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A Users Manual for the Nonlinear Kinematic Hardening Model for Cyclic Loading

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PNGV Technical Report

by

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ABSTRACT

This report describes the implementation of the Chaboche type Nonlinear Kinematic Hardening Model developed for the PNGV SPP (Partnership for the Next Generation Vehicle, Spring-back Predictability Project). The material model includes a nonlinear kinematic and isotropic hardening law, transverse anisotropy, strain range memorization for cyclic hardening/softening and viscoplasticity. This report is a companion to the report: "A Return Mapping Algorithm for Cyclic Viscoplastic Constitutive Models" which concentrates on the theoretical aspects of the model. This report summarizes the necessary parameters for the model, briefly discusses their interpretation and shows some numerical simulations. The report also specifies the data structure requirements for linking the material model software by explicitly referencing the source code delivered to the SPP collaborators.

Introduction

In order to more accurately represent the springback behavior in many sheet forming processes, it has appeared necessary to incorporate some form of nonlinear kinematic hardening to capture phenomena such as the Bauschinger effect. Many uniaxial experiments on the sheet metals commonly used in the automobile industry also support this notion. In this report, a material model developed for the PNGV SPP project that incorporates the following phenomena is described:

1. nonlinear kinematic hardening
2. nonlinear isotropic hardening
3. transversely anisotropic elastic/plasticity
4. power law viscoplasticity
5. strain range memorization

This report describes the necessary parameters to be input for the model and presents the results of the NUMISHEET 93 U-Channel with mild steel using a 2.45 K and 19.6 K binder force. The material model was delivered to the SPP group as a FORTRAN code module in *source code* form so that the collaborators would have an opportunity to modify it at there disposal. The necessary data structures for the material model implementation are also specified in this report.

The document "A Return Mapping Algorithm for Cyclic Viscoplastic Constitutive Models" [1] was also submitted as a report to the PNGV SPP group and is currently in press with the journal *Computer Methods in Applied Mechanics*. The aforementioned document will hereon be referred to as the theory manual. Whereas the presentation herein is a functional description, the theory manual describes the theoretical nature of the material model such as the intrinsic positive dissipation, the *implicit* algorithm for the return mapping and the development of a fully linearized *consistent* tangent.

This report presents the synopsis for the material model theory in Section 1. In Section 2, some results are compared to the work by Zhao [2] for verification of the LLNL model and his work. In Section 3, the results for the Numisheet 93 U-Channel for fully isotropic and fully kinematic idealizations are compared. In the Appendix, the hardcopy of the material model is included for reference. A detailed description of the necessary data structures for implementation is also included at the beginning of the hardcopy.

1. Material Model Description

Yield Surface

This material model can represent the cyclic elasto-viscoplastic behavior of many metals. The model has return mapping algorithms for doing 3D plasticity for brick elements, 2D plane stress plasticity for shells and 1D uniaxial plasticity for beams. The shell model also includes the transversely isotropic behavior often encountered in rolled sheets. The different plasticity theories are manifested in the equation for the yield surface in the following fashion

$$f = \frac{1}{2}\eta^T \boldsymbol{P}\eta - \frac{1}{3}\sigma_y \qquad (1)$$

where the reduced stress is given $\eta = \sigma - \boldsymbol{X}$, $\boldsymbol{X}$ is the backstress, σ_y is the yield stress and the projection matrix $\boldsymbol{P}$ operates on the reduced stress so that the proper stress invariant for 1D,2D and 3D results. The following forms for $\boldsymbol{P}$ are given

$$\boldsymbol{P} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix} \text{ for 3D, } \boldsymbol{P} = \frac{1}{3}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix} \text{ for 2D plane stress.} \qquad (2)$$

Addition forms for $\boldsymbol{P}$ are given in the theory manual. For example, with 2D plane stress, σ_{33}, τ_{23} and τ_{31} are zero. This would give the usual (matrix) form for the 2D elasticity tensor

$$\boldsymbol{C} = \begin{bmatrix} \lambda + 2\mu & \lambda & \\ \lambda & \lambda + 2\mu & \\ & & 2\mu \end{bmatrix} \qquad (3)$$

