UCRL-PROC-200036

LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

Modeling Losses and Interference in Fiber Optic Systems

P. Covello, J. Koning, Jennifer Mariani, Garry Rodrigue

August 28, 2003

2003 AMOS Technical Conference
Wailea, Maui, HI, United States
September 8, 2003 through September 13, 2003

Disclaimer

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

Modeling Losses and Interference in Fiber Optic Systems[1]

Paul Covello, Joe Koning, Jennifer Mariani, Garry Rodrigue
Dept. of Applied Science, University of California @ Davis
Davis, California 95616

1. INTRODUCTION

The transmission characteristics of the optical fiber are of utmost importance for optical telecommunication systems. Indeed, the optical signal experiences all kinds of losses as it propagates and demands the use of regenerators or repeaters along the fiber link, [7]. Attenuation (loss) is a relationship between the optical output power and the optical input power in a fiber optic system. It is a measure of the decay of signal strength, or loss of light power, that occurs as light pulses propagate through the length of the fiber.

The optical guiding properties of optical fiber is based on the principle of total internal reflection, [3]. If we use the mechanism of ray optics whereby light is described by rays that travel in different optical media in accordance with a set of geometrical rules, then one is able to determine incidence angles that light rays must follow in order to be guided. Snell's law states that total internal reflection will occur if the angle of incidence of a light ray within the core of a fiber optic makes with the cladding exceeds the critical angle $\theta_c = \sin^{-1}(n_2/n_1)$ where n_1, n_2 are the indices of refraction of the core and cladding, respectively. Hence, signal attenuation will occur in the waveguide if changes occur in the angle of incidence of the light ray or the indices of refraction of the medium. External factors that could cause either of these to occur include deformations of the fiber such as bends or environmental effects such as electrical fields.

Physical bends break down into two categories: microbending and macrobending, [6]. Microbending is the result of microscopic imperfections in the geometry of the fiber. These imperfections could be rotational asymmetry, changes in the core diameter, rough boundaries between the core and cladding as a result of mechanical stress, pressure, tension, or twisting. Macrobending, on the other hand, occurs when fiber curvatures have diameters on the order of centimeters. The loss of optical power is the result of the less-than-total reflection at the core-to-cladding boundary.

Environmental effects on the optical fiber can cause changes in the indices of refraction of the silica glass comprising either the core or cladding thereby affecting the transmission and reflection properties of the core and cladding, [2]. External electric fields can distort the electron motions (orbits) in the constituent atoms and molecules in such a way that it becomes "more" difficult for the electric field in a light wave to displace electrons parallel to the applied field direction and thereby changing the propagation properties of the optical wave.

In general, the fields of optical waveguides are comprised of two parts. One part represents energy that is guided, i.e., energy that travels in the direction of the wave guide axis; the other part represents radiated energy, i.e., energy that travels obliquely to the axis. It is specifically the radiation loss caused by curvature of a fiber or from the environment that is very important form the point of view of long-distance transmission of optical signals.

Field properties are studied by describing light as an electromagnetic wave phenomenon described by two mutually coupled vector waves, an electric-field wave and a magnetic field wave. The dynamics of these waves in an isotropic media is obtained through the solution of Maxwell's equations:

---

1. This work was performed under the auspices of the US Department of Energy by the University of California, Lawrence Livermore National Laboratory under contract No. W-740S-Eng-48 and supported in part under AFOSR Grant F49620-01-1-0327

$$\frac{\partial \boldsymbol{D}}{\partial t} = \nabla\times\boldsymbol{B} \qquad \frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times\boldsymbol{E}$$
$$\nabla\bullet\boldsymbol{D} = 0 \qquad \nabla\bullet\boldsymbol{B} = 0 \tag{1}$$

