Preprint
UCRL-JC-141551

Use of a Transition Probability/Markov Approach to Improve Geostatistical Simulation of Facies Architecture

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This article was submitted to
American Association of Petroleum Geologists (AAPG)
Hedberg Symposium: Applied Reservoir Characterization Using Geostatistics
The Woodlands, TX
December 3-6, 2000

U.S. Department of Energy

Lawrence
Livermore
National
Laboratory

November 1, 2000

AAPG HEDBERG SYMPOSIUM
"Applied Reservoir Characterization Using Geostatistics"
DECEMBER 3-6, 2000, THE WOODLANDS, TEXAS

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Introduction

Facies may account for the largest permeability contrasts within the reservoir model at the scale relevant to production. Conditional simulation of the spatial distribution of facies is one of the most important components of building a reservoir model. Geostatistical techniques are widely used to produce realistic and geologically plausible realizations of *facies architecture*. However, there are two stumbling blocks to the traditional indicator variogram-based approaches: (1) intensive data sets are needed to develop models of spatial variability by empirical curve-fitting to sample indicator (cross-) variograms and to implement "post-processing" simulation algorithms; and (2) the prevalent "sequential indicator simulation" (SIS) methods do not accurately produce patterns of spatial variability for systems with three or more facies (Seifert and Jensen, 1999). This paper demonstrates an alternative transition probability/Markov approach that emphasizes:

- Conceptual understanding of the parameters of the spatial variability model, so that geologic insight can support and enhance model development when data are sparse.
- Mathematical rigor, so that the "coregionalization" model (including the spatial cross-correlations) obeys probability law.
- Consideration of spatial cross-correlation, so that *juxtapositional tendencies* – how frequently one facies tends to occur adjacent to another facies – are honored.

Transition Probability Approach

Let the indicator variable, $I_j(\mathbf{x})$, for facies j be defined as $I_j(\mathbf{x}) = \{1, \text{if } j \text{ occurs at } \mathbf{x}; 0, \text{otherwise}\}$, where $\mathbf{x}$ is a location. In terms of indicator variables, let the marginal probability, p_j, be defined as $p_j=E\{I_j(\mathbf{x})\}$, and the joint probability, $p_{jk}(\mathbf{h})$, be defined as $p_{jk}(\mathbf{h})=E\{I_j(\mathbf{x})I_k(\mathbf{x}+\mathbf{h})\}$, where $\mathbf{h}$ is a lag vector. Different statistics can be used to measure spatial variability of indicator variables, for example, the indicator (cross-) variogram, indicator (cross-) covariance, or transition probability. Each of these statistics is a function of joint probability and marginal probability statistics. Fundamentally, the joint probability is the purest bivariate measure of spatial variability. However, the transition probabilty, $t_{jk}(\mathbf{h})$, defined here with respect to indicator variables as $t_{jk}(\mathbf{h})=E\{I_j(\mathbf{x})I_k(\mathbf{x}+\mathbf{h})\}/E\{I_j(\mathbf{x})\}$, is the most interpretable. It can be defined in terms of a conditional probability as $t_{jk}(\mathbf{h})=\Pr(k \text{ at } \mathbf{x}+\mathbf{h} \mid j \text{ at } \mathbf{x}\}$. Probability law requires that the row sums of the transition probability matrix, $\mathbf{T}(\mathbf{h})$, sum to one and that the column sums obey $\Sigma_j\, p_j t_{jk}(\mathbf{h}) = p_k$.

Importantly, the transition probability can be defined in an interpretable context:

Given that facies j occurs here, what is the probability that facies k occurs there?

In fact, geologists have commonly used the transition probability to quantify spatial variability of facies since Vistelius (1949), long before indicator geostatistical methods were developed.

Moreover, indicator geostatistical methods can be formulated with respect to the transition probability (Carle and Fogg, 1996).

