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The Kinetic Stabilizer: A Route to Simpler Tandem-Mirror Systems?

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ABSTRACT

This paper discusses a new approach to an MHD stabilizing technique for magnetic fusion systems of the axisymmetric "open-ended" variety. The concept is adaptable to tandem-mirror systems and would result in a major simplification of such systems, accompanied by a substantial improvement in their confinement characteristics. The paper first discusses the present impetus to find a simpler and less expensive route to fusion than that offered by the mainline approach, the tokamak. The history of magnetic fusion research shows that closed and open systems exhibit very different confinement characteristics. Closed systems, such as the tokamak, the stellarator, or the reversed-field pinch have cross-field transport that is dominated by plasma turbulence. By contrast, there are examples of open-systems where turbulence, if present at all, was at such low levels that the transport agreed with "classical" predictions.. The clearest examples are ones in which the field geometry was axiymmetric. However axisymmetric mirror systems are subject to MHD instability. Thus in the years following the famous Ioffe experiment, most open systems have employed asymmetric magnetic fields, with attendant problems of complexity and enhanced cross-field transport. This paper proposes a new means of stabilizing axisymmetric mirror-based systems. The idea, called the "Kinetic Stabilizer" has roots in experiments performed with the axisymmetric Gas Dynamic Trap at Novosibirsk. In these experiments, performed in a high collisionality plasma regime, it was shown that the presence of the effluent plasma in the positive-curvature expanding-field region outside the mirrors was effective in stabilizing a high-beta (30 percent) confined plasma against MHD modes. In the plasmas of tandem-mirror systems the density of the effluent plasma is too low to employ this method of stabilization. The Kinetic Stabilizer solves this problem by using ion beams, injected at small angles up the magnetic gradient outside the end mirrors, to create a localized stabilizing plasma by magnetic compression and reflection of the injected ions partway up the gradient. Theoretical analyses and code calculations are used to show that stability can be achieved even when the kinetic pressure of the Stabilizer plasma is many orders of magnitude lower than that of the confined plasma. It is estimated that axisymmetric tandem mirror systems producing hundreds of megawatts of fusion power could be stabilized using Stabilizer ion beams with a total power of order a few megawatts. If confirmed by further work, the Kinetic Stabilizer idea thus offers a much simpler approach to fusion power than the mainline approaches.

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1. INTRODUCTION

Fifty years of research on the magnetic confinement of fusion plasmas has built an impressive base of physics understanding and of fusion-relevant technology. However, in the view of many there has not as yet emerged an approach that will clearly satisfy the requirements imposed by economics and practicality. We can trace the origins of this concern to several circumstances, but one of them stands out. This circumstance is the historical fact that the main effort in magnetic fusion research has been directed at systems with closed magnetic topology, the first example of which was the stellarator, followed by the tokamak and the reversed-field pinch. As history has shown, each of these approaches has its particle confinement and energy transport dominated by turbulent processes in the plasma. As a result, the size of closed systems, for example the tokamak, must be very large in order to achieve the confinement times required for net fusion power. Not only must the size be large in these devices, but the plasma pressures have necessarily been limited to a few percent of the confining magnetic pressure in order to avoid unacceptable levels of plasma turbulence or catastrophic plasma disruptions. Considering these circumstances one can discern an implicit "research philosophy" for closed systems that might be stated as follows: "To ascertain whether it is possible to gain a sufficient understanding of plasma confinement in the presence of turbulence to define a path to a practical fusion power system."

Given this situation there is a growing movement in the fusion research community, in parallel with the pursuit of the present mainline approach, the tokamak, to look for simpler and smaller approaches, particularly ones where the particle transport is not dominated by turbulence. In this author's opinion, this search is most likely to succeed if it is concentrated in the area of open-ended magnetic systems, examples of which are the mirror and tandem-mirror approaches. The mirror approach to fusion was one of the earliest suggested, but for a variety of reasons it has received far less attention than closed systems. It is hoped that this circumstance can be reversed in the future. This paper is motivated by an attempt to search for better answers to magnetic fusion in the arena of open systems.

In embarking on such a search there are guidelines that can be deduced from the history of research in open systems. Recognizing the subjective nature of any such list, below are listed some guidelines that the author considers important:

- Open systems have demonstrated plasma confinement in near-classical states (i.e., ones not dominated by plasma fluctuations).

- In open systems, axisymmetric confining fields eliminate cross-field particle transport that can arise from bounce-resonant particle drifts that occur in asymmetric confining fields.

- Magnetic fields with positive field-line curvature are effective in suppressing MHD instabilities in open systems.

- In open systems the radial boundary of the confined plasma can be far from material surfaces, thereby permitting the avoidance of turbulence-producing radial temperature gradients and plasma sheath effects that are inherent to closed systems.

- Axial confinement of ions and electrons by ambipolar potentials (as in the tandem-mirror, for example) is effective and is well understood theoretically.

• The origins, and effective means of control, of microinstabilities in open systems are understood theoretically and the theory has been confirmed by experiment.

• The feasibility of employing direct conversion to enhance the efficiency of open-ended systems has been demonstrated in the laboratory.

In addition to these items, there is a general property of axisymmetric open systems that has important implications for particle confinement. In an axisymmetric mirror system the role of the adiabatic invariants μ and J (magnetic moment and longitudinal action integral) insures that trapped particles will remain trapped, i.e. they stay on closed drift surfaces as they traverse back and forth between the mirrors [1]. As a result, in the absence of turbulence or collisional effects these particles would be contained "forever." A classic example of the power of these invariants was the ARGUS experiment, proposed by the late Nicholas Christofilos and carried out in the 1950's. In this experiment a rocket-launched nuclear explosion produced an artificial "Van-Allen Belt" of trapped electrons. An appreciable number of these electrons were still detectable a decade later! This particle-trapping property of axisymmetric mirror systems is very different from closed systems, for example the stellarator, where there exist classes of particles (those temporarily trapped between local field maxima) that would drift across the field and be lost if it were not for collisional detrapping of these particles.

The important role of the adiabatic invariants in axisymmetric mirror systems may help explain some striking early confinement results that were obtained in the Livermore "Table Top" mirror experiment in the 1960s [2]. In this apparatus a spindle-shaped 2-centimeter diameter "hot electron" ($T_e \approx 20$ keV) plasma column was produced by magnetic compression (from an initial field of a few Gauss to a final field of 10,000 Gauss) of a plasma injected from a pulsed plasma gun. The plasma was observed to decay stably with a time constant appropriate to the electron-ion collision rate. During the decay the radial profile of the plasma remained nearly constant. From this observation it was ascertained that the cross-field transport had to be at least 5 orders of magnitude slower than the Bohm-diffusion rate as calculated from the electron temperature, the density gradient, and the strength of the confining field.