Maxwell's equations can be decoupled by taking the curl of the curl-$\boldsymbol{E}$ equation, interchange the order of space and time derivatives on the left side of the resulting equation, replace $\nabla\times\boldsymbol{B}$ by $\partial\boldsymbol{D}/\partial t$, assume $\boldsymbol{D} = \varepsilon\boldsymbol{E}+\boldsymbol{P}$ and $\nabla\times\nabla\times\boldsymbol{E} = \nabla^2\boldsymbol{E}$ to obtain the equation

$$\varepsilon\frac{\partial^2\boldsymbol{E}}{\partial t^2} = \nabla^2\boldsymbol{E} - \frac{\partial^2\boldsymbol{P}}{\partial t^2} \tag{2}$$

Here, $\boldsymbol{P} = \varepsilon(\chi^{(1)}\boldsymbol{E}+\chi^{(2)}:\boldsymbol{EE}+\chi^{(3)}:\boldsymbol{EEE}+\ldots)$ where $\chi^{(i)}$ is the *it* order optical susceptibility and is a tensor of rank $i+1$. A common assumption is to take $\boldsymbol{P} = \varepsilon(\chi^{(1)}\boldsymbol{E}+3\chi^{(3)}|\boldsymbol{E}|^2\boldsymbol{E})$, [2]. Then (2) becomes

$$\tilde{\varepsilon}\frac{\partial^2\boldsymbol{E}}{\partial t^2} = \nabla^2\boldsymbol{E} \tag{3}$$

where $\tilde{\varepsilon} = \varepsilon(1+\chi^{(1)}+3\chi^{(2)}|\boldsymbol{E}|^2)$.

2. ELECTROMAGNETIC MODEL

The vector field $\boldsymbol{E}$ provided by the numerical solution of (3) is the Galerkin approximation

$$\tilde{\boldsymbol{E}}(\boldsymbol{x},t) = \sum_{j=1}^{N} e_j(t)\boldsymbol{W}_j(\boldsymbol{x}) \tag{4}$$

where $\boldsymbol{W}_1, \boldsymbol{W}_2, \ldots, \boldsymbol{W}_N$ are linearly independent vector polynomials, [1]. The coefficients $e_j(t)$ are obtained by forcing the approximation in (4) to satisfy

$$\int_\Omega \tilde{\varepsilon}(\tilde{\boldsymbol{E}})\frac{\partial^2}{\partial t^2}\tilde{\boldsymbol{E}}\bullet\boldsymbol{W}_i d\Omega = \int_\Omega \nabla^2\tilde{\boldsymbol{E}}\bullet\boldsymbol{W}_i d\Omega \qquad i = 1, 2, \ldots, N.$$

This yields the system of ordinary differential equations

$$\boldsymbol{C}\frac{\mathrm{d}^2\boldsymbol{e}}{\mathrm{d}t^2} = \boldsymbol{Ke} \tag{5}$$

where $\boldsymbol{e} = [e_1, e_2, \ldots, e_N]^t$ and the matrices $\boldsymbol{C}, \boldsymbol{K}$ are given by

$$C_{ij} = \int_\Omega \tilde{\varepsilon}\boldsymbol{W}_i\bullet\boldsymbol{W}_j d\Omega \qquad K_{ij} = \int_\Omega \nabla\boldsymbol{W}_i\bullet\nabla\boldsymbol{W}_j d\Omega$$

(5) is then solved by the second order accurate leap-frog scheme

$$\begin{bmatrix}\boldsymbol{e}^{n+1}\\ \boldsymbol{e}^{n}\end{bmatrix} = \begin{bmatrix}\boldsymbol{I}+\Delta t^2\boldsymbol{C}_n^{-1}\boldsymbol{K} & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{O}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}^{n}\\ \boldsymbol{e}^{n-1}\end{bmatrix}$$

where $(C_{ij})_n = \int_{\Omega} \tilde{\varepsilon}(\boldsymbol{e}^n)\boldsymbol{W}_i \bullet \boldsymbol{W}_j d\Omega$.