Expanding (1) for plane stress using $(2)_2$ and assuming there is no backstress $(\boldsymbol{X} = 0)$ yields the usual J_2 form for deviatoric plasticity

$$f = \frac{1}{3}(\sigma_{11}^2 - \sigma_{11}^2\sigma_{22}^2 + \sigma_{11}^2 + 3\tau_{12}^2) - \frac{1}{3}\kappa^2 \qquad (4)$$

The plastic strain is based on the associative flow law and is therefore found from (1) as such

$$\dot{\varepsilon} = \dot{\lambda}\frac{\partial f}{\partial \sigma} = \dot{\lambda}\boldsymbol{P}\eta \qquad (5)$$

The utility of using (1)-(5) for 2D plane stress is that the only unknown strains in the problem are the in plane strains ε_{11}, ε_{22} and γ_{12}. In the past, 3D models where used for 2D plane stress plasticity and all 6 strains had to be computed with the constraint σ_{33}, τ_{23} and τ_{31} = 0 explicitly enforced.

Transverse Isotropy

The anisotropic model extension was not included in the theory manual and will be briefly discussed. For orthotropic plasticity, Hill's criterion reads

$$F(\sigma_{22}-\sigma_{33})^2+G(\sigma_{33}-\sigma_{11})^2+H(\sigma_{11}-\sigma_{22})^2+2L\tau_{23}^2+2M\tau_{31}^2+2N\tau_{12}^2-1=0 \qquad (6)$$

The yield stresses are given by $\sigma_{y1}, \sigma_{y2}, \sigma_{y1}, \tau_{y12}, \tau_{y23}$ and τ_{y31} and are related to the constants *F*-*N* as such

$$F=\frac{1}{\sigma_{y2}^2}+\frac{1}{\sigma_{y3}^2}-\frac{1}{\sigma_{y1}^2}, \quad G=\frac{1}{\sigma_{y3}^2}+\frac{1}{\sigma_{y1}^2}-\frac{1}{\sigma_{y2}^2}, \quad H=\frac{1}{\sigma_{y1}^2}+\frac{1}{\sigma_{y2}^2}-\frac{1}{\sigma_{y3}^2} \qquad (7)$$
$$2L=\frac{1}{\sigma_{y23}^2}, \quad 2M=\frac{1}{\sigma_{y31}^2}, \quad 2N=\frac{1}{\sigma_{y12}^2}$$

For isotropy $6F=6G=6H=2L=2M=2N=3/\sigma_y^2$. For transverse isotropy, properties do not vary in the 1 and 2 planes such that $\sigma_{y1}=\sigma_{y2}=\sigma_y$ and

$$2F=2G=\frac{1}{\sigma_{y3}^2}, \qquad 2H=\frac{2}{\sigma_y^2}-\frac{1}{\sigma_{y3}^2}, \qquad 2N=\frac{4}{\sigma_y^2}-\frac{1}{\sigma_{y3}^2} \qquad (8)$$

Defining the parameter $K=\sigma_y/\sigma_{y3}$ and using it with (7) and (8) in (6) with some algebra produces the following yield function for transverse isotropy

$$f=\frac{1}{3}\Big[\sigma_{11}^2+\sigma_{11}^2+K^2\sigma_{33}^2-K^2\sigma_{33}(\sigma_{11}+\sigma_{22})^2-(2-K)\sigma_{11}\sigma_{22}+ \\ 2L\sigma_y^2(\tau_{23}^2+\tau_{31}^2)+(4-K^2)\tau_{12}^2\Big]-\frac{1}{3}\sigma_y^2=0 \qquad (9)$$

