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# ANALYTIC ICF HOHLRAUM ENERGETICS

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**Abstract.** We apply recent analytic solutions[1] to the radiation diffusion equation to problems of interest for ICF hohlraums. The solutions provide quantitative values for absorbed energy which are of use for generating a desired radiation temperature vs. time within the hohlraum. Comparison of supersonic and subsonic solutions (heat front velocity faster or slower, respectively, than the speed of sound in the x-ray heated material) suggests that there may be some advantage in using high Z metallic foams as hohlraum wall material to reduce hydrodynamic losses, and hence, net absorbed energy by the walls. Analytic and numerical calculations suggest that the loss per unit area might be reduced ~ 20% through use of foam hohlraum walls. Reduced hydrodynamic motion of the wall material may also reduce symmetry swings, as found for heavy ion targets[2].

## I. INTRODUCTION

Radiation heat waves, or Marshak waves[3,4], play an important role in energy transport and in the energy balance of laser, z-pinch and heavy ion beam hohlraums for inertial confinement fusion (ICF) and high energy density physics experiments. In these experiments, a power source, e.g. a laser, delivers energy to the interior of a high Z cavity that is converted to x-rays. Typically, most of the energy is absorbed in a thin, diffusively-heated layer on the hohlraum interior surface, and re-emission from the heated layer sets the radiation temperature T achieved in the hohlraum.

In our recent paper[1], (henceforward referred to as HR) we developed an analytic theory of Marshak waves via a perturbation theory using a small parameter $\varepsilon=\beta/(4+\alpha)$, where the internal energy varies as $T^{\beta}$ and the opacity varies as $T^{-\alpha}$. A consistent theory was built up order-by-order in $\varepsilon$, with the benefits of good accuracy and order-by-order energy conservation. We first derived analytic solutions for supersonic Marshak waves, which remarkably allowed for arbitrary time variation of the surface temperature. We then solved the full set of subsonic equations, though specialized to the case that the surface temperature varies as $t^k$, where self-similar solutions can be found. Our solutions compared very well with exact analytic solutions (for the specialized cases for which they exist) and with radiation-hydrodynamic simulations.

In this paper we apply those results to the following question: Can we save on driver energy by making hohlraum walls out of low density high Z foams, which have less hydrodynamic motion and hence, reduced net absorbed energy by the walls? We answer this question using our HR analytic theory, as well as by numerical simulations. To the degree that the "pure" HR theory diverges from the simulations we explain the non-ideal non-self-similar corrections to the theory that bring it into agreement with the simulations. We do show that low density high Z foams can indeed bring a savings of up to 20% in the required driver energy. For a nominal 5B$ ICF reactor driver of 5 MJ, this is a 1B$ cost saving idea! Reduced hydrodynamic motion of the wall material may also reduce symmetry swings, as found for heavy ion targets[2].

## II. SUPERSONIC SOLUTIONS

For the sake of brevity and clarity we will restrict our study here to an ICF relevant T = 250 eV drive that is constant in time for 4 ns. The basic equation for supersonic, diffusive radiative transport in one dimension is

$$\rho\frac{\partial e}{\partial t}=\frac{4}{3}\frac{\partial}{\partial x}\frac{1}{K\rho}\frac{\partial \sigma T^4}{\partial x} \qquad (1)$$

where $e$ is the internal energy per unit mass, $\rho$ is the density, $T$ is the temperature, $\sigma$ is the Stefan-Boltzmann constant, $K$ is the Rosseland-mean opacity, $t$ is time, and $x$ is the spatial coordinate. $e$ and $K$ are specified functions of $\rho, T$. for given materials.

$$e=fT^{\beta}\rho^{-\mu},\qquad \frac{1}{K}=gT^{\alpha}\rho^{-\lambda}. \qquad (2)$$

with $f$, $g$ constants. Higher density means more recombination – hence more bound electrons to provide line opacity and fewer free electrons which thus reduce the specific heat.

