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Abstract. The rapid, Alfvénic, time scale of erupting solar-prominences has been an enigma ever since they where first identified. Investigators have proposed a variety of different mechanisms in an effort to account for the abrupt reconfiguration observed. No one mechanism clearly stands out as the single cause of these explosive events. Recent analysis has demonstrated that field lines in the solar atmosphere are metastable to ballooning type instabilities. It has been found previously that in ideal MHD plasmas marginally unstable ballooning modes inevitably become "explosive" evolving towards a finite time singularity via a nonlinear 3D instability called "Nonlinear Magnetohydrodynamic Detonation." Thus, this mechanism is a good candidate to explain explosive events observed in the solar atmosphere of our star or in others.

1. Introduction

Coronal mass ejections, which typically release 10^{33} ergs of energy, and energetic solar flares are both closely correlated with solar prominence eruptions. The prominences (also known as filaments) associated with these events appear stable and quiescent for weeks generally lying directly above a magnetic neutral line, where the line-of-sight magnetic field changes direction. Eventually, the prominence violently erupts releasing energy (in the form of plasma heating, particle acceleration, and increased radiation emission) and destroying the structure (i.e. rapid bulk plasma motions) in period of a few hours (Alfvénic time-scales). During the eruption period, a coronal helmet streamer rises above the prominence and a two-ribbon chromospheric flare appears below.

1.1. Do Linear Instabilities Make Sense?

All to often in solar physics rapid time-scale events are interpreted as being the result of reconnection, which converts magnetic energy into thermal (and

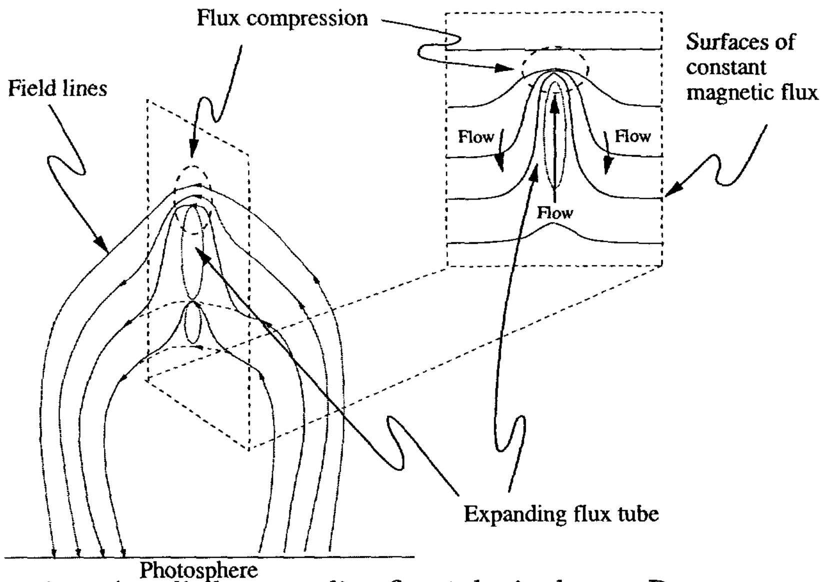

Figure 1. A radially expanding flux tube is shown. Due to conservation of magnetic flux, the Alfvén speed inside the tube shrinks as the expansion proceeds. This effect reduces the stabilizing influence of field line bending making it easier for the tube to expand further. As the tube expands magnetic flux is compressed on the leading edge while flux tubes on the sides of the expanding tube slide around the disturbance.

nonthermal) energy. However, little direct evidence for fast reconnection exists. Due to the very large conductivity of the solar atmosphere (with Lundquist number $S \sim 10^{12-14}$), magnetic energy cannot easily dissipate even with the action of anomalous resistivity (Kulsrud 1998; Shibasaki 2001).

A variety of linear instabilities have also been proposed as explosive eruption mechanisms (e.g. Raadu 1972; Hood & Priest 1979; Hood 1986; Strauss & Longcope 1994). If one could somehow "turn off" the flow of time, place the solar prominence is a highly stressed state, and then allow the flow of time to resume again, linear instabilities could then yield growth rates that are fast enough to explain prominence eruptions. However, solar plasmas (and most natural systems) don't behave this way since they all start in a stable state and *slowly* evolve through marginal stability–slow growth by definition. Therefore, linear instabilities are incapable of producing explosive growth rates in real systems unless they are externally forced at fast (Alfvénic) rates (Cowley & Artun, 1997).

