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Summary

Thomsen's anisotropy parameters for weak elastic and poroelastic anisotropy are now commonly used in exploration, and can be conveniently expressed in terms of the layer averages of Backus. Although there are five effective shear moduli for any layered VTI medium, only one effective shear modulus for the layered system contains all the dependence of pore fluids on the elastic or poroelastic constants that can be observed in vertically polarized shear waves in VTI media. The effects of the pore fluids on this effective shear modulus can be substantial when the medium behaves in an undrained fashion, as might be expected at higher frequencies such a sonic and ultrasonic for well-logging or laboratory experiments, or at seismic frequencies for lower permeability regions of reservoirs.

Introduction

Gassmann's fluid substitution formulas for bulk and shear moduli (Gassmann, 1951) were originally derived for the quasi-static mechanical behavior of fluid saturated rocks. It has been shown recently (Berryman and Wang, 2001) that it is possible to understand deviations from Gassmann's results at higher frequencies when the rock is heterogeneous, and in particular when the rock heterogeneity anywhere is locally anisotropic. On the other hand, a well-known way of generating anisotropy in the earth is through fine layering. Then, Backus' averaging (Backus, 1962) of the mechanical behavior of the layered isotropic media at the microscopic level produces anisotropic mechanical behavior at the macroscopic level. The Backus averaging concept can also be applied to fluid-saturated porous media, and thereby permits us to study how deviations from Gassmann's predictions could arise in an elementary fashion. We consider both closed-pore and open-pore boundary conditions between layers within this model.

We review some standard results concerning layered VTI media in the first section. Then, we discuss singular value composition of the elastic (or poroelastic) stiffness matrix in order to introduce the interpretation of one shear modulus (out of the five shear moduli present) that has been shown recently (Berryman, 2003) to contain all the important behavior introduced by the pore fluid into the shear deformation response. These results are also then related to Thomsen parameters for weak anisotropy. For purposes of analysis, expressions are derived for the quasi-compressional- and quasi-SV-wave speeds. Numerical examples show that the analysis presented is completely consistent with the full theory for layered media.

Notation for elastic and poroelastic VTI media

Elastic and poroelastic notation

We begin by establishing some notation needed in the remainder of the paper. For transversely isotropic media with vertical symmetry axis, the relationship between components of stress σ_{ij} and strain e_{kl} is given by

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} a & b & f & & & \\ b & a & f & & & \\ f & f & c & & & \\ & & & 2l & & \\ & & & & 2l & \\ & & & & & 2m \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix}, \quad (1)$$

where $a = b + 2m$, and $e_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k})$ with u_k being the displacement in direction k.

Definitions of the Thomsen (1986) parameters ϵ, δ, and γ are now well-known. The stiffnesses in (1) are given either exactly or approximately by the relations $a = c(1 + 2\epsilon)$, $m = l(1 + 2\gamma)$, and $f \simeq c(1 + \delta) - 2l$. For P-wave propagation in the earth near the vertical, the important anisotropy parameter is δ. For SV-wave propagation near the vertical, the combination $(\epsilon - \delta)$ plays essentially the same role as δ does for P-waves. For SH-waves, the pertinent anisotropy parameter is γ. All three of the Thomsen parameters vanish for an isotropic medium.

In TI media, c and l are the velocities normal to the layering. Then, ϵ, γ, and δ measure the deviations from these normal velocities at other angles. We present some of the relevant details of the phase velocity analysis later.

Gassmann results for isotropic poroelastic media

To understand the significance of the results to follow, we briefly review some well-known results due to Gassmann (1951). Gassmann's first equation relates the bulk modulus K^* of a saturated isotropic porous medium to the bulk modulus K_{dr} of the same medium in the drained case: $K^* = K_{dr}/(1 - \alpha B)$, where the parameters α and B [respectively, the Biot-Willis parameter (Biot and Willis, 1957) and Skempton's pore-pressure buildup coefficient (Skempton, 1954)] depend on the porous medium and fluid compliances. For the shear moduli of the drained (μ_{dr}) and saturated (μ^*) media, Gassmann's quasi-static theory gives $\mu^* = \mu_{dr}$. Note that the fluid effect is all contained in the parameter $\lambda^* = K^* - \frac{2}{3}\mu^*$, where λ and μ are the well-known Lamé parameters.