Markov Chain Model

The most fundamental 1-D stochastic model for categorical variables is the Markov chain. In the familiar discrete-lag formulation, a 1-D Markov chain assumes that $\mathbf{T}(h+\Delta h)=\mathbf{T}(h)\mathbf{T}(\Delta h)$ for lag h. In the continuous-lag formulation, the Markov chain is defined by $\mathbf{T}(h)=\exp(\mathbf{R}h)$, where $\mathbf{R}$ is the transition rate matrix. The matrix exponential function, $\exp(\mathbf{R}h)$, is computed by an eigenvalue or spectral decomposition (Agterberg, 1974). Transition rates (entries in $\mathbf{R}$) can be interpreted in a geologic or geometric context. The diagonal entries in $\mathbf{R}$ are inversely related to the negative of the *mean length* (e.g., mean thickness in the vertical direction), and the off-diagonal entries are proportional to the juxtapositional tendencies. Therefore, if one has an idea of what would be plausible facies proportions, mean lengths, and juxtapositional tendencies, one can easily formulate a Markov chain model of spatial variability. Markov chains have proven to be excellent models for vertical transition probability measurements of clastic sedimentary facies (Carle et al., 1998; Weissmann et al., 1999). A 3-D model of spatial variability can be formulated by interpolating 1-D Markov chain models for the principal directions (Carle and Fogg, 1997).

Conditional Simulation

The conditional simulation algorithm consists of two steps: (1) cokriging-based SIS; and (2) simulated quenching. Both steps use the 3-D Markov chain as the spatial variability model. In the SIS step, a transition probability-based formulation of indicator cokriging is used to estimate the local facies probabilities conditional to nearby data and already simulated cells. Because of the inherent singularities caused by row and column summing constraints required by probability law, the indicator cokriging system of equations is solved with singular value decomposition instead of a standard linear equation solver (Carle and Fogg, 1996). Although the indicator cokriging-based SIS vastly improves the conditional simulation of multi-category systems relative to the traditional indicator kriging approach, the SIS algorithm still does not reproduce the spatial variability prescribed by the model. The simulated quenching step improves the match between modeled and simulated spatial variability. Simulated quenching is accomplished by cycling through every cell in the realization and inquiring whether change in facies will improve the match between modeled and simulated spatial variability; if so, the change is accepted.

Application

The transition probability/Markov approach is applied to the "**true.dat**" data set supplied by Deutsch and Journel (1998) as categorized by Goovaerts (1997). The four "facies" from **true.dat** are mapped as "reality" shown in Figure 1. This data set provides an interesting challenge to a facies-based geostatistical approach because there are strong juxtapositional tendencies of 2-1-3-4 and the reverse; the facies are not randomly distributed in space.

A sixty-point subset of **true.dat** conditions the realizations and is mapped as "data" in Figure 1. The four realizations on the right of Figure 1 were generated using the transition probability/Markov geostatistical approach. Each realization displays a pattern of spatial variability that is practically identical to "reality."