2. A Nonlinear Mechanism?

A natural nonlinear mechanism that exhibits explosive behavior is called "Detonation" (Hurricane, Fong, & Cowley 1997 hereinafter *HFC*). This explosive scenario can occur in an arbitrary equilibria which is locally near the marginal ballooning-Rayleigh-Taylor-Parker mode stability boundary. This novel mechanism is referred to as "detonation," since it is a magnetic analog to the metastable aspect of a chemical explosive.

Instability is driven by density gradients opposing gravity while field-line bending provides some stabilization–diagrammatically this imbalance can be expressed as a simple dispersion relation

$$\Gamma^2 = k_{\parallel}^2 v_A^2 - \frac{g}{L_\rho} - \frac{g}{H} - \frac{c_0^2}{RL_p} \tag{1}$$

where Γ is the growth rate, $k_{\parallel}$ is the wavenumber along the field, v_A is the Alfvén speed, γ is the polytropic index, c_0 is the ion sound speed, R is the local radius of curvature, L_p is the pressure gradient scale length, g is the acceleration due to gravity, $H = \gamma p/\rho g$ the atmospheric scale height, and L_ρ is the density gradient scale length. From left to right, the physical meaning of the terms in Eq. (1) are field line bending, the Rayleigh-Taylor drive, Parker drive (Parker 1967), and the ballooning (pressure) drive. All of these terms nearly balance around marginal stability. Expansion increases the cross-sectional area of the rising flux tube (see Figure 1) since it is moving from a region of high pressure to low pressure. From conservation of magnetic flux, the magnetic field strength in the tube is reduced ($\phi = BA = const.$). This nonlinear effect results in the weakening of the stabilizing field line bending term (v_A, is reduced). Thus, even if the system was marginally linearly stable it is nonlinearly metastable. Ultimately, the stabilizing contribution of magnetic field-line bending is rapidly reduced allowing the unstable drive terms to take over.

3. Nonlinear Magnetohydrodynamic Detonation

3.1. The Essential Physical Picture

The assumptions of the model are that the equilibrium is locally near the marginal stability boundary and that the expected mode of instability has a fast variation in one direction (the "y" direction) across the magnetic field, **B**. It is shown that the plasma displacement, ξ, is largely radial across gradients in pressure and/or density (along x, being the magnetic flux function).

A non-linear ballooning mode envelope equation, that describes in detail the behavior diagrammed in the previous section, can be obtained from the usual MHD equations via a lengthy multiple-scale ordering procedure *assuming the system is close to marginal stability* (see *HFC*)

$$\frac{\partial^2 \xi}{\partial t^2} = \left(1 - \frac{x^2}{\Delta^2}\right)\xi + \frac{\partial^2 \int u dy}{\partial x^2} + \xi \frac{\partial^2 \overline{\xi^2}}{\partial x^2} + \left(\xi^2 - \overline{\xi^2}\right) + \nu \frac{\partial^2}{\partial y^2}\frac{\partial \xi}{\partial t}. \tag{2}$$

Eq. (2) gives the evolution of the scaled plasma displacement $\xi(x, y, t) = \partial u/\partial y$, across field-lines and in time. Equilibrium dependence only enters through the width of the linear growth rate (Δ) and the scaled viscosity (ν). Here, viscosity due to charge-exchange interactions with neutrals, has been added to the ideal equation since it is the chief non-ideal effect in solar plasmas. Generally, instability described by Eq. (2) begins in a localized region. At low amplitude two nonlinear terms become important. One nonlinearity–the "explosive