Poroelastic Thomsen parameters

Backus averaging

Backus (1962) presented an elegant method of producing the effective constants for a thinly layered medium composed of either isotropic or anisotropic elastic layers. This method applies either to spatially periodic layering or to random layering. For simplicity, we assume that the physical properties of the individual layers are isotropic, in which case the coefficients in equation (1) take the values $a = c = \lambda + 2\mu$, $b = f = \lambda$, and $2l = 2m = 2\mu$ for each individual layer. The key idea presented by Backus is that these equations can be rearranged into a form where rapidly varying coefficients multiply slowly varying (really constant) stresses or strains.

The derivation has been given many places including Schoenberg and Muir (1989). Using brackets $\langle x \rangle$ to indicate the volume (or equivalaently the one-dimensional layer) average of the quantity x in the simple layered medium under consideration, the anisotropy coefficients in equation (1) are related to the layer parameters by the following expressions: $c = \left\langle \frac{1}{\lambda+2\mu} \right\rangle^{-1}$, $f = c\left\langle \frac{\lambda}{\lambda+2\mu} \right\rangle$, $l = \left\langle \frac{1}{\mu} \right\rangle^{-1}$, $m = \langle \mu \rangle$, $a = \frac{f^2}{c} + 4m - 4\left\langle \frac{\mu^2}{\lambda+2\mu} \right\rangle$, and b = a - 2m. When the layering is fully periodic, these results may be attributed to Postma (1955), while for more general layered media including random media they should be attributed to Backus (1962).

Recall that these equations reduce (Backus, 1962) to isotropic results with $a = c$, $b = f$, and $l = m$, if the layer shear modulus is a constant ($= \mu$), regardless of the behavior of λ.

Singular value decomposition

The singular value decomposition (SVD), or equivalently the eigenvalue decomposition in the case of a real symmetric matrix, for (1) is relatively easy to perform. We can immediately write down four eigenvectors:

$$\begin{pmatrix} 0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \begin{pmatrix} 0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \begin{pmatrix} 0\\0\\0\\0\\0\\1 \end{pmatrix}, \quad \begin{pmatrix} 1\\-1\\0\\0\\0\\0 \end{pmatrix}, \quad (2)$$

and their corresponding eigenvalues, respectively $2l$, $2l$, $2m$, and $a-b = 2m$. All four correspond to shear modes of the system. The two remaining eigenvectors must be orthogonal to all four of these and therefore both must have the general form $(1, 1, X, 0, 0, 0)^T$, with the corresponding eigenvalue $\chi = a + b + fX$. The remaining condition that determines both X and χ is $\chi X = 2f + cX$, which, after substitution for χ, leads to a quadratic equation having the solutions

$$X_\pm = \frac{1}{2}\left(-\left[\frac{a+b-c}{f}\right] \pm \sqrt{8 + \left[\frac{a+b-c}{f}\right]^2} \right). \quad (3)$$

The ranges of values for $X_\pm$ are $0 \le X_+ \le \infty$ and, since $X_- = -2/X_+$, $-\infty \le X_- \le 0$. The interpretation of the solutions $X_\pm$ is simple for the isotropic limit where $X_+ = 1$ and $X_- = -2$, corresponding respectively to pure compression and pure shear modes. For all other cases, these two modes have mixed character, indicating that pure compression cannot be excited in the system.The pertinent functional $F(x) = \frac{1}{2}\left[-x + \sqrt{8 + x^2}\right]$ is easily shown to be a monotonic function of its argument x. So it is sufficient to study the behavior of the argument $x = (a+b-c)/f$. Space limitations preclude further discussion of this here, but it is straightforward to do the analysis and the main result is that the shear modulus fluctuations giving rise to the anisotropy due to layering are (as expected) the main source of deviations of x from unity. There are both linear and quadratic contributions from these fluctuations.

Carrying this analysis forward and using a shear energy criterion, Berryman (2003) shows, furthermore, that

$$G_{eff} \equiv (a + c - m - 2f)/3 \quad (4)$$

is the fifth shear modulus of interest in VTI media, and the only one that can contain effects of fluids in undrained poroelastic media via $\lambda \to \lambda^*$.

Dispersion relations for seismic waves

The general behavior of seismic waves in anisotropic media is well known, and the equations are derived in many places including Berryman (1979) and Thomsen (1986). The results are

$$\rho\omega_\pm^2 = \frac{1}{2}\left\{(a+l)k_1^2 + (c+l)k_3^2 \pm \sqrt{[(a-l)k_1^2 - (c-l)k_3^2]^2 + 4(f+l)^2k_1^2k_3^2}\right\}, \quad (5)$$

for compressional (+) and vertically polarized shear (−) waves and

$$\rho\omega_s^2 = mk_1^2 + lk_3^2, \quad (6)$$

for horizontally polarized shear waves, where ρ is the overall density, ω is the angular frequency, k_1 and k_3 are the horizontal and vertical wavenumbers (respectively), and the velocities are given simply by $v = \omega/k$ with $k = \sqrt{k_1^2 + k_3^2}$. The SH wave depends only on elastic parameters l and m, which are not dependent in any way on layer λ and therefore will play no role in the poroelastic analysis. Thus, we can safely ignore SH-waves here.