Propagating modes in a step-indexed fiber are analytically constructed by juxtaposing the solutions of (5) in the cladding and the core in such a manner that the characteristic condition of tangential continuity across core-cladding interface is preserved. Consequently, it is important that the numerical solution also obey this characteristic condition. This is accomplished by using the proper vector polynomials $\boldsymbol{W}_i$ in the Galerkin approximation of (4). Specifically, if Ω_e is an arbitrary hexahedral element in a grid describing a fiber, see Fig. 1, then there exists a unique one-to-one mapping ϕ_e from the reference element $\Omega_0 = -1 \le x, y, z \le 1$ onto Ω_e. Specifically, define on Ω_0 the vector polynomials

$$\begin{aligned}
\boldsymbol{W}_1^{(0)} &= [1 - y - z + yz, 0, 0]^t & \boldsymbol{W}_4^{(0)} &= [z - yz, 0, 0]^t & \boldsymbol{W}_7^{(0)} &= [yz, 0, 0]^t \\
\boldsymbol{W}_2^{(0)} &= [0, 1 - x - z - xz, 0]^t & \boldsymbol{W}_5^{(0)} &= [0, z - xz, 0]^t & \boldsymbol{W}_5^{(0)} &= [0, xz, 0]^t, \\
\boldsymbol{W}_3^{(0)} &= [0, 0, 1 - x - y - xy]^t & \boldsymbol{W}_6^{(0)} &= [0, 0, x - xy]^t & \boldsymbol{W}_4^{(0)} &= [0, 0, xy]^t
\end{aligned}$$

see Fig. 1. Note that each vector $\boldsymbol{W}_i^{(0)}$ is associated with a specific edge of Ω_0.

Fig. 1. Isoparametric mapping from reference element to hexahedral element. $\boldsymbol{W}_4^{(0)}$ on reference element and $\boldsymbol{W}_4$ on hexahedral element

The definition of the vector polynomials on Ω_e is given by $\boldsymbol{W}_i = J_e^{-t}\boldsymbol{W}_i^{(0)}$ where J_e is the determinant of the Jacobian of ϕ_e, see Fig. 1. Thus, the value of N in (4) is the number of edges in the entire grid. The fact that the approximation $\tilde{\boldsymbol{E}}$ in (4) preserves tangential continuity across material interfaces is established in [8].

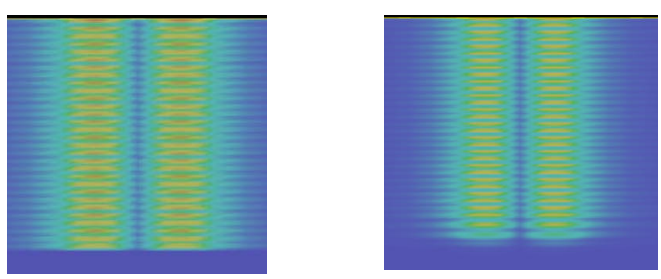


Fig. 2. (a) Exact Solution (b) Computed Solution

2.1 Bent Waveguides

The properties of optical waves in a straight step-indexed fiber are well-known and have been studied extensively, [3]. However, the situation for bent step indexed fibers is not as good. A common approach is to describe Maxwell's equations in terms of local coordinates of the fiber and then transform the equations back to Cartesian coordinates so that analysis of the bent fiber becomes one of studying the modes of the transformed equations on an equivalent straight waveguide, [6]. The difficulty is that the transformed equations cannot be conventionally analyzed and an asymptotic expansion around a perturbation parameter of the form $O(R^{-1})$, $R =$ radius of curvature , is used to eliminate terms in the equations so that conventional mathematical tools can be applied. Thus the analysis is only valid for large curvature radii. The case for not-so-large radii still remains open.