Additional evidence that axisymmetric open systems can confine plasma with cross-field transport rates not dominated by turbulence comes from experiments performed in 1968 in the 8 meter theta pinch experiment at Culham, U. K. [3]. In these experiments careful measurements were made of the rate of diffusion of plasma across the confining field. It was found that both the transport rates and the plasma radial profile agreed with that predicted on the basis of classical processes, and disagreed by at least two orders of magnitude with the rates predicted from Bohm diffusion.

A more recent experiment, the Gas Dynamic Trap [4] at Novosibirsk, Siberia, has provided additional confirmation of classical confinement in an axisymmetric mirror system. To quote a statement in the reference, "The comparison between the measured and the calculated energy contents of the fast ions shows that within the accuracy of the measurements the experimental and the simulated data are identical." The "simulated data" referred to was obtained from computer-modeling of the experiment using classical rates of diffusion.

To summarize, there are seemingly unambiguous examples taken from the history of research on axially symmetric open systems where cross-field transport occurs at rates that are not inconsistent with those expected from classical diffusion. These results are clearly at variance with the turbulence-dominated transport rates consistently exhibited by closed systems.

As was suggested for closed systems, we could also define a "philosophy of research" for open systems, one that could assist us in our search for simpler approaches to magnetic fusion. This philosophy could be stated as: "Finding new ways to exploit the ability of open systems to confine plasma in low-turbulence states, at the same time taking into account their known limitations." In the next section we will discuss these limitations, and some suggested ways to ameliorate them. Underlying this plan of action is the author's conviction that the surest route to "simpler, smaller" magnetic fusion systems is to be found through the investigation of a magnetic field topology that has been shown to permit confinement that is not dominated by turbulent processes.

2. OPEN SYSTEMS: LIMITATIONS AND OPPORTUNITIES

In looking for new avenues to fusion to be based on an open-ended magnetic field topology it is important to understand the weaknesses of this topology, weaknesses that must be overcome en route to utilizing its strengths. The outstanding problem of open systems, known from the start, is that of controlling end losses. A second problem, important if axisymmetric fields are to be employed, is the MHD stability of the confined plasma column. Finally, there is an issue that has been brought up repeatedly in the past, that of electron thermal conduction along the field lines between the chamber ends and the confined plasma. We will discuss these issues in turn, together with approaches that can be taken to deal with them, and concluding with a discussion of the Kinetic Stabilizer concept and its possible role.

2.1 End Losses

Open-ended systems based solely on the use of the mirror effect to control end losses have, from the start, exhibited confinement times that were dominated by these losses. In the usual low-collisionality fusion regimes this end-loss-limited confinement time, marginal from the standpoint of fusion requirements, is of the order of the classical ion-ion collision time, increasing only logarithmically with the mirror ratio (ratio of the field at the mirror throat to the central field). Early attempts to deal with the end-loss problem included the use of direct conversion of the energy of escaping ions (both fusion reaction products and fusion fuel ions) [5]. If the efficiency of direct conversion could be made high enough, the result would be, in effect, to increase the reaction "Q" value of the fusing plasma by an order of magnitude. A better solution, put forth by Dimov [6] and by Fowler and Logan [7] is, of course, the tandem-mirror idea. In the tandem mirror small mirror cells at the ends of a long central cell are used to create a potential well for the plasma ions trapped in the central cell. The electrons of the plasma are trapped at the same time by the overall positive ambipolar potential that arises naturally from the disparate rate of ion-ion and electron-ion collision processes. If it can be implemented practically, confinement by the plasma potentials in the tandem mirror offers an almost ideal answer to the problem of end losses. As the title of this paper suggests, the Kinetic Stabilizer idea is being proposed as a new way to implement the tandem mirror idea.

2.2 MHD Instability of an Axisymmetric Mirror Cell

Suggested early on by Teller, and confirmed theoretically by Rosenbluth and Longmire [8], plasma confined in an axisymmetric mirror cell is subject to MHD instabilities of the "interchange" variety, leading to motion of the plasma column across the confining field.

In later calculations Rosenbluth, Krall, and Rostoker [9] showed that in a sufficiently long plasma column "finite-orbit" effects stabilize all but the lowest order (m = 1) mode of this instability. This mode corresponds to a sideways drift of the plasma column as a whole. The MHD problem was addressed in the famous Ioffe experiment of the 1960s. In this experiment it was shown that the use of non-axisymmetric fields that are characterized at all azimuths by positive field line curvature at the radial boundary of the plasma (something possible only in open systems) suppressed the MHD interchange instability. Following the Ioffe experiment most research groups studying open systems adopted some form of "magnetic well" that incorporated this idea. Although striking results were obtained, including the stable confinement of a fusion-relevant plasma at plasma beta values approaching unity (in the 2XIIB experiment at Livermore [10]), there was a negative consequence: In asymmetric fields there can be classes of particles that exhibit enhanced cross-field transport owing to resonance-enhanced drifts associated with their bouncing motion back and forth between the mirrors. In addition, the complexity of the fields and the magnet coils that generated them, particularly when they were employed in tandem-mirror systems, was another concern.

A way out of this problem is discussed in a paper by Ryutov [11] and the theory presented there was confirmed in the axisymmetric Gas Dynamic Trap experiment [12]. According to the theory an otherwise MHD-unstable plasma confined between mirrors can be stabilized if there exists a sufficient density of effluent plasma on the expanding (positive-curvature) field lines outside the mirrors. Since the GDT experiment operates in a high-collisionality regime in order to enhance its mirror confinement, the effluent plasma density was high enough to stabilize the interior plasma, which had a beta value of 30 percent. Unfortunately, owing to the low effluent plasma density in such cases, this elegant stabilizing method will not work under the lower-density, lower-collisionality regimes in which a tandem-mirror fusion system would be expected to operate. The Kinetic Stabilizer idea, to be discussed, offers a way out of this dilemma.

