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COMPARISON OF UP-SCALING METHODS IN POROELASTICITY AND ITS GENERALIZATIONS

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OUTLINE

- Biot's Poroelasticity Theory Is Correct!
  - Laboratory data
  - Finite element approach to modeling
- Four Methods for Up-Scaling
  - Effective medium theories
  - Mixture theory
  - Homogenization theory
  - Volume averaging methods
- Conclusions

Biot's (1962) Strain Energy Functional

$$2E = He^2 - 2Ce\zeta + M\zeta^2 - 4\mu I_2$$

where H, C, M, and μ are poroelastic constants,

$e = \nabla \cdot \vec{u} =$ frame dilatation,

$\zeta = -\nabla \cdot \vec{w} =$ increment of fluid content,

$\phi =$ porosity,

$\vec{u} =$ solid frame displacement,

$\vec{u}_f =$ pore fluid displacement,

$\vec{w} = \phi(\vec{u}_f - \vec{u}) =$ relative displacement, and

$I_2 = e_x e_y + e_y e_z + e_z e_x - \frac{1}{4}(\gamma_x^2 + \ldots) =$ a strain invariant.

Biot's Equations of Dynamic Poroelasticity

$$\omega^2 \rho \vec{u} + (H - \mu)\nabla e + \mu \nabla^2 \vec{u} = -\omega^2 \rho_f \vec{w} + C \nabla \zeta,$$

$$\omega^2 q(\omega) \vec{w} - M \nabla \zeta = -\omega^2 \rho_f \vec{u} - C \nabla e,$$

where

$\omega = 2\pi f =$ angular frequency,

$\rho = \phi \rho_f + (1 - \phi)\rho_m =$ the average density,

$q(\omega) = \rho_f [\tau/\phi + iF(\xi)\eta/\kappa\omega]$, and

$p_f = -M \nabla \cdot \vec{w} - C \nabla \cdot \vec{u} =$ fluid pressure.

Some Relations Among Poroelastic Constants

$$H = K_u + \tfrac{4}{3}\mu,$$

$$C = BK_u,$$

$$M = BK_u/\alpha = C/\alpha,$$

where

$\alpha = 1 - K/K_m =$ the effective stress coefficient, and

K_u is the undrained or Gassmann bulk modulus of the system.

Dispersion Relations

- For shear wave:

$$k_s^2 = \omega^2(\rho - \rho_f^2/q)/\mu$$

- For fast and slow compressional waves:

$$k_\pm^2 = \tfrac{1}{2}\left[b + f \mp [(b-f)^2 + 4cd]^{1/2}\right]$$

$$b = \omega^2(\rho M - \rho_f C)/\Delta, \quad c = \omega^2(\rho_f M - qC)/\Delta$$
$$d = \omega^2(\rho_f H - \rho C)/\Delta, \quad f = \omega^2(qH - \rho_f C)/\Delta$$

where

$$\Delta = HM - C^2.$$

SOME UP-SCALING RESULTS

via effective medium theory or homogenization methods

- Electrical Conductivity (scale invariant)

$$J = \sigma E \rightarrow \langle J \rangle = \sigma^* \langle E \rangle$$

- Navier-Stokes equation $\rightarrow$ Darcy's equation

  definitely not scale invariant!

- Linear elasticity + Navier-Stokes equations $\rightarrow$

  Biot's equations of poroelasticity

- Heterogeneous Biot $\rightarrow$????

  Possibly to the double-porosity model in a variety of circumstances (last year's talk!).

First Method: Effective Medium Theory

Effective medium theory is designed to produce estimates of coefficients in the equations of motion.

Various good alternatives are available:

- Average T-matrix (Mori-Tanaka, Kuster-Toksöz)
- Self-consistent (SC or CPA)
- Differential effective medium (DEM)
- Also, rigorous bounding methods are known.

Second Method: Mixture Theory

Mixture theory is designed to keep careful track of the energy in the system. So this approach includes:

- Hamiltonian and Lagrangian methods
- Biot's original method
- Drumheller and Bedford's method

This method is especially powerful for nonlinear problems, but also provides a good method to derive Biot's linear equations.

Third Method: Homogenization Theory

Homogenization theory is probably the newest of the methods, being first developed in the 1970s. Other methods can be traced back to earlier periods of history. Periodic boundary conditions are normally used to implement the method.

Development is designed to determine rigorously *the form of the equations* in some fixed frequency regime. So it may not determine how the equations change as frequency is varied widely.

Fourth Method: Volume Averaging

Volume averaging was apparently first developed
in the 1960s for application to Darcy flow.
There are similarities to homogenization theory,
but does not require periodic boundary conditions.

Uses rigorous identities concerning volume
integration in 3D to smooth the equations of interest.
Not restricted to a fixed frequency domain,
but requires supplementary information to obtain
estimates of the coefficients.

Other Methods: Were Any Left Out?

- There are other methods I have not talked about today, including:
  - Double-porosity up-scaling (my talk last year)
  - Numerical methods
  - Hybrid methods — using two or more methods simultaneously: for example, mixture theory supplemented with effective medium theory is one very powerful combination.

CONCLUSIONS

I take a very democractic viewpoint concerning all these methods. I have never seen an up-scaling method I did not like. (Well, almost never!)

All these up-scaling methods have some advantages and some disadvantages.

I have stressed the advantages today.

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