Now consider the plane stress case where σ_{33}, τ_{23} and τ_{31} = 0 and define the parameter *R* to be the ratio of strains

$$R=\frac{\dot{\varepsilon}_{22}}{\dot{\varepsilon}_{33}} \qquad (10)$$

for a uniaxial tensile deformation in the 1 direction. Using the associative flow rule (5) and taking the appropriate derivatives in (9), the ratio (10) with $\sigma_{22} = 0$ can be shown to be constant and given by

$$R = \frac{2}{K^2} - 1 \tag{11}$$

Solving for *K* in terms of *R* in (10) and substituting into (9) provides the yield function for transverse anisotropy

$$f = \frac{1}{3}\left[\sigma_{11}^2 + \sigma_{11}^2 - \frac{2R}{R+1}\sigma_{11}\sigma_{22} + 2\frac{2R+1}{R+1}\tau_{12}^2\right] - \frac{1}{3}\sigma_y^2 \tag{12}$$

Equation (12) can be rewritten in the form (1) where $\boldsymbol{P}$ is given by

$$\boldsymbol{P} = \frac{1}{3}\begin{bmatrix} 2 & -2\dfrac{R}{R+1} & 0 \\ -2\dfrac{R}{R+1} & 2 & 0 \\ 0 & 0 & 4\dfrac{2R+1}{R+1} \end{bmatrix} \tag{13}$$

Using (13) and the reduced stress $\eta = \sigma - \boldsymbol{X}$ in(12) provides a kinematic hardening model for transverse isotropy.

Hardening

The constitutive model is based on multi-component forms of kinematic and isotropic hardening variables in conjunction with the yield criterion (1) for rate-independent plasticity. The nonlinear evolution of each of the multi-component kinematic hardening variables is based on Armstrong-Frederick type rule and is given by

$$\dot{\boldsymbol{X}}_i = \frac{2}{3}\alpha_i\,(C_i\eta - \sigma_y\,D_i\,\boldsymbol{X}_i)\,\dot{\lambda} \qquad \forall \quad i = 1,2,...,numkin \tag{14}$$

where $\boldsymbol{X}_I$ denotes an independent kinematic hardening variable, C_i and D_i denote the *numkin* material constants, α_i = 1 without strain range memorization and $\dot{\lambda}$ denotes the consistency parameter[1]. A saturation type hardening rule is used to describe the nonlinear evolution of each of the isotropic hardening variables. Mathematically, the evolution of an independent isotropic hardening variable R_i is given by

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[1] Note that the theory manual uses a different notational form for the kinematic hardening $\dot{\boldsymbol{X}}_i = \frac{2}{3}D_i\,(Q_{xi}\eta - \kappa\,\boldsymbol{X}_i)\,\dot{\lambda}$.

$$\dot{r}_i = \frac{2}{3} b_i \left(Q_i - r_i\right) \sigma_y \dot{\lambda} \qquad \forall \quad i = 1, 2, ..., numiso \tag{15}$$

where b_i and Q_i denote isotropic material constants. For rate-independent plasticity, the elastic domain is defined by the yield criterion (1) as

$$f = \frac{1}{2} \eta^T \boldsymbol{P} \eta - \frac{1}{3} \sigma_y \leq 0 \qquad \text{where} \qquad \sigma_y = \sigma_{yo} + r \tag{16}$$

The center $\boldsymbol{X}$ and the change in the size of elastic domain r are defined in terms of independent kinematic and isotropic hardening variables as

$$\begin{aligned} \boldsymbol{X} &= \sum_i \boldsymbol{X}_i \\ r &= \sum_i r_i \end{aligned} \tag{17}$$

For rate-independent plasticity, the plastic multiplier $\dot{\lambda}$ is obtained through the consistency condition. On the other hand, in the case of viscoplasticity, the plastic multiplier may be obtained as

$$\dot{\lambda} = \left\langle \frac{f}{K} \right\rangle^n \tag{18}$$

where K and n denote the viscosity coefficient and exponent respectively. With viscoplasticity, the inequality in (16) is not be enforced and (18) is used in (5), (14) and (15).