Note that

$$\rho\omega_+^2 + \rho\omega_-^2 = (a+l)k_1^2 + (c+l)k_3^2, \quad (7)$$

So we can make the identifications

$$\rho\omega_+^2 \equiv ak_1^2 + ck_3^2 - \Delta \qquad \text{and} \qquad \rho\omega_-^2 \equiv lk^2 + \Delta, \quad (8)$$

with Δ still to be determined. Then, we find that

$$\Delta \simeq \frac{[(a-l)(c-l)-(f+l)^2]}{(a-l)/k_3^2+(c-l)/k_1^2}. \qquad (9)$$

The numerator of this expression is known to be a positive quantity for layered materials (Postma, 1955; Berryman, 1979). Furthermore, it can be rewritten in terms of Thomsen's parameters as

$$[(a-l)(c-l)-(f+l)^2] = 2c(c-l)(\epsilon-\delta). \qquad (10)$$

Combining these results in the limit of $k_1^2 \to 0$ (for relatively small horizontal offset), we find that

$$\rho\omega_+^2 \simeq ck^2 + 2c\delta k_1^2, \qquad (11)$$

and

$$\rho\omega_-^2 \simeq lk^2 + 2c(\epsilon-\delta)k_1^2, \qquad (12)$$

with $\Delta \simeq 2c(\epsilon-\delta)k_1^2$, to a very good approximation.

TABLE 1. Layer parameters for the three material simple layered medium used to produce the examples in Figures 1 and 2.

Constituent	K (GPa)	μ (GPa)	z (m/m)
1	9.4541	0.0965	0.477
2	14.7926	4.0290	0.276
3	43.5854	8.7785	0.247

TABLE 2. The VTI elastic coefficients and Thomsen parameters for materials (see Table 1) used in the computed examples of Figures 1 and 2.

Elastic Parameters and Density	*Case* $B=0$	*Case* $B=\frac{1}{2}$	*Case* $B=1$
a (GPa)	33.8345	50.3523	132.7003
c (GPa)	33.1948	50.4715	134.2036
f (GPa)	22.2062	38.5857	120.7006
l (GPa)	4.0138	4.0138	4.0138
m (GPa)	6.7777	6.7777	6.7777
G_{eff} (GPa)	5.2797	5.8841	6.2417
δ	-0.0847	-0.0733	-0.0399
$\epsilon-\delta$	0.0943	0.0745	0.0343
γ	0.3443	0.3443	0.3443
ρ (kg/m^3)	2120.0	2310.0	2320.0

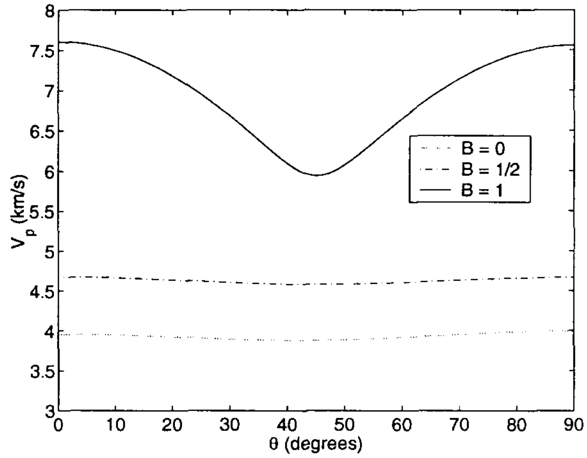

Fig. 1: Compressional wave speed V_p, as a function of angle θ from the vertical. Two curves shown correspond to choices of Skempton's coefficient $B=0$ for the drained case (dashed line) and $B=1$ for the undrained case (solid line). The case $B=\frac{1}{2}$ (dot-dash line) is used to model partial saturation conditions as described in the text. The Biot-Willis parameter was chosen to be $\alpha=0.8$, constant in all layers.

