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Abstract. We present simple scaling relations of the brightness a Thomson x-ray source on the electron and laser beam parameters in the case of a head (180 degree) interaction geometry. In particular, it is shown that a direct relation exists between the x-ray brightness and the electron beam brightness. These relations are discussed in the context of the PLEIADES Thomson x-ray source, where 10^7 photons per pulse, with photon energies as high as 140 keV, have been produced by colliding a 0.25 nC, picosecond electron bunch with a 500 mJ, 50 fs, 800 nm laser pulse. The estimated peak brightness of the source is about 10^{16} photons/s/mm^2/mrad^2/0.1%b.w. A comparison of the current performance of the source and the predicted performance using optimized parameters is presented.

INTRODUCTION

Thomson scattering of a short, intense laser pulse by a relativistic electron bunch is a promising method for producing high brightness, hard x-ray pulses capable of probing the structural dynamics of high-Z materials at ultra-fast (femtosecond) time scales. The success of such sources depends heavily on the ability to produce low emittance electron beams with sub-picosecond pulse durations. A standard figure of merit of x-ray source performance is the peak brightness, which is defined as the number of photons/second/unit area/unit solid angle/unit bandwidth. In this paper we present relatively simple scaling relations of a Thomson x-ray source peak brightness on the incident laser and electron beam parameters. In deriving these relations, we focus on the case of a 180 degree interaction geometry (i.e. head on collision). While previous experiments have successfully operated with a 90 degree (side-on) interaction geometry [1], the 180 degree geometry offers maximum interaction between the laser and electron beams, resulting in a larger x-ray dose for a given electron beam bunch charge and laser energy. We also assume that the bunch durations of both the laser and electron beams are less than the diffraction length (Rayleigh length) of the laser pulse and the beta function of the electron beam. This insures maximum interaction between the laser and electron bunches for a given interaction spot size, and is readily met by current high brightness electron beams and tera-watt laser systems. It is shown that a direct relation exists between the x-ray brightness and the electron beam brightness, illustrating the importance of high brightness electron beam production in x-ray source development. These results are discussed in the context of the PLEIADES Thomson x-ray source, where up to 10^7 photons per pulse, with photon energies as high as 140 keV, have been produced by colliding a 0.25 nC, picosecond electron bunch with a 500 mJ, 50 fs, 800 nm laser pulse[2][3].

THOMSON SCATTERING THEORY

Thomson backscattered x-rays are produced when a incident photon with angular frequency ω_0 scatters off an electron with relativistic Lorentz factor γ such that the angular frequency of the scattered photon is given by

$$\frac{\omega_s}{\omega_0} \approx \frac{2\gamma^2 (1-\beta\cos(\theta_0))}{1+(\gamma\theta)^2}, \tag{1}$$

where θ_0 and θ are respectively the angles of the incident and scattered photon with respect to the electron direction. The angular distribution of the scattered photons is described by the differential Thomson cross section. In the case of a relativistic electron beam ($\gamma \gg 1$), this is expressed approximately as[4]

$$\frac{d\sigma}{d\Omega} \approx r_0^2 \frac{4\gamma^2}{\left(1+\theta^2\gamma^2\right)^2}\left[1-\frac{4\theta^2\gamma^2}{\left(1+\theta^2\gamma^2\right)^2}\cos^2(\phi)\right] \tag{2}$$

where r_0 is the classical electron radius, θ and ϕ are the polar and azimuthal scattering angles of the x-rays with respect to the electron direction, and it is assumed the laser is polarized in the plane where $\phi = 0$. Equations (1) and (2) can be combined to derive the photon number spectrum integrated over all solid angles,

$$\frac{dN_x}{d\omega_s} = \frac{N_x \sin(\theta)}{\sigma}\frac{d\theta}{d\omega_s}\int_0^{2\pi}\frac{d\sigma}{d\Omega}d\phi \approx \frac{N_x}{\omega_s(0)}\left[\frac{3}{4}+3\left(\frac{\omega_s}{\omega_s(0)}-\frac{1}{2}\right)^2\right], \tag{3}$$

where N_x is the total number of scattered x-ray photons, $\omega_s(0)$ represents the maximum on axis x-ray energy, and it is assumed that $\omega_s \leq \omega_s(0)$. It has also been assumed $\gamma \gg 1$ and $\theta \ll 1$. Similarly, the energy spectrum of the x-ray pulse integrated over all solid angles can be expressed as

$$\frac{dU_x}{d\omega_s} = \hbar\omega_s \frac{dN_x}{d\omega_s} \approx \frac{3}{2}N_x\hbar\left(\frac{\omega_s}{\omega_s(0)} - 2\frac{\omega_s^2}{\omega_s^2(0)} + 2\frac{\omega_s^3}{\omega_s^3(0)}\right). \tag{4}$$

Note that while the energy spectrum is maximum at the maximum photon energy, the photon count spectrum is actually symmetric about $\omega_s(0)/2$.