The concept of memory surface is used to describe the strain range dependent material memory effects that are induced by the prior strain histories. This is usually manifested as cyclic hardening and sometimes cyclic softening. The strain range dependent memory effects on isotropic and kinematic hardening evolve the parameters α_i and Q_i in equations (14) and (15) respectively as such

$$\alpha_i = \alpha_{Mi} + \left(\alpha_{0i} - \alpha_{Mi}\right) e^{-2\delta_i q} \qquad \forall \quad i = 1, 2, ..., numkin \tag{19}$$

and

$$Q_i = Q_{Mi} + \left(Q_{0i} - Q_{Mi}\right) e^{-2\omega_i q} \qquad \forall \quad i = 1, 2, ..., numiso \tag{20}$$

The parameter q is related to the maximum plastic strain achieved during loading. The reader is referred to the theory manual for a detailed description of the strain range memorization.

Input Parameters

The following is a summary of the input for the material model. The *prop*(48) array (see source code description in the last section) stores these parameters sequentially in the following order:

Young's modulus, E	prop(1)
Poisson's ratio, ν	prop(2)
Yield stress, σ_{yo}	prop(3)
Viscosity coefficient, K	prop(4)
Viscosity exponent, n	prop(5)
Number of kinematic hardening rules, ($numkin \geq 1$)	prop(6)
Number of isotropic hardening variables, ($numiso \geq 1$)	prop(7)
Memory effects variable, (*memeff*)	prop(8)
EQ. 0: do not include	
EQ. 1: include	
C_1, D_1, C_2, D_2, …. C_{numkin}, D_{numkin}	prop(8)-prop(n1), n1 = 8+2*numkin
b_1, Q_1, b_2, Q_2, …. b_{numiso}, Q_{numiso}	prop(n1+1)-prop(n2), n2 = n1+2*numiso
Q_{01}, Q_{M1}, ω_1, Q_{02}, Q_{M2}, ω_2, …. $Q_{0numiso}$, $Q_{Mnumiso}$, ω_{numiso}	prop(n2+1)-prop(n3), n3 = n2+2*numiso
D_{01}, D_{M1}, δ_1, D_{02}, D_{M2}, δ_2, …. $D_{0numkin}$, $D_{Mnumkin}$, δ_{numkin}	prop(n3+1)-prop(n4), n4 = n3+2*numkin

Note, if *memeff* = 0 then all input past *prop*(8) is ignored. If *memeff* = 1, then input values for Q_1- Q_{numiso} in *prop*(n1+1)-*prop*(n2) are ignored and the value for Q_i is taken from (20). Note, a maximum of 48 parameters can be input to the material model since the dimension for the *prop* array is hard coded.

The anisotropic hardening parameter R is not included in the *prop*(48) array but is instead passed through the argument list (see source code section).

Unixaxial Loading

The first order differential equations given by (14) and (15) can be easily solved for constant strain rate loading. The solution of which can be employed when curve fitting experimental data.

Example

In this example, a solution for the saturation stress is found for uniaxial monotonic loading in the 1 direction for a material with one kinematic hardening law. The first thing to note is that the backstress $\mathbf{X}$ in (14) is *not* deviatoric. Therefore, using

$\mathbf{X} = \{X_1,X_2,X_3\}^T$, examination of (14) will show that $X_2 = X_3 = 0$. Furthermore, $\eta = \{\sigma_1 - X_1, 0, 0\}^T$. The, plastic strain in the 1 direction is found from (5) and (2) or (13)

$$\dot{\varepsilon}_1 = \dot{\lambda}(\boldsymbol{P}\,\eta)_1 = \dot{\lambda}\frac{2}{3}\eta_1 = \dot{\lambda}\frac{2}{3}(\sigma_1 - X_1) = \frac{2}{3}\dot{\lambda}\sigma_y \tag{21}$$

Using (21) in (14), the backstress can then be found

$$\dot{X}_1 = C\dot{\varepsilon}_1^p - D\dot{\varepsilon}_1^p X_1 \tag{22}$$

and

$$X_1 = \frac{C}{D}(1 - e^{-D\varepsilon_1^p}) \tag{23}$$

The saturation stress is then found from (23) to be

$$\sigma_y^{saturation} = \sigma_{yo} + \frac{C}{D} \tag{24}$$

This can be generalized for multiple backstress *and* isotropic hardening components to be

$$\sigma_y^{saturation} = \sigma_{yo} + \sum_{i=1}^{numkin}\frac{C_i}{D_i} + \sum_{i=1}^{numiso} Q_i \tag{25}$$