2.3 Electron-Conduction Heat Losses

Finally, another frequently raised concern about open-ended systems is that of thermal conduction, by the electron population, between the confined plasma and the physical boundaries at the chamber end. If this conduction were to occur at the classical rate there would be no hope for maintain a plasma at fusion temperatures in a mirror or tandem-mirror system. Fortunately there are strong limits on electron thermal conduction that arise naturally and that suppress the conduction by orders of magnitude, to the point that it should not be a concern. The first of these limits is the one imposed by the presence of the positive ambipolar potential of the plasma with respect to the region outside the mirrors. This potential, arising naturally as a result of the disparate collisional rates of the electrons and ions, forces the electron loss rate to equal that of the ions, with a corresponding orders-of-magnitude drop in the rate of energy loss via the electron channel. However, a possible remaining energy exchange mechanism between the confined plasma and the end walls is that of secondary electrons. These could return along the field lines, exchange with the hot electron, and thus cool the plasma. Again theory [13] has shown a naturally arising solution to the problem, confirmed again in an elegant experiment in the GDT [14]. The solution is the following one: If the expansion ratio of the magnetic field out to the end walls exceeds the square-root of the ion-to-electron mass ratio, there will arise naturally a retarding potential for any electrons coming down the field lines from the end walls that will keep these electrons from flowing into the confined plasma. The confirming experiment in the GDT was performed by installing a large electron-emitting thermionic cathode outside the mirror. This cathode was movable axially from a position close to the mirror to one beyond the critical expansion ratio mentioned above. When the cathode was heated and was located near the mirror, its emitted electrons exchanged with the trapped electrons, causing cooling of the confined plasma. When the cathode was retracted beyond the critical distance it had no influence, whether emitting or not. It should be noted that in the design of earlier major mirror experiments the field expansion ratio requirement given above was not satisfied, so that one would expect that secondary electrons from the end walls could have played a role in depressing the electron temperature of the confined plasma.

2.4 Recapitulation

The limitations of open systems have been discussed en route to proposing new fusion systems based on this geometry. The tandem-mirror concept offers a practical solution to the problem of end losses, and the issue of electron thermal conduction has been addressed and shown to be controllable. Finally, to take advantage of the superior confinement that should be obtainable through the use of axisymmetric confining fields, the issue of the MHD interchange mode must be dealt with. As demonstrated in the GDT experiment, stabilization by effluent plasma present on the expanding field lines outside the mirrors is effective in the high-collisonality regime of that experiment. This proven stabilization means could be employed to solve the MHD problem of axisymmetric systems, provided it can be implemented in a practical way in the low collisionality regimes of tandem mirror systems. In the next section we will discuss the Kinetic Stabilizer idea as a means to this end.

3. THE KINETIC STABILIZER CONCEPT

The Kinetic Stabilizer idea evolved from an earlier idea, the Kinetic Tandem [15]. The Kinetic Tandem concept was proposed as a means to utilize the tandem mirror idea in a confining field having the form of a long solenoid whose axisymmetric field is constant in the confinement region, and then decreases uniformly at each end (i.e. no mirrors are employed). The confining field is itself thus MHD-stabilizing, every field line having positive curvature. To create the potential peaks needed to confine the fusion plasma ion beams are injected up the magnetic gradient at the ends. The ion beams are aimed at small angles to the field lines so the ion density is increased greatly (relative to that at the ion sources) by magnetic compression and by the ions being slowed and reflected as they approach the top of the magnetic gradient. These localized density peaks thus generate a positive ambipolar confining potential by the same Boltzmann-like mechanism as in the original tandem mirror concept. Analytical and computer-code calculations were performed that demonstrated the generation of fusion-relevant plasma parameters using realistic ion beam current densities and beam energies. Although the Kinetic Tandem appeared to be capable of becoming a fusion power system, it came at a price: In order to "pay" for the beam power required to maintain the plugs the length of the central solenoid had to be many kilometers, casting doubt on the economic feasibility of the idea.

The Kinetic Stabilizer borrows the ion-beam-produced plasma peak idea of the Kinetic Tandem and marries it to the MHD stabilization technique demonstrated in the GDT experiment. That is, a mirror or tandem mirror system is to be constructed using only circular coils, so as to form axisymmetric mirror cells. Ion beams are to be aimed up the magnetic gradient at the ends so as to form a plasma density peak located partway up the gradient, i.e. in the region of positive field-line curvature outside the outermost mirror. This peaked plasma density then stabilizes the confined plasma by its presence. In this way it resolves the "too-low-an-effluent-density-to-stabilize" problem associated with present tandem mirror systems, as alluded to earlier.

As will be shown, the beam power requirements of the Kinetic Stabilizer are orders of magnitude lower than the beam powers needed for the Kinetic Tandem. This circumstance should allow the design of tandem mirror systems that are far smaller and less expensive than a net-power-producing Kinetic Tandem is projected to be. The most significant point is, however, that the Kinetic Stabilizer offers a way to design mirror or tandem-mirror systems using only axisymmetric fields. If history is a guide, axisymmetric-field systems should show superior performance with respect to greatly reduced turbulence and near-classical cross-field transport. Furthermore, the use of circular coils to create the confining fields should greatly reduce the complexity and cost of the magnet system, including the achievement of higher mirror fields than would be feasible with the complex-geometry coils typically being employed in present tandem-mirror systems.

3.1 The MHD Stability Criterion

For an axisymmetric field the criterion for MHD stability against the interchange mode can be expressed as an integral condition [11]. The integrand of this integral is the plasma pressure multiplied by product of the second derivative of the radius of the plasma and the cube of the plasma radius. The integral is to be performed over the entire length of the system between its physical ends. The condition is as follows:

$$I = \int_{-L}^{L} a^3 \frac{d^2a}{dr^2} \left(p_{perp.} + p_{par.} + \rho \langle v^2 \rangle\right) dz \qquad (1)$$

In the present context stability will be achieved if the positive contribution to the integral from the Kinetic Stabilizer plasma created outside the mirrors exceeds the negative contribution to the integral that arises from the plasma contained between the mirrors. Because of the strong weighting of the stability integral (by the radius cubed and by the positive curvature of the lines) in the expander region outside the mirror, it follows that the pressure of the Stabilizer plasma can be much lower than the central plasma pressure and yet can still be effective in stabilization. Also, because the Kinetic Stabilizer plasma can be created at a chosen location on the magnetic gradient, and because the expander field can be tailored so as to maximize its curvature at the same location, the possibility for optimizing the stabilization action exists. This optimization will be illustrated in examples given later.