By supersonic, we mean that the velocity of the heat front is much greater than the speed of sound in the heated material. This will occur in low density high Z foams. We consider the case of constant $\rho$ since, in the supersonic limit, hydrodynamic motion is too slow to give rise to density changes. Inserting eq. (2) into eq. (1), together with $\rho$ = constant gives

$$\begin{aligned}&\frac{\partial}{\partial t}T^{\beta}=C\frac{\partial^2}{\partial x^2}T^{4+\alpha}\\ &C=\frac{16}{(4+\alpha)}\frac{g\sigma}{3f\rho^{2-\mu+\lambda}}\end{aligned} \qquad (3)$$

The parameter $\varepsilon$ defined as $\beta / (4+\alpha)$ plays an important role in our theory, as we employ $\varepsilon$ as our expansion parameter. It is also useful to introduce a dimensionless spatial variable $y = x / x_F$, where $x_F$ is the time-dependent heat front position. In HR we solved for T(t,y), for $x_F$, for the absorbed flux and energy, and then successfully compared our analytic solutions to numerical results from the radiation-hydrodynamics code HYDRA [5]. For this purpose we used a fit to the opacity and equation of state for gold in the temperature range 1 - 3 HeV (1 HeV = 100 eV) with temperature in HeV units and $\rho$ in g/cc.

$$\begin{aligned}&f=3.4\ \text{MJ/g}\quad \beta=1.6\quad \mu=0.14\\ &g=\frac{1}{7200.}\ \text{g/cm}^2\quad \alpha=1.5\quad \lambda=0.2\end{aligned} \qquad (4)$$

If time is in ns units, then $\sigma$ =1.03 x $10^{-2}$ MJ/ns/cm$^2$. For these parameters, $\varepsilon$ =0.291 and the constant C is 4.08 x $10^{-7}/\rho^{2.06}$ cm$^2$/ns. For those parameters we found $x_F^2 = [(2+\varepsilon)/(1-\varepsilon)]\ C\ T^{4+\alpha-\beta}\ t$ where C is defined in eq. (3). For our case then, $x_F$ $(cm)$ = 0 .0012 $T^{1.95}\ t^{.5} / \rho^{1.03}$. Our solutions there also lead to a prediction here for energy per unit area absorbed by the gold wall:

$$E / A = 0.0029\ T^{3.55}\ t^{.5} / \rho^{0.17}\ (\text{MJ / cm}^2) \qquad (5)$$

## III. SUBSONIC SOLUTIONS

In HR we constructed perturbation solutions to the subsonic equations for the case that the surface temperature varies as $t^{k}$, where self-similar solutions exist. The basic equations in Lagrangian form are

$$\begin{aligned}&\frac{\partial V}{\partial t}=\frac{\partial u}{\partial m}\\ &\frac{\partial u}{\partial t}=-\frac{\partial P}{\partial m}\\ &\frac{\partial e}{\partial t}+P\frac{\partial V}{\partial t}=\frac{4}{3}\frac{\partial}{\partial m}\frac{1}{K}\frac{\partial \sigma T^4}{\partial m}\end{aligned} \qquad (6)$$

where $V=\frac{1}{\rho}$ is the specific mass, $u$ is the flow velocity, $P$ is the pressure and the mass variable $m=\int \rho dx$. The effectively-infinite density at the ablation front [6-9] means that we have the boundary conditions $u,V,T\rightarrow 0$ at the heat front as well as $T(0,t)=T_S(t)$. Our subsonic solutions included the hydrodynamic flow solution as the density changes in time and space. In the subsonic case, the specific heat, (pressure/density) and opacity are each assumed to vary as density to a small power, of order $\varepsilon$, as presented in Eqs. ( 2 ) and ( 4 ). We again assume power-law dependence of opacity and equation of state variables as above with the additional condition that P=re/V. The parameter $r$ is assumed to be of order $\varepsilon$ (a typical value of r for gold at 1-3 HeV is 0.25). Employing the self-similar ansatz, the quantity, y = m / $m_F$ with $m_F$ the mass coordinate of the heat front, becomes the similarity variable (analogous to y = x / $x_F$ above). We again solved for T(t,y), for $m_F$, for the absorbed flux and energy, and then successfully compared our analytic solutions to numerical results from the radiation-hydrodynamics code. Those solutions gave self-similar time dependencies for the ablated mass, and absorbed flux of