The material model can replicate linear isotropic and kinematic hardening as follows. Set the following (*numkin* = 1 and *numiso* = 1) and

$$\begin{aligned} &C_1 = (1-\beta)E_1 && D_1 = 0 \\ &(b_1 Q_1) = \beta\, E_p && b_1 \to \infty,\ Q_1 \to \infty \end{aligned} \tag{26}$$

where β is the mixed hardening parameter (ratio of isotropic to kinematic hardening) and E_p is the plastic modulus. For the linear isotropic hardening, b_1 is set sufficiently big and Q_1 is set sufficiently small.

Curve Fitting

Curve fitting experimental data using solutions of the form (23) is a non-linear least squares problem. In lieu of that, one can choose a priori a set of D_i and then use a linear least squares fitting process to choose C_i. The first guess for D_i can be made by recognizing from (23) that a given i component quantity is nearly saturated at $\varepsilon_i = 1/D_i$.

2. Cyclic Uniaxial Loading

An example of cyclic loading and saturation is given. The material parameters are taken from Zhao [2] for SPCEN mild steel.

Isotropic hardening	Q	37.7 MPa	b	67.8
Kinematic hardening	C	23.7 GPa	D	416
Anisotropic ratio	R	1.53		
Yield stress	σ_{yo}	108 Mpa		
Young's Modulus	E	153 GPa		

Figure 1 shows the stress strain using the material model for the uniaxial cyclic loading. These results match Zhao [2]. Using (25) and the above parameters, saturation should be achieved at the stress

$$\sigma^{saturation} = 108 + 23{,}700/416 + 37.7 = 202.67 \qquad (27)$$

and can be verified in Figure 1.

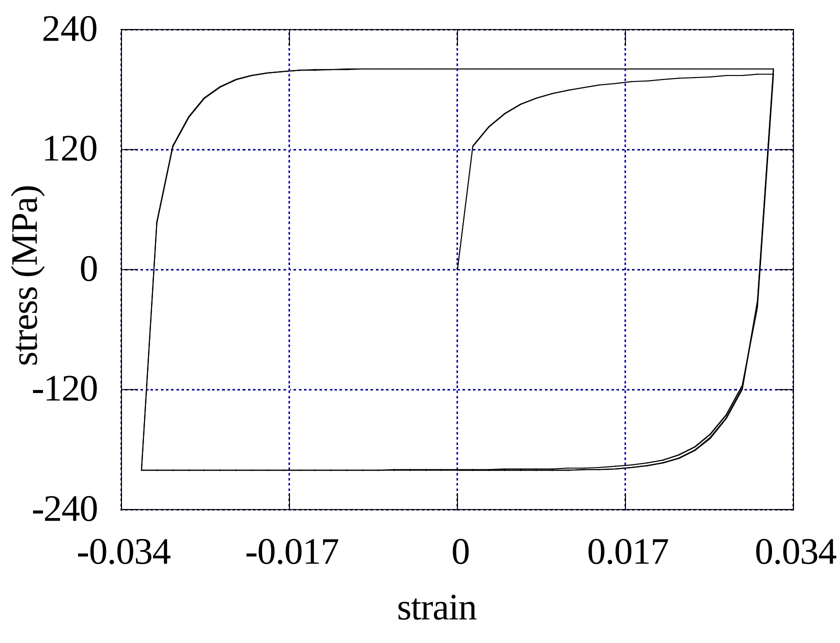

Figure 1 Stress verses strain for SPCEN mild steel.