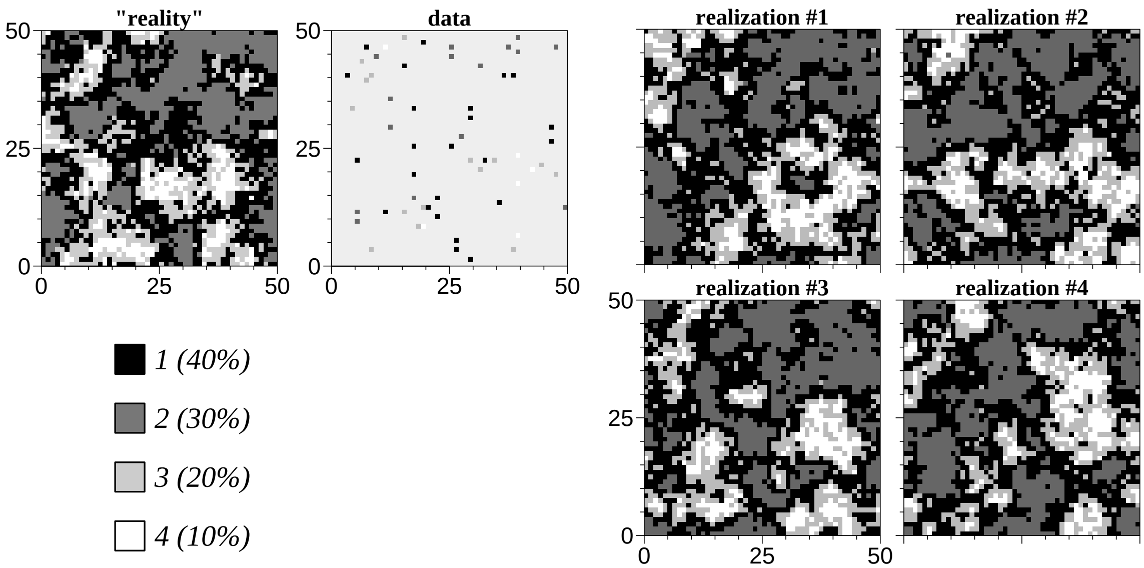

Figure 1. Comparison of "reality," the exhaustive data set, with four realizations generated by the transition probability/Markov geostatistical approach.

In practice, geostatistical simulation techniques must work effectively in 3-D. The 3-D realization shown in Figure 2 shows that the transition probability/Markov approach effectively reproduces the 3-D pattern of spatial variability.

Granted, an exhaustive data set was used to obtain measured transition probabilities for development of a Markov chain model. Alternatively, a similar Markov chain model could be developed with insights on proportions, mean length, and the juxtapositional tendencies, i.e. the strong 2-1-3-4 spatial ordering of the facies. Transition rates would be set to zero or very low for facies pairs that tend not to occur near each other, and relatively high for facies pairs that tend to occur adjacent to each other.

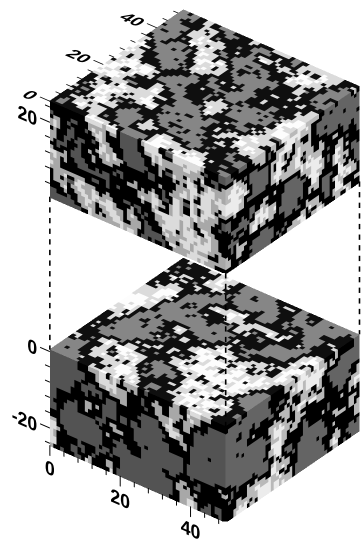

Figure 2. 3-D realization generated by transition probability/Markov approach with pattern of spatial variability similar to "reality" in Figure 1.

In this application, isotropy was assumed. Anisotropy and, importantly, asymmetry can easily be built into the Markov chain model. Asymmetry means that the juxtapositional tendencies in one direction are not necessarily the same in the opposite direction, for example, fining-upward cyclothems of fluvial depositional facies (Allen, 1970). The transition probability allows for asymmetry, whereas the indicator cross-variogram intrinsically assumes symmetry. Therefore, the traditional indicator cross-variogram approach may not be ideal for characterizing spatial variability in geologic systems that exhibit asymmetric juxtapositional tendencies, such as fluvial systems.

Comparison with the Traditional Indicator Kriging Method

The traditional SIS indicator simulation algorithm uses indicator kriging to estimate local conditional facies probabilities. The indicator kriging approach does not make full use of the cross-correlation information contained in the data (Deutsch and Journel, 1998). As a result, the indicator kriging-based simulation method can not necessarily reproduce nonrandom juxtapositional tendencies. The three realizations shown in Figure 3 were produced by the SISIM program (Deutsch and Journel, 1998) using variogram models similar to those modeled by Goovaerts (1996). These realizations show patterns of heterogeneity similar to the "SIS realization #1" of Goovaerts (1996); these realizations clearly do not reproduce spatial variability evident in "reality" shown in Figure 1.