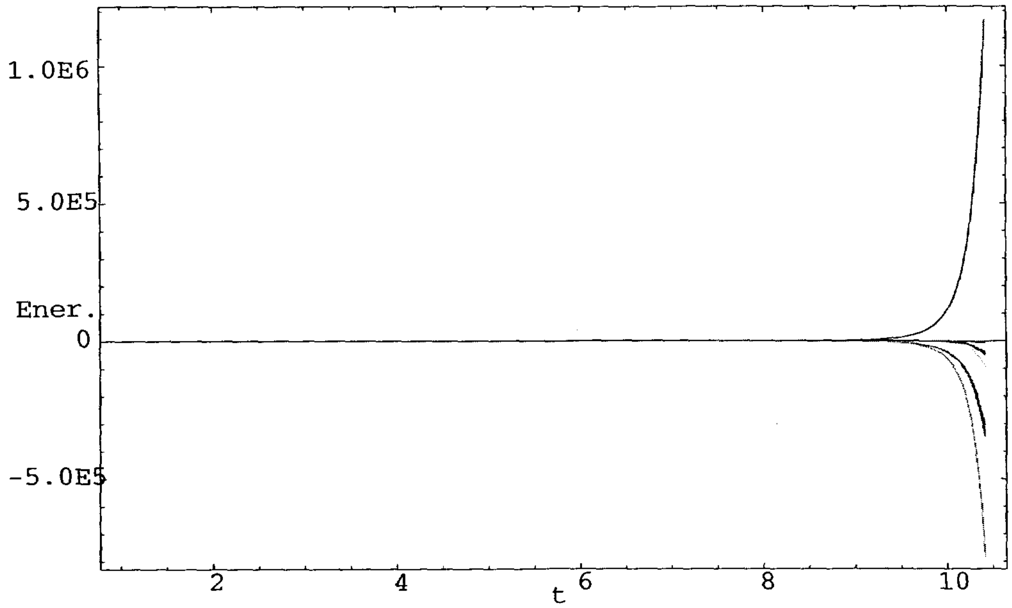

Figure 2. A time evolution plot of the energies of the system, as predicted by Detonation is shown. From the top down the curves are the nonlinear energy, the total energy, the linear energy, the viscous energy, the kinetic energy, and the quasi-linear energy. Positive (negative) sign indicates energy sources (sinks). The behavior is finite-time singular.

nonlinearity"– causes the mode to grow explosively and narrow into finger-like structures across the field (fourth term on the right). The other nonlinearity–the "quasi-linear nonlinearity"– flattens the profiles and broadens the mode into the linearly stable region (third term on the right). The linearly stable region is in fact metastable due to the explosive nonlinearity. As the unstable fingers spread they destabilize the metastable region– thus "detonating" the plasma.

Given an arbitrary equilibrium one can compute what regions in the equilibria are most susceptible to instability, the full three dimensional spatial structure of the linear and early nonlinear stage of the instability, and the expected time evolution. Mathematically, the total radial displacement is given by the form $\xi_x = \xi(x, y, t)H(l)$ where $H(l)$ gives the perturbation structure along a field line The eigenfunction, $H(l)$ obeys the linear ballooning equation, which is *schematically* given in Eq. (1), but the actual form is a complex set of coupled second order linear differential equations along magnetic field lines (given explicitly in *HFC*).

In order to solve the instability equations for realistic prominence equilibria, we have constructed initial states using a method presented in Fong, Hurricane, & Cowley (2001). Both normal polarity flux-rope and inverse polarity flux-rope solutions were found. By explicit solution of the ballooning equations, both normal and inverse flux-rope solutions were shown to be unstable in localized regions using common values for prominence parameters with $\sim$80% of the linear instability forcing coming from the Rayleigh-Taylor drive and $\sim$20% of the