3. 2D Draw Bending

The following example assess the effects of kinematic hardening on the springback of a stamped sheet metal part. The 93 Numisheet 2D Draw Bending problem using mild steel is simulated for two case:

1. Isotropic hardening exclusively
2. Kinematic hardening exclusively

The forming setup is shown in Figure 2 (all dimensions are in mm). The sheet is 0.78 mm thick and two different binder forces are examined: F=19.6 kN and F=2.45 kN. The parameters for both the isotropic and kinematic hardening were chosen to fit the experimentally derived power law curve (28) for mild steel using the heuristic method described above.

$$\sigma = 565.32(0.00717 + \varepsilon)^{0.2589} \qquad (28)$$

The curve fit is shown in Figure 3. The material parameters are shown in Table 1 and the uniaxial loading and unloading curves are shown in Figure 4. A portion of the Baushinger effect can be seen upon unloading for the kinematic hardening case in Figure 4. In Figure 5., the results are shown before and after the tooling is removed for the 2kN binder force case. The amount of springback is measured by the deflection of the flange angle (upper right hand portion of the sheet) from horizontal. The simulated flange angles are shown in Table 2 and are compared with the experimental average results. As expected, the kinematic hardening provides more springback. In the case of the 19.6 kN binder force, the kinematic hardening gave too much springback.

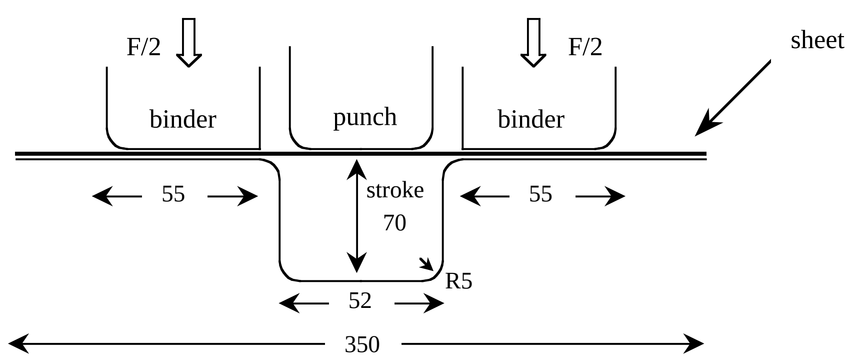

Figure 2. Numisheet 93 2D Draw Bending

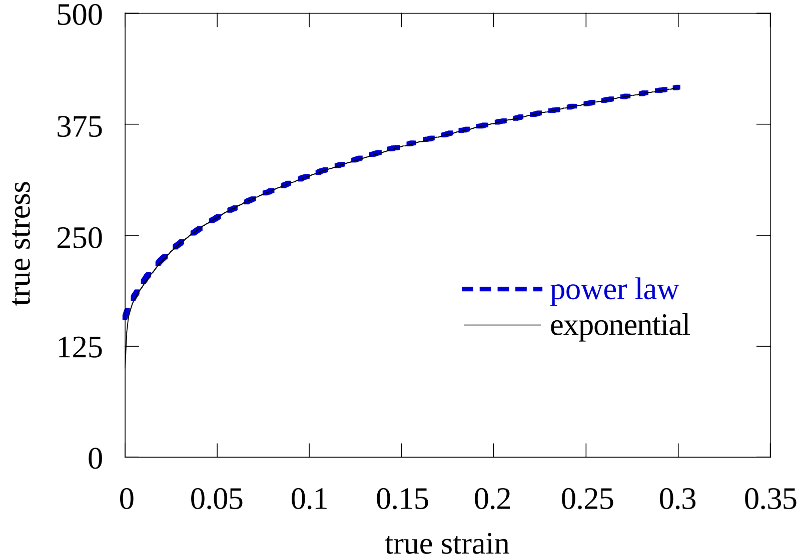

Figure 3. Curve fit using parameters in Table 1.

Table 1.

E = 206 Gpa, ν = 0.3, σ_y = 100.462, Friction coefficient = 0.144

<u>Isotropic Hardening Case</u>

b_1,Q_1	3.3333333	272.936
b_2,Q_2	20	29.0895
b_3,Q_{13}	50	57.2771
b_4,Q_4	1000	57