3.2 Evaluation of the Stability Integral in the Central Cell and in the Expander

The eventual utility of the Kinetic Stabilizer idea in MHD-stabilizing an axisymmetric mirror-based system will depend on quantitative and economic factors. That is, it will depend on optimizing the ratio of the fusion power produced in the confined plasma to the beam power required to stabilize that plasma. The computations involved in determining this ratio involve two separate determinations. The first of these is to evaluate the stability integral, Equation 1, for the plasma within the confinement region. In a tandem mirror this first evaluation would involve contributions from both the central cell and the end cells containing the potential-forming "plug" plasmas. The result would be a negative numerical quantity reflecting the MHD-unstable character of these plasmas. The second computation would be the evaluation of the stability integral taking into account the presence of the Kinetic Stabilizer plasma in the expander region. Here the computation will also involve two kinds of determinations. The first of these is to calculate the pressure and spatial extent of the plasma peak that is formed by the magnetic compression and stopping of the ion beams injected at the end of the expander region. The second determination will be to fold the computed pressure profile of the Kinetic Stabilizer plasma into the stability integral, Equation 1, to determine the positive contribution to the integral from the presence of the Kinetic Stabilizer plasma. Comparing the two results, that is, the negative term from the confined plasma, and the positive term from the Kinetic Stabilizer plasma, one can determine the beam power required to stabilize the fusing plasma and compare this power with the calculated fusion power output. As will be shown, by optimizing the expander geometry and the beam injection angles the calculations indicate the possibility of stabilizing a 100 Megawatt-level fusion power system using Stabilizer beams whose power requirements are very small compared to this power.

The analytical and computer-code-based computational methods for performing the above-indicated calculations are detailed in previous papers on the Kinetic Tandem and the Kinetic Stabilizer. In this paper we will present only the results of calculations that have been performed using these methods. Two examples will be presented. The first of these, taken from a previous paper [16], is that of a single mirror cell containing a fusion plasma, stabilized by beam-produced plasmas located at each end. A simple system such as this one might be useful as a neutron source for materials testing.

The second example will illustrate the stabilization of the plugging plasma of a tandem mirror system based on the original tandem-mirror concept of Dimov and Fowler and Logan. In this first and simplest form of the tandem mirror, the plugging potentials are generated simply by maintaining the density of the plasmas in the end cells at a substantially higher value than the density of the central plasma. The first U.S. tandem-mirror experiments [17] were performed using this approach and it was found that the end-loss plugging that was observed agreed closely with that predicted by theory. However, in part owing to the complexity of the non-axisymmetric fields that had to be employed in these experiments, the implied limitations on the fusion power density that characterized such systems was felt to be a severe economic constraint. As a result, the idea of the "thermal barrier" [18] was introduced as means to allow operation at higher central–cell plasma densities relative to the end-cell density. The price that was paid was a still further increase in the complexity of the magnetic fields and of the plasma technology required to implement the thermal barrier concept. What we are proposing here is to employ the Kinetic Stabilizer in conjunction with the use of axisymmetric fields and magnet coils in tandem mirror systems. In this way it may be possible to increase the magnetic field strength and reduce the cost of the magnet coils of the end cells to the point that tandem mirror systems based on the original simple concept could compete economically. Our second example will explore this possibility, building the results of previous paper studies of fusion power plants based on the tandem mirror idea.

4. EXAMPLE I: SINGLE MIRROR CELL WITH KINETIC STABILIZERS

This example was discussed in an earlier paper [16] and will only be reviewed briefly here. The system consists of a long central cell at each end of which is an expander region. The Kinetic Stabilizer plasma is formed partway up the magnetic gradient, chosen so as to maximize its stabilizing effect. Farther down the magnetic gradient there are located the ion sources that launch ions up the magnetic gradient. In transit up the gradient these ions are magnetically compressed, slowed, and then reflected, thereby producing the Stabilizer plasma peak. The ions are launched at injection angles chosen optimally to produce the Stabilizer plasma at the chosen location.

As noted previously, this example illustrates how the Kinetic Stabilizer concept might be used in conjunction with a simple mirror cell for the purpose of generating a high flux of 14 MeV neutrons for materials testing. It is not presented as a net power-producing fusion system. It will illustrate, however, the fact that the power required by the Kinetic Stabilizer beams in such a system is small compared to the fusion power output. Table I summarizes the main parameters of Example I.

Table I

Fusion Parameters of Example I

Central cell magnetic field	5.0 Tesla
Mirror ratio of central cell	2.0
Fusion plasma ion density (50-50 DT)	6.2×10^{20} m^{-3}
Ion and electron temperature of central-cell plasma	15 keV
Beta value of central-cell plasma	0.3
Length of fusion plasma	25.0 m.
Diameter of fusion plasma	0.15 m.

For this example a simple form for the flux surface of the expander was assumed, as calculated in the paraxial approximation [19] for a magnetic field that has a Gaussian variation with z at r = 0, i.e. $B(z) = B_0 \, Exp[-(z/z_0)^2]$, where B_0 is the strength of the magnetic field at the mirror. Figure 1 shows the calculated shape of the expander flux surface, with an initial radius (at the mirror) of .05 m. The characteristic distance, z_0, in this case is 1.0 m.

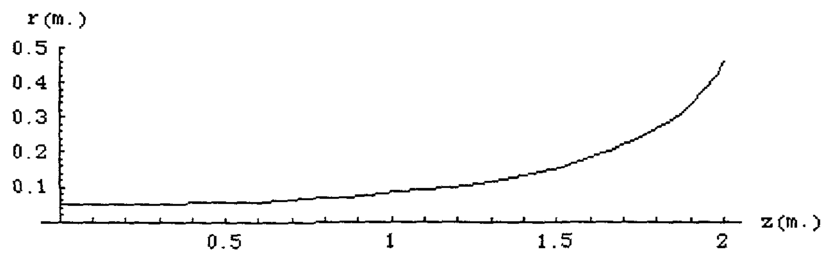

Figure 1: Expander flux surface calculated for a Gaussian field variation

The parameters of the Kinetic Stabilizer are summarized in Table II below:

Table II

Parameters of the Kinetic Stabilizer of Example I

Mean diameter of Kinetic Stabilizer plasma	1.0 m.
Area of Kinetic Stabilizer ion source region	4.0 m^2
Energy of Kinetic Stabilizer ions (Cs^+)	1.0 keV

With the above set of parameters and assumptions the calculated D-T fusion power released was 45 MW, to be compared with a total Kinetic Stabilizer beam power (both ends) of 200 kW. The fact that the power required to maintain the stabilizing plasmas is so small relative to the fusion power output comes directly from the strong dependence of the stabilization on the positive curvature and the radial expansion of the field lines in the expander region. In the second example we will show how it is possible to optimize the shape of the flux surfaces in the expander region so as to take even further advantage of this scaling. This example will involve the stabilization of the end cells of a tandem-mirror system whose parameters could make it a serious contender for a fusion power generating system.