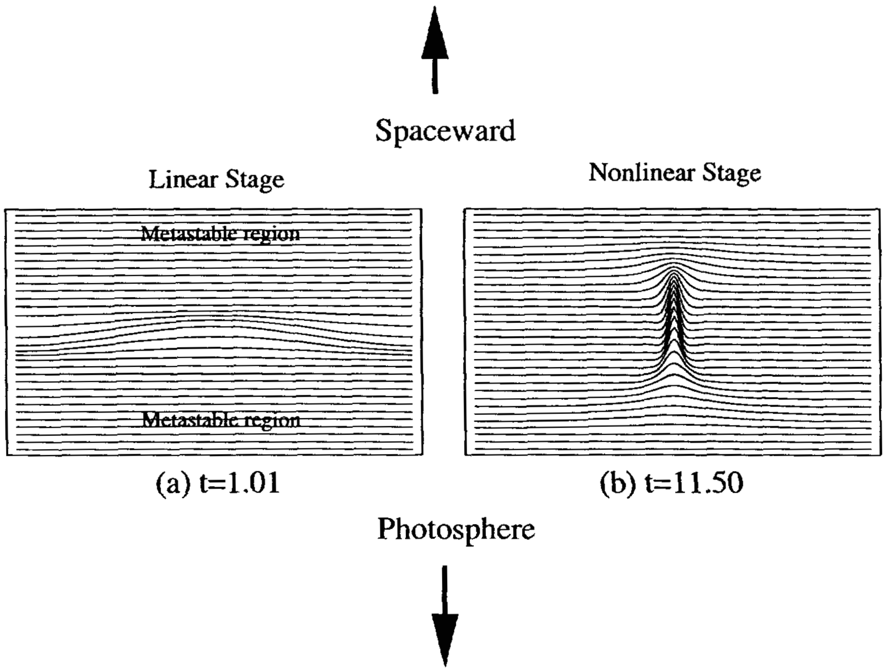

Figure 3. The spatial structure, across field lines, predicted by detonation evolve from usual ballooning mode characteristics (a) to very narrow finger-like structures (b). Surfaces of constant magnetic flux are shown. The magnetic field lines are normal to the page. In this case, the system was locally linearly unstable in a narrow radially localized region. Nonlinearly, instability has spread both upward and towards the photosphere with the finger pointing upward. This figure corresponds to the cut across the flux tubes shown in Figure 1.

forcing coming from the Parker drive. These slowly growing linear modes trigger the nonlinear detonation.

4. Detonation Predictions

A variational technique is elegant and instructive way of solving the nonlinear stability equation, Eq. (2), since a statement of energy conservation can be formed from the nonlinear equation by multiplying by $\dot{\xi}$ and integrating over x and y:

$$\frac{d}{dt}(K+U) = -2F$$

$$K = \int dxdy \frac{1}{2}\left(\frac{\partial \xi}{\partial t}\right)^2; \quad F = \frac{\nu}{2}\int dxdy \left(\frac{\partial^2 \xi}{\partial y \partial t}\right)^2 \tag{3}$$

$$U = \int dxdy \left[-\frac{1}{2}\left(1 - \frac{x^2}{\Delta^2}\right)\xi^2 + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{4}\left(\frac{\partial \xi^2}{\partial x}\right)^2 - \frac{1}{3}\xi^3 \right]$$

We choose a trial function that is a qualitative fit to both the linear and expected nonlinear behaviors:

$$\xi = A(t)e^{-\alpha(t)x^2}[1 - w(t)] \sum_{m=0}^{\infty} w^m(t) \cos\left[(m+1)\frac{2\pi y}{L}\right] \tag{4}$$

with $|w(t)| < 1$ and $\alpha(t) > 0$. $A(t)$ is the mode amplitude, while $\alpha(t)$ and $w(t)$ are directly related to characteristic mode widths across the magnetic field. The choice of trial function allows us to obtain each term of the energy conservation statement *in closed form:*

$$K = \frac{L}{2}\sqrt{\frac{\pi}{2\alpha}}A^2\frac{1-w}{1+w}\left[\left(\frac{\dot{A}}{A} + \frac{\dot{w}}{w-1} + \frac{w\dot{w}}{1-w^2} - \frac{1}{4}\frac{\dot{\alpha}}{\alpha}\right)^2 \right.$$
$$\left. + \frac{1}{8}\frac{\dot{\alpha}^2}{\alpha^2} + \frac{\dot{w}^2}{(1-w^2)^2}\right] \tag{5}$$

$$U = \frac{L}{4}\sqrt{\frac{\pi}{\alpha}}A^2\frac{1-w}{1+w}\left[-\frac{1}{\sqrt{2}}\left(1 - \frac{1}{4\alpha\Delta^2}\right) + \frac{\alpha}{4}\frac{1-w}{1+w}A^2 \right.$$
$$\left. - \frac{1}{3\sqrt{3}}\frac{wA}{1+w} + \frac{L^2}{4\pi^2}\frac{\alpha}{\sqrt{2}}\frac{1-w^2}{w^2}Li_2(w^2)\right] \tag{6}$$

$$F = \frac{\nu\pi^2}{L}\sqrt{\frac{\pi}{2\alpha}}A^2\frac{(1-w)^2}{(1-w^2)^3}\left\{\left(\frac{\dot{A}}{A} + \frac{\dot{w}}{w-1}\right)^2(1+w^2)\right.$$

$$+8\left(\frac{\dot{A}}{A} + \frac{\dot{w}}{w-1}\right)\dot{w}w\left(\frac{1+\frac{1}{2}w^2}{1-w^2}\right) + 4\dot{w}^2 P_2\left(\frac{1+w^2}{1-w^2}\right) \tag{7}$$