<u>Kinematic Hardening Case</u>

D_1,C_1	3.3333333	909.78667
D_2,C_2	20	581.79
D_3,C_3	50	2863.85
D_4,C_4	1000	57000

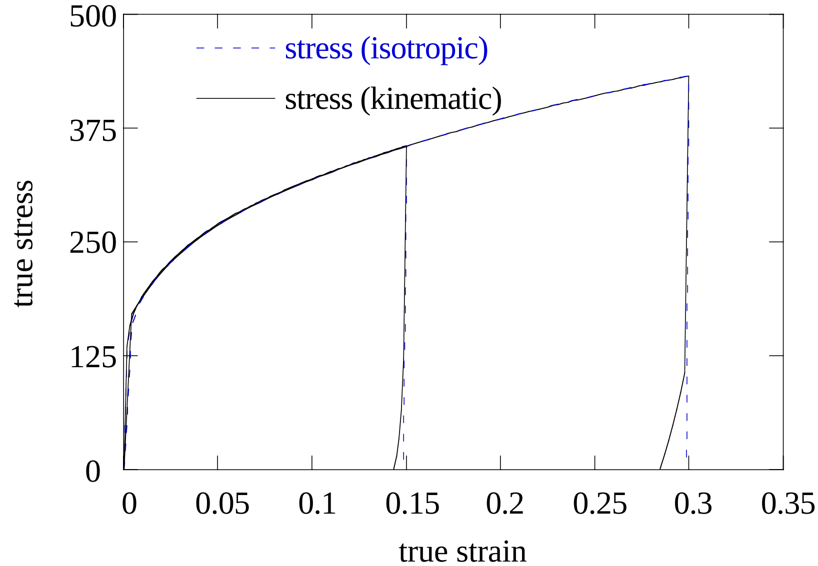

Figure 4. Loading and unload for isotropic and kinematic material models.

binder force	Isotropic hardening	kinematic hardening	experimental avg.
2.45 kN	17.20	17.57	17.1
19.6 kN	12.10	16.30	12.6

Table 2. Flange angles measured in degrees after springback for different cases.

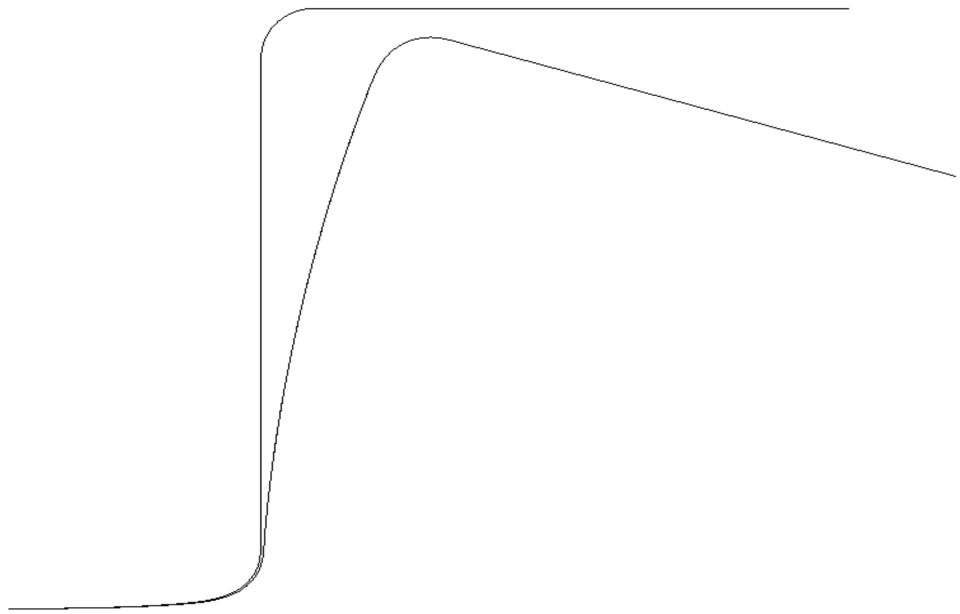

Figure 5. Sheet before and after tooling is removed. The springback is shown for the 2 kN kinematic hardening case.

References

1. Nukula P., “A Return Mapping Algorithm for Cyclic Viscoplastic Constitutive Models,” *Computer Methods in Applied Mechanics* (to appear).
2. Zhao K., “Cyclic Stress-Strain Curve and Springback Simulations,” Ph.D Dissertation, (1999).