5. EXAMPLE II: A NEW LOOK AT AN OLDER TANDEM-MIRROR FUSION POWER SYSTEM

Our second example will explore the possibility of implementing the original tandem-mirror concept in an axisymmetric magnetic field. The use of circular coils to produce the mirror and confining fields should allow substantially higher magnetic fields and a smaller plasma volume in the end cells as compared to tandem-mirror systems employing non-axisymmtric fields. Thus, in addition to reducing cross-field transport, the use of axisymmetric fields should ameliorate the economic penalty of having to maintain an electron density in the end cells that is an order of magnitude higher than that in the central cell. This density requirement must be met in order to implement the original form of the tandem mirror. In this example we will present a first look at this new kind of tandem mirror system.

In presenting this example we will utilize some of the results of previous paper studies of fusion power systems based on the tandem-mirror concept. One such study, performed in 1985, was dubbed the "MINIMARS" study [20], following as it did an earlier study, the MARS fusion power plant [21]. MINIMARS was aimed at determining if the tandem mirror idea could be utilized in a smaller-sized fusion power plant than previous studies. It was targeted at achieving an electrical power output of 600 Mwe. Its smaller size was achieved in part by redesigning the end-cells, replacing the quadrupole field coils by octupole fields. However the limitations implicit in use of these multipole fields ruled out the employment of the original tandem-mirror idea that would have required maintaining plasmas in the end cells at a higher density than that of the central cell plasma. Instead MINIMARS, as did MARS, assumed the use of a "thermal barrier" [18] that would inhibit the exchange of energy between the central cell electrons and those in the end-cells. This requirement added complexity and introduced the need for many megawatts of microwave power to heat the electrons in the thermal barrier and end cell region. Our example II will replace all the coils in MINIMARS with circular coils, eliminate the need for a thermal barrier, and provide MHD stabilization by employing Kinetic Stabilizers.

In Table II there is given a brief review of the fusion-related parameters of MINIMARS which we will carry over into our Example II.

Table II
Fusion Parameters of MINIMARS

Fusion Power (MW)	1200
Electrical power output (MWe)	600
Neutron wall loading (MW-m^{-2})	2.7
Central-cell magnetic field (Tesla)	3.0
Choke coil field (Tesla)	26.0
Length of central cell (m.)	95.
Plasma radius (m.)	0.42
Ion temperature (keV)	30
Plasma beta	0.6
Mirror ratio of central cell (beta-enhanced)	13.7

In addition to the above parameters, an important parameter of MINIMARS was the microwave and beam power inputs required to produce the thermal barriers and to maintain the plasmas in the end cells. This power input came to a total of 15.5 MW.

5.1 Stabilizing a High Electron Density Tandem-Mirror End Cell

In considering the destabilizing terms to the MHD stability integral, Equation 1, that are contributed by the central cell and the end cells of our axisymmetric tandem-mirror system, it becomes clear that the main contributions will be made from the end cells. This circumstance has two origins. First, the plasma pressure in the end cells will be higher than that in the central cell because of its higher density and its higher mean ion energy. Second, to minimize the plasma volume in the end cells relative to the central cell plasma volume the length of these cells must be much shorter than that of the central cell. Owing to the contribution to the integrand from the negative field line curvature, the stability integral will have a larger negative value in a short cell than in a longer one. Our main concern will therefore be to stabilize the end cells. If this result is accomplished the negative contribution to the stability integral from the central cell can always be reduced relative to the end cells by increasing its length sufficiently, something that is already required in order to achieve a favorable power balance.

Considering these circumstances it is clear that in designing our new tandem mirror it is necessary to reach a compromise between opposing demands. That is, to reduce the plasma volume in the end cells they should be made as short as possible. However, shortening the cell will have the effect of increasing its negative contribution to the stability integral, requiring therefore a larger positive contribution from the Kinetic Stabilizer plasma, hence a higher beam power will be required to form the Stabilizer plasma. A reasonable compromise would be one where the power required to maintain the Stabilizer plasma is of the same order as that required to maintain the plasma in the end cells, with the expectation that the fusion power released in the much-larger-volume central cell would be large compared to either of these power requirements.

The generation of a potential barrier for the central cell ions in our present example depends on the existence of an electron density in the end cell that is higher than the electron density of the central cell plasma. When this is the situation the ambipolar potential of the end cells will be higher than that of the central cell in accordance with the Boltzmann relationship:

$$e\phi_{plug} - e\phi_{c.\ cell} = kT_e \log_e \left[\frac{n_e(plug)}{n_e(c.\ cell)} \right] \qquad (2)$$

This circumstance presents us with another compromise to be made between the power required to maintain the ions in the plugs against mirror losses and the beam power required to generate the Kinetic Stabilizer plasma. First, to reduce the power losses associated with mirror losses from the end cells it is desirable to maintain the ion temperature in these cells at a higher value than that of the fusing plasma. However, higher ion temperatures correspond to higher ion pressures, and thus to the need for a higher pressure Stabilizer plasma. These facts suggest that there may be an optimum choice of ion species for the plugs, one which maximizes the electron density in the end cells while minimizing the ion pressure and at the same time has an acceptable rate of power loss through mirror losses. For mirror losses arising from ion-ion collisions the scaling law of the power losses is given by the equation:

$$P_{loss} = C_{F.P.} \left(Z^4 / A^{1/2}\right) E_i^{-1/2} \int n_i^2 dV \qquad \text{Watts} \qquad (3)$$

Here $C_{F.P.}$ is a constant determined by solving the Fokker-Planck equation, Z and A are the atomic number and atomic weight of the ions, respectively, and E_i is the ion energy. The integral of the square of the ion density, n_I, is to be performed over the volume of the bounding flux surface between the mirrors.

We note now that the plugging action of the end cells depends on the ratio of the electron density of the end cell to that in the central cell. Thus, if we assume that the end-cell ions are fully stripped the electron density will be equal to Zn_i. Thus we can rewrite Equation 3 as follows:

$$P_{loss} = C_{F.P.} \left(Z^2 / A^{1/2}\right) E_i^{-1/2} \int n_e^2 dV \qquad \text{Watts} \qquad (4)$$

On the basis of this result it appears that some possible choices for the ions in the end cells are D^+, for which $Z^2/A^{1/2} = 0.707$, protons, for which $Z^2/A^{1/2} = 1.0$, and He-4^{++}, for which $Z^2/ A^{1/2} = 2.0$. For the present example we will choose D^+, however recognizing that there may be cases where the use of protons or helium ions (for example, to reduce the neutron production in the end cells) could be advantageous.

Finally, there is one additional choice to be made in optimizing the end cells, namely the mirror ratio, since the end losses depend inversely on the logarithm of the mirror ratio. On the other hand, the negative contribution to the stability integral from the end cell increases with the mirror ratio. As a result a compromise value is called for, which we estimate to lie between R = 2.0 and R = 3.0.