$$\left. -\frac{1}{2}\frac{\dot{\alpha}}{\alpha}\left[\left(\frac{\dot{A}}{A} + \frac{\dot{w}}{w-1}\right)(1+w^2) + 4\dot{w}w\left(\frac{1+\frac{1}{2}w^2}{1-w^2}\right)\right] + \frac{3}{4}\frac{\dot{\alpha}^2}{\alpha^2}\dot{w}^2 P_2\left(\frac{1+w^2}{1-w^2}\right)\right\}$$

where $L = 4\pi\Delta/3$, Li is the dilogarithm function and P_2 is a Legendre polynomial of order 2. Eq. (6) is particularly useful as it specifies the three-dimensional parameter space boundary between stability, $U(A, w, \alpha) > 0$, and instability, $U(A, w, \alpha) < 0$ (Fong, Cowley, & Hurricane 1999).

We need to obtain equations for $A(t)$, $w(t)$, and $\alpha(t)$ to complete the solution. Letting $L = K - U$, Euler-Lagrange evolution equations for $A(t)$, $w(t)$, and $\alpha(t)$ are obtained from:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{A}}\right) + \frac{\partial F}{\partial \dot{A}} = \frac{\partial L}{\partial A} \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{w}}\right) + \frac{\partial F}{\partial \dot{w}} = \frac{\partial L}{\partial w} \tag{9}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) + \frac{\partial F}{\partial \dot{\alpha}} = \frac{\partial L}{\partial \alpha} \tag{10}$$

Due to the complexity of Eqs. (5)-(7), the above equations of motion, Eqs. (8)-(10) are most easily obtained and solved with symbolic manipulation software. The beauty of this solution method is that we've bypassed many of the difficulties associated direct numerical integration of a nonlinear time dependent partial differential equation (e.g. artificial dissipation) that destroy fine scale spatial structure. The problem has been reduced to the solution of three nonlinear coupled ordinary differential equations.

4.1. Energies

The equations (8)-(10) have been solved numerically. A plot of the various terms that compose this energy conservation statement are shown in Figure 2 as a function of time for a typical numerical integration. The behavior is finite time singular. Scaling analysis of the numerical results have shown that the kinetic energy of the system increases as $\sim (t_0 - t)^{-5.5}$, t_0 being the detonation time–the value of which depends upon the equilibrium.

4.2. Spatial Structure and Metastability

Instability begins in a localized (in x and y) region where the configuration is closest to the marginal stability boundary. If the system has crossed the linear stability boundary, then the linear instability grows in the localized region and has the characteristics of a usual ballooning mode (Figure 3a).

At nonlinear stages, the mode grows explosively and narrows into finger-like structures while simultaneously spreading into linearly stable regions. As the finger spreads into the linearly stable region it destabilizes the region nonlinearly (Figure 3b) since the *linearly stable region is in fact only metastable.* At the front of the fingers, strong gradients develop as the finger penetrates into the metastable region suggesting the possible formation of a localized shock. Of course the analysis behind the construction of nonlinear equation fails at large amplitudes, so further work is needed to understand the possible formation of shock-like fronts and the large amplitude evolution (direct 3D simulation would be useful for this).

If the equilibrium is entirely linearly stable, then a field line must be displaced by an amount sufficient to overcome the stabilizing field line bending forces. Once sufficiently displaced, the nonlinear behavior is the same (i.e. also explosive) as that discussed above.

5. Conclusion & Speculative Remarks

Explosive behavior in solar plasmas has been an anomaly in physics for decades. The particular class of instability chosen here–those driven by pressure or gravity–are fine perpendicular scale MHD instabilities. Note that even though they are fine scale they can, in the detonation scenario, have global consequences. Macroscopic instabilities, like the kink mode, are not known to have explosive behavior–indeed in some cylindrical situations simple pitchfork bifurcations occur (Rutherford, Furth, & Rosenbluth 1971). Detonation exhibits elements of both transcritical and pitchfork like bifurcation behaviors although the actual situation is more complex than these labels imply due to the three-dimensional spatial structure of the instability.

Detonation demonstrates that explosive behavior is a natural and fundamental part of MHD for arbitrarily configured plasma equilibria. This explosive property of MHD promises an exciting explanation for the explosive nature of solar filaments.

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