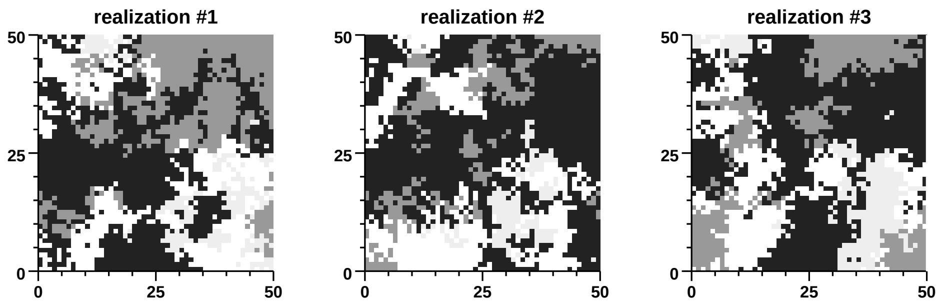

Figure 3. Three realizations produced by the indicator kriging-based SIS algorithm.

Conclusions

Indicator geostatistical approaches offer practical and effective means for generating simulations of facies architecture. The transition probability/Markov approach provides a modeling framework that encourages the integration of geologic insight and makes practical the development of coregionalization models. Realizations obtained from the transition probability/Markov approach exhibit more consistency with the desired patterns of spatial variability as compared to realizations obtained from the traditional indicator variogram-based approach.

Acknowledgment

This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract N. W-7405-Eng-48.

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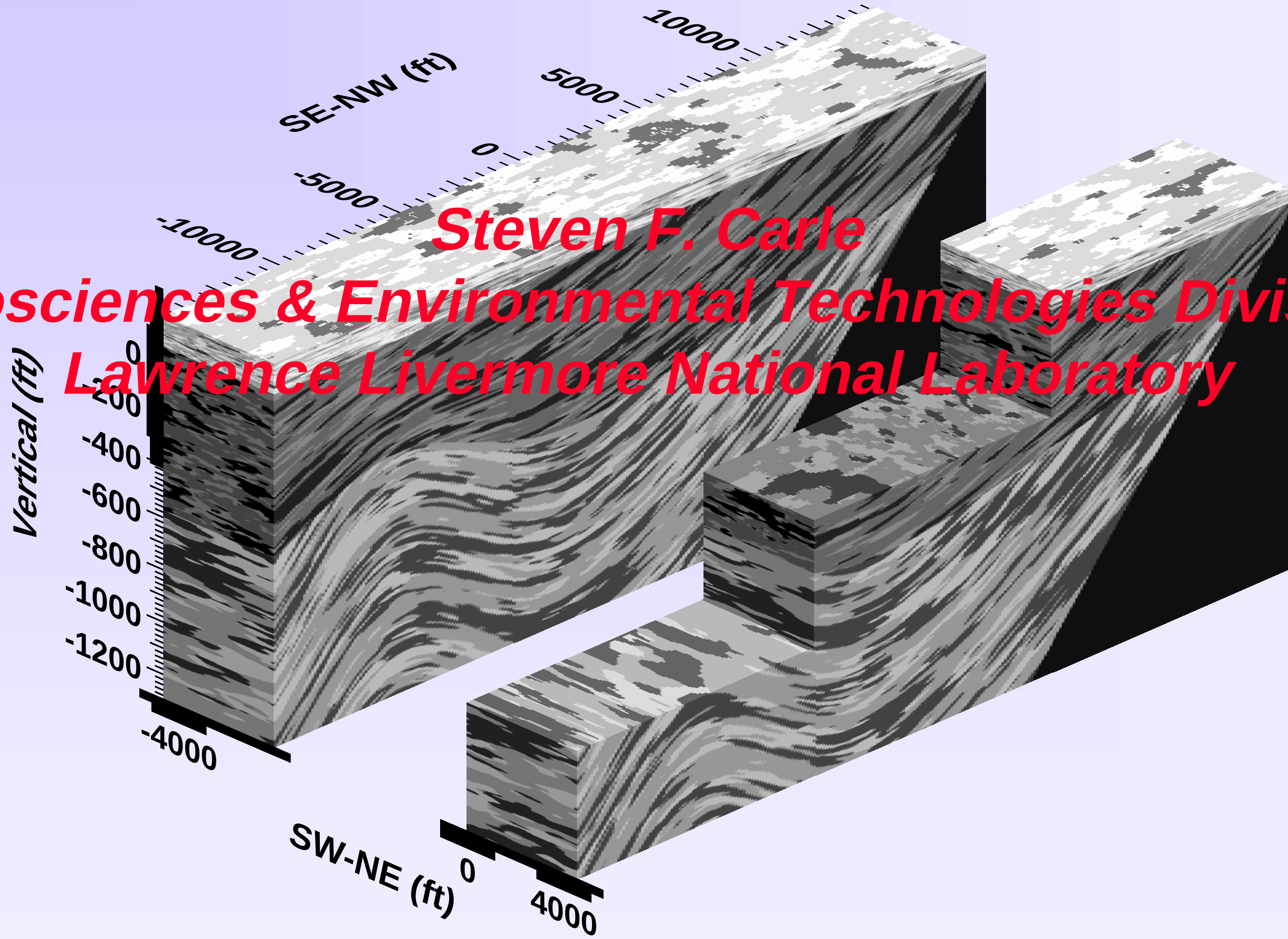