$$m(t) = m_0 T_S(t)^{1.914} t^{0.5156}$$
$$F(t) = F_0 T_S(t)^{3.346} t^{-0.4115} \qquad (7a)$$

with $T_S(t) = T_0 t^k = T_0 t^{\frac{q_0}{5.5}}$ for $T_0$ in HeV and $t$ in ns. The quantities $m_0, F_0$, are given below for q=0.

$$m_0 = 9.90 \times 10^{-4}\ \mathrm{g/cm^2}$$
$$F_0 = 3.40 \times 10^{-3}\ \mathrm{MJ/ns/cm^2} \qquad (7b)$$

Thus for our case (and doing the rather simple $E = \int F dt$ calculation) we get

$$E/A = 0.0058\, T^{3.346}\, t^{.5885} \quad (\mathrm{MJ/cm^2}) \qquad (8)$$

## IV. DISCUSSION

Comparing Eqs (5) and (8) we see that for densities in the neighborhood of 0.3 gm/cc there is clearly less wall loss for the supersonic case. Lowering densities further decreases opacity and increases specific heat, both in the undesirable direction of more loss to internal energy. Raising densities would be desirable as that would lower wall losses even further, but unfortunately it would take us into the subsonic regime. The sound speed, $C_S$, at 250 eV in gold is about 56 μm/ns, which (using the expression for $x_F$ that precedes Eq. (5)) exceeds the supersonic heat front velocity at 4 ns when $\rho_0$ = 0.4 gm/cc (and $C_S t$ equals $x_F$ (at t=4 ns) when $\rho_0$ = 0.6 gm/cc).

In Fig. (1) we plot E/A vs. initial $\rho_0$ of the wall from Eq. (5) (with T=2.5, t=4.) and we put the subsonic ("infinite density") result of Eq. (8) at $\rho_0$=100. We also plot the numerical simulation results.

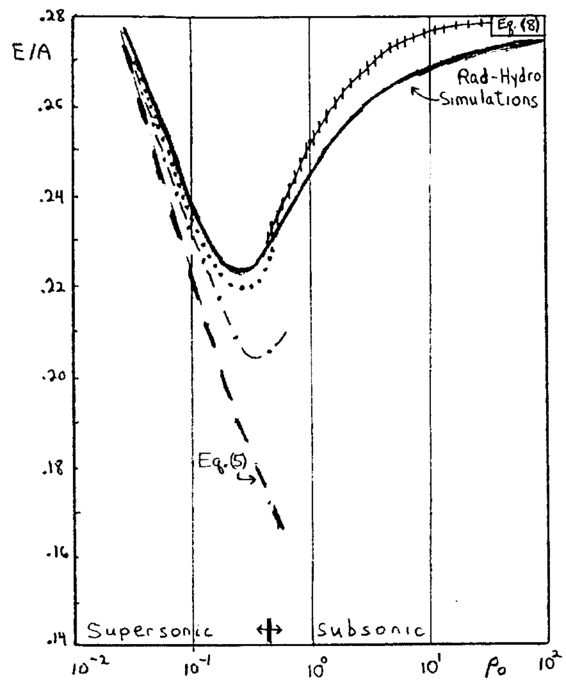


**Figure 1. Wall loss (MJ/cm$^2$) vs. initial wall density (gm/cc).** Dashed line: Eq. (5). Solid: Simulations. Dot-Dashed: Eq. (5) + kinetic energy of rarefaction. Dotted: Dot-dashed + increased internal energy in lower density rarefaction. Cross Hatched: Subsonic E/A corrected for early time episode of supersonicity

Note that Eq. (5) closely matches the full physics numerical simulations, deep in the supersonic regime (at very low ρ) when little hydrodynamic motion is expected. When hydrodynamic motion is artificially turned off in numerical simulations (not shown here), Eq. (5) closely matches those artificial simulations for all densities.