5.2 Optimizing the Expander

As mentioned earlier, the scaling of the stability integral, Equation 1, with plasma radius and radius of curvature, suggests the possibility of optimizing the flux surfaces in the expander so as to maximize the contribution of the Kinetic Stabilizer plasma. This optimization is facilitated by the fact that the location of the Stabilizer plasma can be pre-determined by adjustment of the angle of injection and angular spread of the ion beams from the Stabilizer ion sources. This being the case it appears that an optimum shape for the expander could be one with the following characteristics:

Upon leaving the mirror the flux surface is conical in shape (zero second derivative). After expansion of the field to a value somewhere between the field value at the mirror and the chosen field value at the location of the ion sources, the conical angle changes to a larger angle, with a sharply outwardly curving (large second derivative) transition region between the two. The ion sources, located farther out, on the second conical region, are aimed so as to concentrate the Stabilizer plasma in the transition region. When this optimized geometry was compared to the simple Gaussian-expander geometry of Example I an increase in the stabilizing effect by about an order of magnitude was found. Figure 2 is a computer-generated representation of an expander flux surface having the above-described optimized shape.

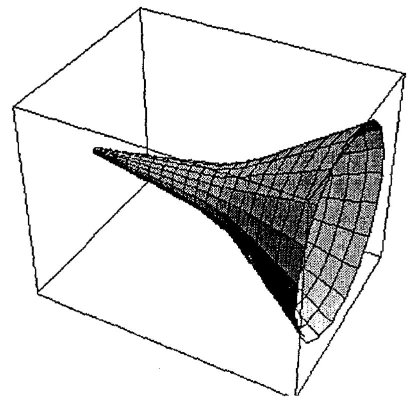

Figure 2: Computer-generated representation of an optimized expander flux surface

5.3 Evaluating the Stability Integral for the End Cells

The first step in the process of determining the parameters of a Kinetic Stabilizer for a tandem-mirror end cell is to evaluate the stability integral, Equation 1, within an end cell and to determine the scaling of the results found as a function of the length and volume of plasma in the end cell.

Before the stability integral can be evaluated it is necessary to determine the axial profile of the plasma pressure in the end cell. For the evaluations here this variation was determined by employing the "normal-mode" profile as determined by Fokker-Planck analysis of mirror confinement. Using an eigenvalue obtained by variational methods [22], the axial profile of the plasma density was determined as a function of magnetic field between the midplane and the mirrors in the end-cell. A transformation of variables was then performed, using an analytic form for the mirror fields in terms of trigonometric functions and Bessel functions [23]. Figure 3 shows the computed plasma density distribution in a mirror cell with a mirror ratio of 2:1 and a length between the mirrors of 3.0 meters.

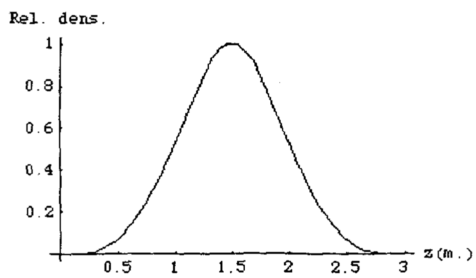

Figure 3: Plasma density distribution in end-cell with mirror ratio 2.0 and length 3.0 m.

The stability integral was calculated by numerical integration of the same analytic expression that was used above to calculate the mirror cell fields, as weighted by calculated plasma density distributions, an example of which is shown in Figure 3. The results of these calculations are depicted in Figures 4 and 5 below. Figure 4 is a plot of the absolute value of the stability integral as a function of the length of the cell, for a cell with a mirror ratio of 2:1. Figure 5 is a plot of the value of the stability integral as a function of mirror ratio, for cells with a length of 3.0 meters. For these plots the plasma radius at the mirror throat has been assumed to be 0.1 m.

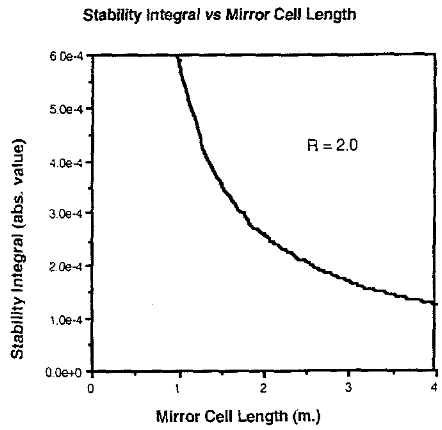

Figure 4 Absolute value of stability integral as a function of mirror cell length, mirror ratio 2.0.

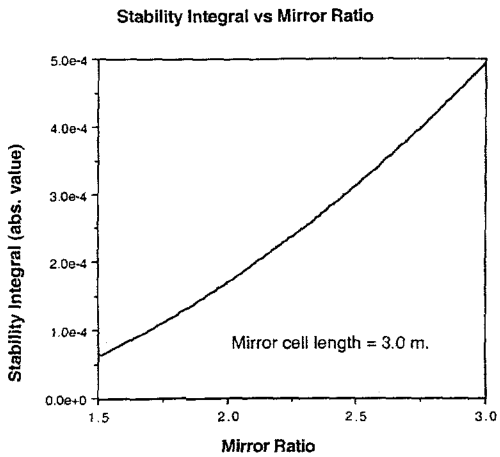

Figure 5: Absolute value of stability integral as a function of mirror ratio

The information on these plots was used to select a set of parameters for the end cells of the tandem-mirror system of Example II.

5.4 Evaluating the Stability Integral for the Expanders

A version of the conical-geometry expander of the type shown in Figure 2 was employed in evaluating the contribution to the stability integral from the beam-formed Stabilizer plasma of Example II. An optimized case was found for beams injected with angles of injection spread between 10° and 18°. To perform the calculation a source angular distribution, approximating that from a real ion source, was used. This angular distribution is shown in Figure 6.

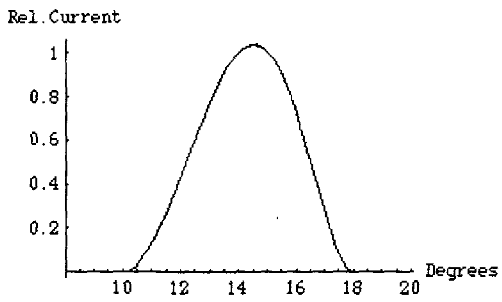

Figure 6: Kinetic Stabilizer ion-source angular distribution employed in Example II

The code used to evaluate the stabilizing contribution to the stability integral first calculates the density distribution of the Stabilizer plasma that is produced on the expander magnetic field, upstream of the ion sources, at the location where the second derivative of the flux-surface radius has its maximum value. This density distribution is shown in Figure 7. Note the increase in density caused by the magnetic compression.