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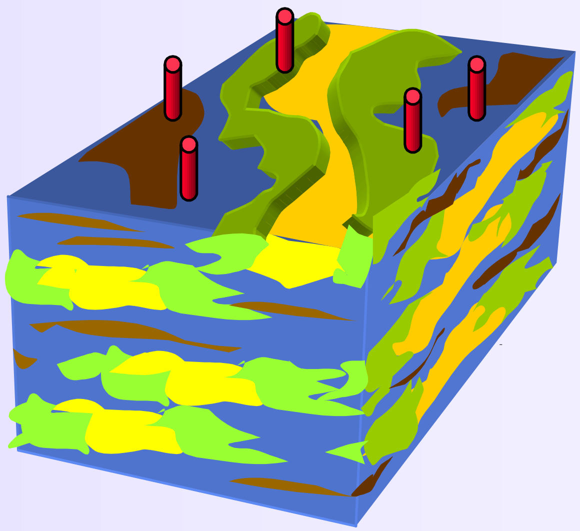
Why use a Transition Probability/Markov Approach to Improve Geostatistical Simulation of Facies Architecture?

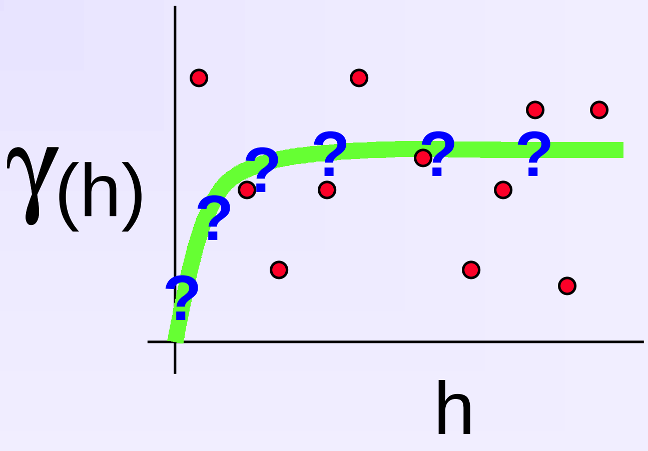


Facts:

- Heterogeneity is related to facies architecture.
- Geostatistics can be used to simulate heterogeneity at relevant scales.

Issues:

- We rarely have enough data to apply the traditional empirical geostatistical approach in 3-D
- Geology should be integrated into geostatistics.