In the supersonic regime at higher $\rho_0$, rarefactions do in fact eat into the wall at the drive boundary and hydro motion ensues. An isothermal rarefaction has kinetic energy per unit area of $\rho_0\, C_S^{\ 3}\, t$. We can add that extra energy sink in, on a case by case basis (it breaks the self similarity). For our parameters this adds about $.07\rho_0$ MJ/cm$^2$ and this matches the full physics simulation's opinion of the kinetic energy. That is the dot-dashed correction curve in Fig. (1). Also the lower density in the rarefaction leads, via Eq. (2) to a higher specific heat. This adds an additional $\mu/(1-\mu)$ fraction of internal energy to that part of the heat front overtaken by the rarefaction. For our parameters this adds about $.037\rho_0^{.86}$ MJ/cm$^2$. These 2 effects together (the dotted curve in Fig. (1) calculated out to the high $\rho_0$ edge of the supersonic regime) largely reproduce the E/A full physics numerical simulation curve throughout the entire supersonic regime. While these additional energy sinks reduce the full "bonus" of being supersonic that Eq. (5) naively promises, we still note a 20% reduction from the solid wall result.

Note too that Eq. (8) closely matches the full physics numerical simulation at the very high end of the initial-wall-density x axis, deep in the subsonic regime. However, in the lower density part of the subsonic regime the simulations differ from the infinite density result. We speculate here that that may be due to the period of time early in the simulation when indeed the heat wave is supersonic. As the initial density, $\rho_0$, decreases, an increasingly longer early-time duration of supersonicity exists. We can correct for this, again on a case by case basis, as it again, breaks the self similarity. We find $t_{sonic}$ , the time of transition from super to sub sonic (when $C_S$ t becomes larger than the $x_F$ (t) that precedes Eq. (5)). We then subtract the subsonic E/A ($t=t_{sonic}$) of Eq. (8) from E/A (t = 4 ns) of Eq. (8) and add in its stead the supersonic E/A($t=t_{sonic}$) of Eq. (5), plus the two corrections to that as described in the previous paragraph. For our parameters, the procedure outlined above leads to a simple expression for the correction: E/A (MJ/cm$^2$) = $0.28 - 0.027 / \rho_0^{1.2}$ and the result largely reproduces the E/A simulation curve throughout the entire subsonic regime, as seen in the cross hatched curve of Fig. (1).

## V. SUMMARY

On the basis of our HR theory, as well as on the basis of numerical simulations, we have shown that hohlraum walls made of low density ($\rho=0.3$ gm/cc) high Z foams can decrease wall loss by 20%. While our previous work allowed us to correctly predict the wall loss at the two extremes of initial wall density, as shown in Fig. (1), we discovered that at intermediate densities there were discrepancies. We showed here how to account for non-ideal effects in the middle regime, near the sonic transition and thereby restore agreement with the numerical simulations therein. As a bottom line conclusion, there is a real advantage, energy wise, to consider low density high Z foam for use as hohlraum walls for driving ICF targets. For a nominal 5B$ ICF reactor driver of 5 MJ, this 20% reduction is a 1B$ cost saving idea!

There may be a further advantage in going this route. Reduced hydrodynamic motion of the wall material may also reduce symmetry swings, as found for heavy ion targets[2]. Detailed calculations will need to be done to assess this aspect more definitively.

## ACKNOWLEDGMENTS

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