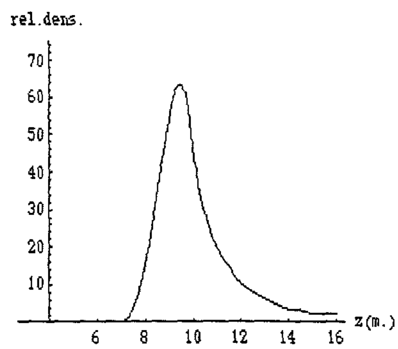

Figure 7: Magnetically compressed Stabilizer plasma density distribution vs position, produced by Stabilizer ion beams with the angular distribution shown in Figure 6.

The Stabilizer plasma density distribution shown in Figure 7 was then used as input to the code to calculate the positive contribution to the stability integral made by the Stabilizer plasma. The answer obtained therefore represents a value that is normalized to unit ion pressure at the location of the sources. This number, together with the area of the sources, can then be used to evaluate the required total Kinetic Stabilizer ion beam current needed to stabilize a given plasma in the end cell.

The code results gave a value for the value of the stability integral of +48.4. Comparing this value with the typical (negative) values of the stability integrals as shown in Figures 4 and 5, whose absolute values are of order 10^{-4}, it can be seen that the kinetic ion pressures at the surface on which the ion sources are located can be nearly six orders of magnitude smaller than the plasma pressure in the end cell and still be effective in stabilizing that plasma

The final input from the code needed to evaluate the Stabilizer beam power is the outer radius of the surface on which the ion sources are to be located. From the calculated shape of the optimized flux surface this radius was found to be 4.48 meters. This dimension, plus the other results from the code, can now be used to evaluate the specific tandem mirror system of Example II.

To evaluate the positive contribution to the stability integral from the Kinetic Stabilizer ion beams it is necessary to choose the type of ion to be injected and the energy at which these ions are to be injected. Since the only requirement from the Kinetic Stabilizer system is that it should produce a given plasma pressure at a chosen location on the expander the choice of injected ions can be made on the basis of minimizing the beam power required to produce this plasma. It is therefore advantageous to choose heavy ($A>>1$) ions of moderate energy to produce the Stabilizer plasma as these ions will require less power to maintain the plasma than would lighter, higher-energy, ions. Among possible ions would be heavy noble gas ions, such as Xenon ($A = 131$), or an alkali metal such as Cesium ($A = 133$). For Example II we will choose the latter ion, easy to produce by thermal means.

A further consideration in evaluating the positive contribution to the stability integral of the Stabilizer plasma is its location and radial width. As pointed out by Ryutov [24], in an axisymmetric mirror cell containing a plasma whose radial profile is relatively flat out to its boundary, the negative contribution to the stability integral, Equation 1, arises only from the region where there is a negative density gradient, i.e., from the boundary. In the absence of a detailed MHD code estimation of this effect, we will here assume that the Stabilizer plasma need only be maintained over the outer 10 percent of the transverse area of the Stabilizer plasma. This circumstance implies that only the outer 10 percent of the area of the surface on which the sources are located need be filled with ion sources. This radius being 4.48 m. the area occupied by ion sources is therefore equal to 6.3 m^2.

5.5 Evaluating the End-Cell Parameters.

As was noted earlier, in designing the new tandem mirror system it is necessary to satisfy competing requirements. Specifically the challenge is to find a combination of dimensions and plasma parameters for the end cells that satisfies the potential requirements for tandem-mirror confinement and at the same time leads to power requirements for maintaining the end-cell plasma and the Kinetic Stabilizer plasma that are an acceptably small fraction of the fusion power output of the central cell. To perform this kind of optimization with sufficient rigor would require the exercise of much more sophisticated computer codes than the codes employed here. What follows therefore should be considered only as a first cut at such a design, one mainly aimed to establish an initial level of credibility for a kinetically stabilized tandem-mirror fusion system.

The fusion parameters of The MINIMARS fusion power system, which are to be carried over into the new design, define the requirements for potential plugging. In the tandem mirror system of Example II this requirement is to be met by increasing the electron density of the end cells over that in the central cell by a sufficient factor. This circumstance defines the required electron density in the end cells. For this example this density is to be 1.0×10^{21} m^{-3}. This density is a factor of five higher than the electron density of the plasma in the central cell so that it will lead to a plasma potential that is approximately a factor of $\log_e(5) = 1.6$ higher than the electron temperature in the central cell, as appropriate to this form of tandem-miror system. From the theory of potential confinement of ions in a tandem-mirror system [25], and assuming $T_I = T_e$ in the central cell, this value of the depth of the potential well would lead to an enhancement of the ion confinement over the confinement in a simple mirror cell by a factor given approximately by the relationship: $(e\phi/kT_e)\mathrm{Exp}(e\phi/kT_e) = 8.1$. This factor, taken together with the beta-enhanced mirror ratio of the central cell, should lead to a Q-value for the system that is acceptably high compared to unity, provided the power loss from the end cells is sufficiently low. As will be shown later, this requirement should be achievable with the new system.

This first, simplest, form of the tandem mirror that we are considering functions more effectively if the mean kinetic energy of the ions in the end cells is substantially higher than that of the ions in the central cell. In this way the power required to balance the mirror losses of the end cells is lowered, as indicated in Equation 4. However, higher ion energy in the end cells implies a higher plasma pressure that must be stabilized by the Kinetic Stabilizer, requiring that a compromise value be found. For Example II we will assume D^+ ions at a mean ion energy in the end cells of 100 keV.

Our next assumption for the end-cell parameters is that the end-cell mirror ratio should be 2:1 and that the cell length should be 3.0 m. For the contributions to the stability integral associated with these choices see Figures 4 and 5. Finally, we need the value of the n^2-weighted volume of the end cell for these parameters for insertion into Equation 4. From the code this value is .0465 m^3.

Given these values we can now estimate the power requirements of the end cells associated with mirror losses from those cells. The Fokker-Planck coefficient in Equation 4 was set at a value appropriate for a beam-injected plasma confined in a mirror cell with a mirror ratio of 2.0. In terms of the n^2-weighted volume of the end cells this power loss is given by Equation 5 below.

$$P_{End\text{-}loss} = 1.77 \times 10^{-34} \int n_i^2 dV \quad \text{Watts} \qquad (5)$$

The peak ion density in the end cells is set at 1.0×10^{21} m^{-3}. Inserting this value and the n^2-weighted volume of .0465 m^3 into Equation 5, the end-loss power from each end cell is found to be 8.2 MW, for a total loss power of 16.4 MW from the two end cells. This loss power is comparable to the microwave and beam power that was required to maintain the thermal barriers and the end-cell plasmas in MINIMARS, suggesting that the new system should require about the same power as MINIMARS to create its confining potentials, provided the power required to maintain the Kinetic Stabilizer plasmas is reasonably small. In the next section we will evaluate this power, inserting the end-cell plasma parameters and the stability integral contributions.