Another difficulty presented by the bent fiber is that the index of refraction is no longer the same as it was for the original straight fiber. The core and cladding are stressed when the fiber is bent and the index of refraction needs to reflect this. Several approaches in the literature analyze Maxwell's equations on a straight fiber with an index of refraction that $n + O(R^{-1})$ where n is the index of refraction of the original straight fiber. Again, this is only valid for large curvature radii. This approach also does not take into account optical losses as predicted from ray optics for bent fibers.

2.2 Bent Step-Indexed Fiber

We first consider the case of propagating an optical wave in a bent optical fiber. Fig. 3 illustrates a 1.8 million element approximation of a 52.8 micron length of a bent fiber with 55.13 micron bend radius. 5.66 million unknowns are numerically determined at each time step. The computation requires 72 hours on 38 processors of the MHPCC Huinalu Linux based supercomputer.

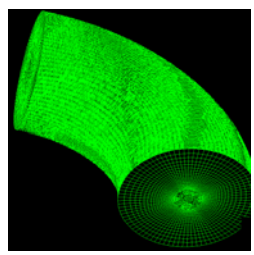


Fig. 3. Finite element approximation of a bent step-indexed fiber

The Gaussian pulse of a TE_{01} mode can be seen in Fig. 4(a) where the core is purple and the cladding green. A close-up of the pulse interacting with the core-cladding interface is shown in Fig. 4(b)-(c) where the core is again purple and the cladding is blue.

(a) (b) (c)

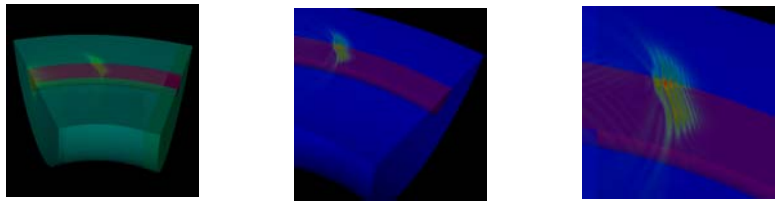


Fig. 4. Propagation of a Gaussian pulse in a bent step-indexed fiber

2.3 External Fields

If a strong electric field is applied to an otherwise optically isotropic material such as glass, then the change in the refractive index will be due to the Kerr effect. Specifically, the refractive index is described by the relation

$$n = n_0 + 2n_2|\boldsymbol{E}|^2$$

where n_0 represents the usual refractive index and n_2 is a new optical constant that gives the rate at which the refractive index increases with increasing intensity.

We study the effect on the propagation of an optical wave by the external impingement of a highly intense microwave on a step-indexed optical fiber. Fig. 5 illustrates a 301,000 element approximation of a step-indexed optical fiber in air. A plane microwave impinges the optical fiber from the top at 3 sec. and continues for another 10 sec. before being terminated. The pulse is then left to interact with the propagating

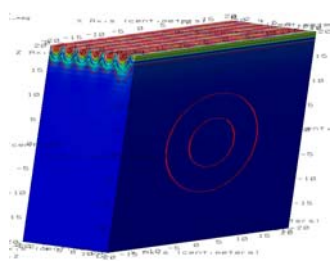
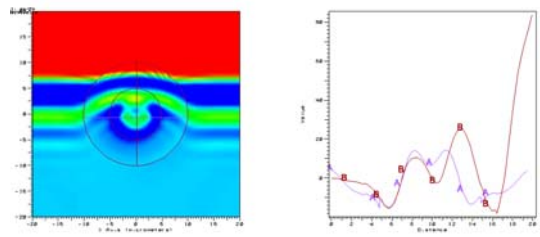


Fig. 5. Microwave impinging on optical fiber from top

(a)



(b)