Computed Examples

From previous work (Berryman, 2003), we know that large fluctuations in the layer shear moduli are required before significant deviations from Gassmann's quasi-static constant result showing that the shear modulus is independent of fluid properties will become noticeable. To generate a model that demonstrates these results, I made use of a code of V. Grechka [used previously in a joint publication (Berryman *et al.*, 1999)] and then I arbitrarily picked one of the models that seemed to be most interesting for the present purposes. The parameters of this model are displayed in TABLE 1. The results for the various elastic coefficients and Thomsen parameters are displayed in TABLE 2. The results of the calculations for V_p and V_{sv} are shown in Figures 1 and 2.

The model calculations were simplified in one way: the value of the Biot-Willis parameter was chosen to be a uniform value of $\alpha=0.8$ in all layers. We could have actually computed a value of α from the other layer parameters, but to do so would require another assumption about the porosity values in each layer. Doing this seemed an exercise of little value because we are just trying to show in a simple way that the formulas given here really do produce the types of results predicted and to get a feeling for the magnitude of the effects. Furthermore, if α is a constant, then it is only the product αB that matters. Whatever choice of constant $\alpha \le 1$ is made, it mainly determines the maximum value of the product αB for B in the range $[0,1]$. So, for a parameter study, it is only important not to choose too a small value of α, which is why the choice

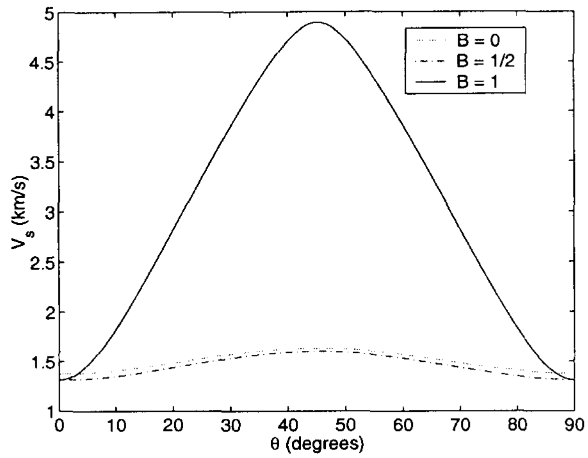

Fig. 2: Vertically polarized shear wave speed V_{sv}, as a function of angle θ from the vertical. Two curves shown correspond to choices of Skempton's coefficient $B = 0$ for the drained case (dashed line) and $B = 1$ for the undrained case (solid). The case $B = \frac{1}{2}$ (dot-dash line) is used to model partial saturation conditions as described in the text. The Biot-Willis parameter was chosen to be $\alpha = 0.8$, constant in all layers.

$\alpha = 0.8$ was made. This means that the maximum amplification of the bulk modulus due to fluid effects can be as high as a factor of 5 for the present examples.

We took the porosity to be $\phi = 0.2$, and the overall density to be $\rho = (1-\phi)\rho_s + \phi S \rho_l$, where $\rho_s = 2650.0$ kg/m^3, S is liquid saturation ($0 \leq S \leq 1$), and $\rho_l = 1000.0$ kg/m^3. Then, three cases were considered: (1) Liquid saturation $S = 0$ and $B = 0$, which is the drained case, assuming that the effect of the saturating gas on the moduli is negligible. (2) Liquid saturation $S = 1$ and $B = 1$, which is the fully undrained case, assuming that a fully saturating liquid has the maximum possible stiffening effect on the locally microhomogeneous, poroelastic medium. And (3) liquid saturation $S = 0.95$ and $B = \frac{1}{2}$, which is intended to model a case of partial liquid saturation, intermediate between the other two cases. For smaller values of liquid saturation, the effect of the liquid might not be noticeable, since the gas-liquid mixture when homogeneously mixed will act much like the pure gas in compression, although the density effect is still present. When the liquid fills most of the pore-space, and the gas occupies less than about 3% of the entire volume of the rock, the gas starts to become disconnected, and we expect the effect the liquid to start becoming more noticeable, and therefore we choose $B = \frac{1}{2}$ to be representative of this case.

Conclusions

One of our main results is that, although there are five effective shear moduli for any layered VTI medium, there is the one and only one effective shear modulus G_{eff} for the layered system that contains all the dependence of pore fluids on the elastic or poroelastic constants that can be observed in vertically polarized shear waves in VTI media. The pore-fluid effects on this effective shear modulus can be substantial when the medium behaves in an undrained fashion, as might be expected at higher frequencies such a sonic and ultrasonic for well-logging or laboratory experiments, or at seismic frequencies for lower permeability regions of reservoirs. These results are clearly illustrated in Figure 2.

Acknowledgments

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