5.6 Estimating the Kinetic Stabilizer Beam Power Requirements

In this section we will present an example calculation of the beam power requirements for the Kinetic Stabilizer which employs Cs^+ ions in its ion sources. These sources are presumed to be located at the outer end of the expander field, occupying the outer 10 percent of the end area. As noted earlier, since the radius of this surface is 4.4 meters, this area is 6.3 m^2. From the relative values of the stability integral, i.e. 48.4 for the Kinetic Stabilizer per unit kinetic pressure at the source plane, and 1.7×10^{-4} per unit peak pressure in the end cells, the required pressure ratio between the expander and the end cell is equal to 3.51×10^{-6}. We will assume that the energy of the Cs^+ ions of the Kinetic Stabilizer beams is 500 eV, and we will ignore the pressure of the neutralizing electrons in estimating the beam power required for stabilization.

With the above assumptions the kinetic pressure, $\rho\langle v^2\rangle$, at the sources is equal to .0357 Pascals per Ampere/m^2 of accelerated Cs^+ ions. Inserting the required pressure ratio between the sources and the end cell, this figure translates to the stabilization of a peak pressure of 1.02 x 10^4 Pascals per Ampere/m^2 of Stabilizer ions. Since the peak plasma pressure in the end cells is equal to 1.6 x 10^7 Pascals, the required current density is 1.57 x 10^3 Amperes/m^2 (157 ma/cm^2). At an ion energy of 500 eV this current density corresponds to a beam power of 790 kW/m^2. Since the total source area is 6.31 m^2, the Stabilizer beam power per cell is 5.0 MW, or about 60 percent of that required to maintain the end-cell plasmas. The total beam power required, i.e., end-cells plus Kinetic Stabilizer beams, even assuming no recovery of the energy of either beam ions, is thus of the same order as the power that was required to maintain the thermal barriers and the end cells of MINIMARS.

5.7 Recapitulation

In Example II we have presented a first-cut at the design of a tandem mirror fusion power system whose fusion parameters are essentially the same as those assumed in the MINIMARS study, where the target was to design a fuson power plant with a power output of order 500 MWe. We have shown that employment of the Kinetic Stabilizer concept should allow the use of axisymmetric magnetic fields throughout, generated by solenoids and circular coils. This latter circumstance was shown to facilitate a return to the first-proposed, simpler, form of the tandem mirror idea, one for which the plasma physics issues are well understood, and one which did not require thermal barriers for its operation. It was shown that the power required to maintain and MHD-stabilize the end cells was small compared to the fusion power output, being of the same order as the power required to maintain the thermal barriers and the end-cell plasmas in MINIMARS. As was the case with the MINIMARS study, it has been implicitly assumed that the cross-field particle transport in the central cell and in the end cells is small enough to be ignored in calculating the power balances. This assumption, while seemingly supported by the earlier-cited experiments with open systems, would clearly have to be addressed and verified if tandem mirror systems based on the use of the Kinetic Stabilizer concept are to be pursued further.

In the next section we will discuss other aspects of the Kinetic Stabilizer concept and possible means for optimizing its effectiveness.

6. FURTHER COMMENTS ON THE KINETIC STABILIZER AND ITS APPLICATIONS

The previous examples of the use of the Kinetic Stabilizer concept in mirror and tandem-mirror fusion power systems have been given mainly to show that its use can open up promising new possibilities for these types of systems. However, in applying the concept to real fusion systems it will be important to optimize its performance to meet economic goals. We have previously mentioned the optimization possible through the design of the expander fields. We list below some possible additional ways to optimize the efficiency or increase the effectiveness of the Kinetic Stabilizer.

1. Form the Stabilizer plasma peak closer to the mirror by aiming the ion beams more nearly parallel to the field lines in the expander. In this way a local potential peak could be formed outside the mirror in a region of high positive field-line curvature. "Warm" plasma trapped between the mirror and this potential peak then would provide an additional MHD stabilizing effect.
2. Employ microwave power to heat the neutralizing electrons of the Kinetic Stabilizer plasma so as to increase its stabilizing effect.
3. Employ gridded direct converters to recover energy from the once-reflected ions of the Kinetic Stabilizer plasma so as to reduce the power requirements..

4. Utilize the ponderomotive force of r.f. fields to enhance the confinement of the Kinetic Stabilizer plasma on the expander field. The fact that the Stabilizer plasma can be of very low density and still be effective may mean that this early-proposed confinement technique could be put to a new use.

Finally, it should be noted that the fact that the Kinetic Stabilizer should only be required to be maintained on the outer portion of the flux surfaces of the expander field. This circumstance means that the inner flux surfaces will be unencumbered and therefore it should be possible to incorporate direct converters at the ends of the system to recover energy from escaping fuel ions or charged reaction products, thereby increasing the fusion "Q" of the system.

7. CONCLUSION

The thesis underlying this paper is that there is a great need to find simpler, smaller approaches to magnetic fusion power systems than those represented by closed-magnetic-field systems such as the tokamak. Fifty years of magnetic fusion research have shown that closed systems, without known exception, have confinement times that are dominated by the effects of plasma turbulence. This fact, in turn, dictates that these systems should be large and that the pressure of their confined plasmas must be kept small relative to the energy density of the confining fields in order to achieve fusion-relevant confinement times.

The other lesson that we have learned in fifty years of fusion research is that magnetic fields of open topology, especially those whose fields are axisymmetric, are capable of confining fusion-relevant plasmas, at plasma energy densities approaching that of the confining magnetic fields, for times approaching "classical" values. That is, plasma turbulence, if present at all, can drop to such low levels as to have little influence on the confinement. The issue then becomes whether it is possible to overcome the main problem of open systems, namely end losses, so that the inherent advantages of a confinement geometry that is not dominated by turbulence can be exploited. The tandem-mirror approach offers a practical answer to the end-loss problem, especially if that approach can be implemented in an axisymmetric form. The Kinetic Stabilizer concept offers a possible avenue to this implementation, provided that the calculations that have been presented here are confirmed by further analyses and proof-of-principle experiments.

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