be very useful in the design of faster and more reliable evolutionary algorithms. Besides facilitating comparisons between different algorithms, and providing a convenient tool to adjust the selection pressure, the results of this paper would be a critical component of the framework.



(a) Select best, delete worst

(b) Select best, delete randomly

(c) Tournaments, delete worst

(d) Tournaments, delete randomly

(e) Ranking, delete worst

(f) Ranking, delete randomly

Figure 2: Generations until convergence using different selection and replacement strategies and varying the generation gap. The lines are the theoretical predictions using equation 15, and the dots are experimental results.

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