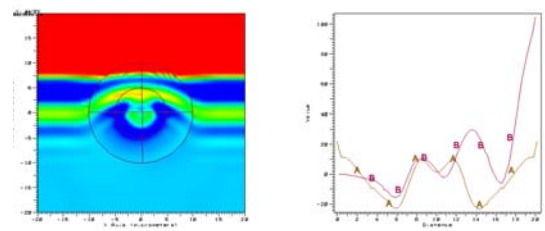


Fig. 6. Plot of x-component of optical field. (a) Without Kerr effects. (b) With Kerr effects

guided wave. Fig. 6 illustrates the results after 220 sec. If no Kerr effect is invoked (i.e., $n_2 = 0$), then the Fig. 6(a) illustrates the behavior whereas Fig. 6(b) illustrates the behavior with the Kerr effect. The intensities on the drawn horizontal (A) and vertical (B) lines in the fiber are plotted on the right. Fig. 7 illustrates a side-view of the result at 75 sec. (Fig. 7(a)-(c)) and 216 sec. (Fig. 7(b)-(d)). Fig. 7(e) illustrates the propaga-

(a) (b)

(c) (d)

(e)

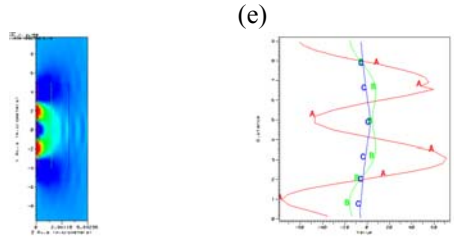
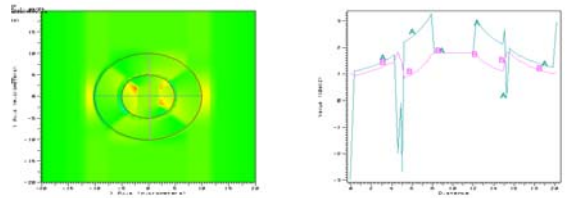


Fig. 7. Plot of z-component of optical field. (a)-(b) Without Kerr effects. (c)-(d) With Kerr effects. (e) No microwave impingement

(a)



(b)

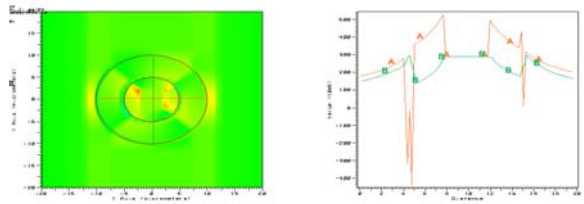


Fig. 8. Intensity plots. (a) Without Kerr effects; (b) With Kerr effects

tion of the optical wave at 266 sec. without any microwave impingement. Intensity plots along the vertical lines are on the right.

Fig. 8 illustrates the intensity of the signal at the end of the fiber where the values of the intensity on the horizontal (A) and vertical (B) lines are plotted on the right.

3. GEOMETRIC OPTICAL MODEL

The rays of geometric optics in an isotropic media are obtained by drawing normals to the wavefront surfaces. Although the artifact of rays breaks down in the vicinity of sharp boundaries where diffraction effects can no longer be ignored, the rays continue to be useful as they convey information about the magnitude and direction of energy flow. An approximation to the rays of geometric optics is provided from an asymptotic analytical approach to Maxwell's equations when the wavelength is small. Specifically, if we assume

$$\boldsymbol{E}(\boldsymbol{x}, t) = \mathrm{Re}\,[\boldsymbol{E}(\boldsymbol{x})e^{-i\omega t}] = \mathrm{Re}[\boldsymbol{A}e^{i\omega[S(\boldsymbol{x})-t]}], \tag{6}$$

and then substitute (6) into (3), then for large values of ω we see that $S(\boldsymbol{x})$ satisfies the eikonal equation

$$\|\nabla S\|^2 = \tilde{\varepsilon}, \tag{7}$$

[3]. The values $\boldsymbol{E}(\boldsymbol{x}, t)$ are determined from the level sets of S. That is, the family of surfaces $S(\boldsymbol{x}) = \text{constant}$ gives the successive positions of the optical wave in $\boldsymbol{x}$-space. Since ∇S is normal to the level surfaces of S, a front marching scheme can be developed by treating the normal vector as an optical ray to recursively generate the surface $S(\boldsymbol{x}) = t^{n+1}$ from the surface $S(\boldsymbol{x}) = t^n$, [4].