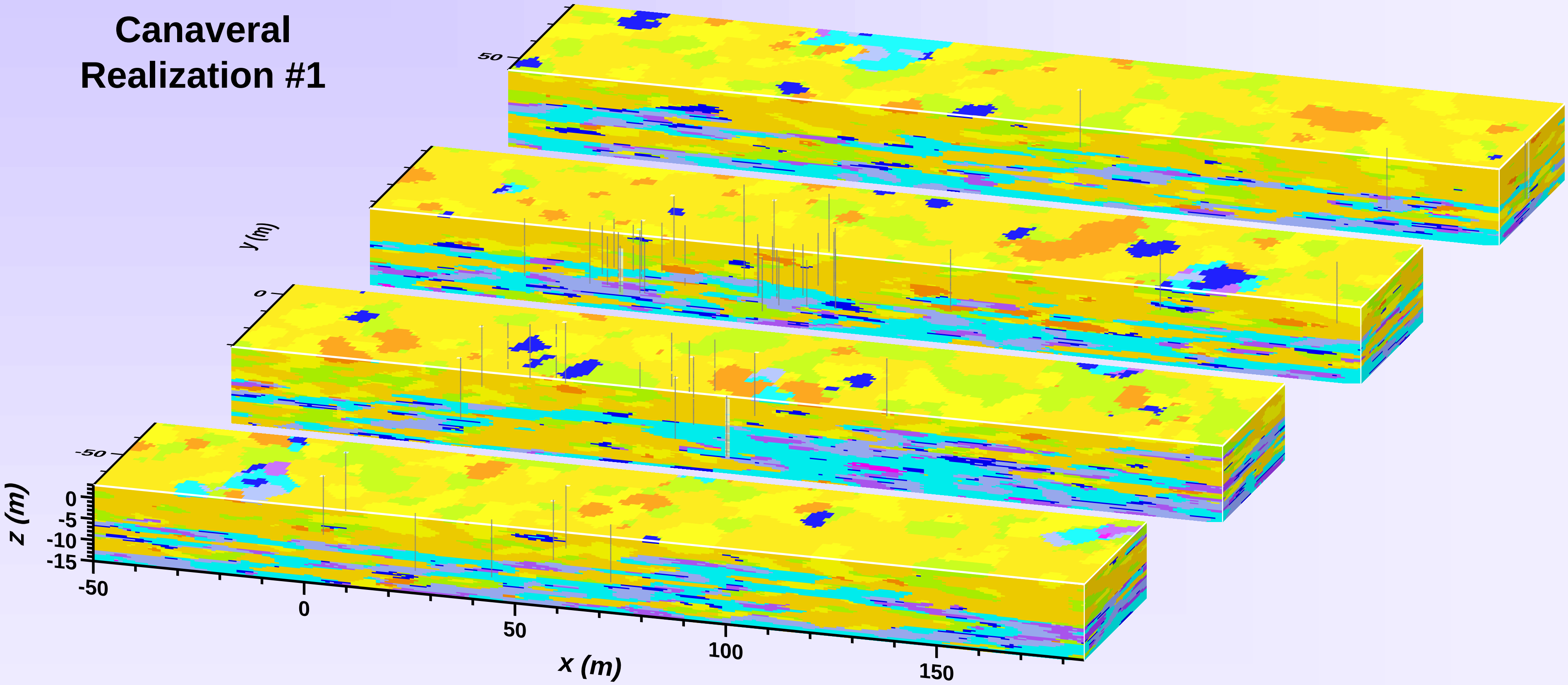


Approach:

- Transition Probability/Markov

Geostatistical Simulation of Shallow Marine Sand Facies at Cape Canaveral, Florida

Canaveral Realization #1



m-c sand	f-m sand	f-m sand <0.5sh	f-m sand >0.5sh
silty m-c sand	silty f-m sand	silty f-m sand <0.5sh	silty f-m sand >0.5sh
silt/clay			

Conclusions:
The Transition Probability/Markov Approach

- Categorical (indicator) geostatistics can be implemented with the transition probability.
  - More interpretable than the indicator variogram.
  - Fully considers cross-correlations, including asymmetry.
  - Amenable to mathematically simple yet theoretically powerful Markov chain model.
  - Formulates cokriging and simulated annealing (quenching).
  - Accurately reproduces spatial auto and cross-correlations in conditional simulations.

- The transition probability/Markov approach has found many successful applications.
  - Data can be abundant or sparse.
  - Markov chain parameters relate to fundamental observable attributes.