The exact solution of (1) for the straight optical fiber can be obtained and is illustrated in Fig. 9(b). Although geometric optics is useful for determining very general properties of light propagation, it is the simplicity of ray optics that limits it use in understanding the exact properties of the attenuation of light.

Fig. 9(a) illustrates the solution as computed using ray optics where it is clearly seen that optical field in the cladding is not represented as it is using the finite element electromagnetic model.

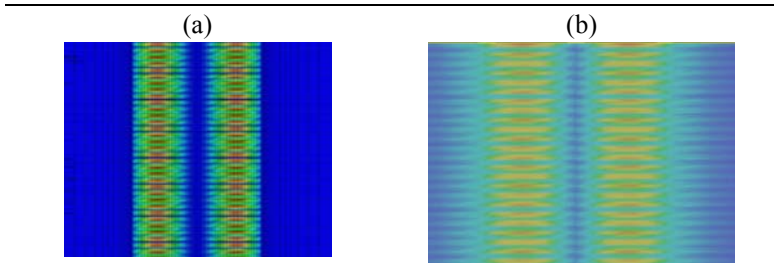


Fig. 9. (a) Computed Solution using geometric optics (b) Exact solution

The main advantage of the geometric optic approach is the speed at which a computation can be done. Fig. 10 illustrates the computation of an optical pulse in a bent fiber. The computation took on the order of minutes to complete in contrast to the finite element approach which would require hours.

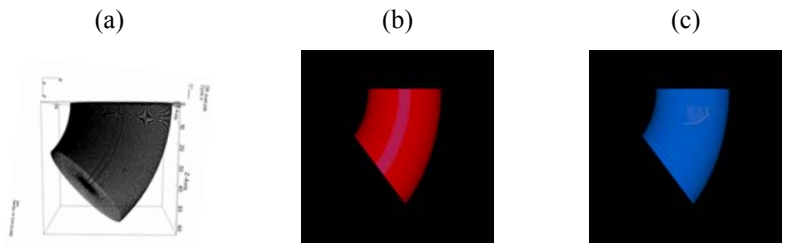


Fig. 10. (a) Hexahedral grid representation of bent step-indexed fiber. (b) Core in blue; cladding in red. (c) Leakage of optical pulse into cladding

4. REFERENCES

1. Rodrigue,G. and White,D., *A Vector Finite Element Time-Domain Method for Solving Maxwell's Equations on Unstructured Hexahedral Grids*, SIAM J. Sci. Comp., 23(3), 683-706, 2001.
2. Boyd, R.W., *Nonlinear Optics*, 2nd Edition, Academic Press, San Diego, CA, 2003.
3. Iizuka, K., *Elements of Photonics*, Volume II, Wiley-Interscience, New York, NY, 2002.
4. Covello, P., and Rodrigue, G., *A Generalized Front Marching Algorithm for the Solution of the Eikonal Equation*, J. of Comp. and Appl. Math., 156, 371-388, 2003.
5. Tai, H. and Rogowski, R., *Optical Anisotropy Induced by Torsion and Bending in an Optical Fiber*, Optical Fiber Technology, 8, 162-169, 2002.
6. Vassallo, C., *Optical Waveguide Concepts*, Elsevier Science Pub., Amsterdam, The Netherlands, 1991.
7. Grote, N. and Venghaus, H., *Fiber Optic Communication Devices*, Springer-Verlag, Berlin, Germany,2001
8. Nedelec, J.C., *A New Family of Mixed Finite Elements in R3*, Numer. Math., 50,57-81, 1986