The rate of scattered photon production is given by the spatial overlap integral of the product of the electron and photon densities multiplied by the total Thomson cross section, σ. For the case of a head on collision, this can be expressed as

$$\frac{dN_x}{dt}(t) = 2\sigma c \iiint n_\gamma(\mathbf{x},t)\, n_e(\mathbf{x},t)\, d^3\mathbf{x}\,, \tag{5}$$

where c is the speed of light, and n_γ and n_e are the photon and electron densities respectively. In the paraxial approximation, the photon density for a Gaussian laser pulse is given by

$$n_\gamma(r,z,t) = \frac{N_\gamma}{(2\pi)^{\frac{3}{2}} c\Delta t_L x_L^2 \left(1+\frac{z^2}{z_0^2}\right)} \exp\left[-\left(\frac{t-z/c}{2\Delta t_L}\right)^2\right] \exp\left[\frac{-r^2}{2x_L^2\left(1+\frac{z^2}{z_0^2}\right)}\right] \quad (6)$$

where Δt_L is the root mean square (rms) longitudinal pulse length, x_L is the rms transverse beam size, z_0 is the Rayleigh length, and N_γ is the total number of photons in the pulse. Similarly, the the electron bunch density can be expressed as

$$n_e(r,z,t) = \frac{N_e}{(2\pi)^{\frac{3}{2}} c\Delta t_e} \frac{1}{x_e^2\left(1+\frac{z_e^2}{\beta_e^2}\right)} \exp\left[-\left(\frac{t-z_e/c}{2\Delta t_e}\right)^2\right] \exp\left[\frac{-r_e^2}{2x_e^2\left(1+\frac{z_e^2}{\beta_e^2}\right)}\right], \quad (7)$$

were β_e is the minimum beta function of the electron bunch. Assuming a 180 degree incidence angle between the laser pulse (assumed to be travelling in the z direction) and the electron bunch, the electron coordinates can be transformed to the laser coordinates simply by setting $z_e = -z$ and $r_e = r$.

Inserting Eqs. (6) and (7) into Eq. (5) and integrating over space and time, we get after a fair amount of manipulation that

$$N_x = \frac{N_e N_\gamma \sigma_T}{2\pi\left(x_L^2+x_e^2\right)}\left[\sqrt{\pi\alpha}e^{\alpha}\,\mathrm{erfc}\left(\sqrt{\alpha}\right)\right], \quad (8)$$

where we have defined

$$\alpha \equiv \frac{2\left(x_L^2+x_e^2\right)}{c^2\left(\Delta t_L^2+\Delta t_e^2\right)\left(\frac{x_L^2}{z_0^2}+\frac{x_e^2}{\beta_e^2}\right)}, \quad (9)$$

In the limit that both z_0 and β_e are longer than the duration of both the laser pulse and the electron bunch, i.e.

$$\beta_e, z_0 > c\sqrt{\left(\Delta t_L^2+\Delta t_e^2\right)}, \quad (10)$$

α will be a large number and the expression in the brackets in Eq. (8) will approach unity, leading to the much simplified expression

$$N_x \approx \frac{N_e N_\gamma \sigma_T}{2\pi\left(x_e^2+x_L^2\right)}. \quad (11)$$

X-RAY BRIGHTNESS SCALING

In terms of the x-ray beam parameters, the peak brightness can be described by

$$B_x(\omega_s) = \frac{N_x K_{\omega_s}}{(2\pi)^{5/2}\Delta t_s x_s^2 x_s'^2} \quad (12)$$

where Δt_s, x_s, and x_s' are the rms bunch duration, spot size, and divergence of the x-rays centered around ω_s, while K_{ω_s} refers to the fraction of photons with wavelength within a 0.1% bandwidth of ω_s. In the scaling relations presented here, we will assume that ω_s corresponds to the peak on axis photon energy, $\omega_s(0)$, since this will typically correspond to the highest beam brightness. In general, K_{ω_s} and x_s' will be dependent on the x-ray bandwidth resulting from the laser bandwidth and electron beam emittance and energy spread. However, for the cases where the energy spread and laser bandwidth are small (i.e. less than a few percent) the spatially integrated spectrum of the x-ray pulse will be largely independent of the details of the electron and laser distributions, and K_{ω_s} can be determined from Eq. (3), resulting in $K_{\omega_s} = 0.0015$ for $\omega_s = \omega_s(0)$. Additionally, for the cases where the x-ray spectrum at a given observation direction is dominated by the electron beam emittance, we can make the simplifying assumption that the divergence of the x-ray photons with energy approximately equal to the peak on axis energy, $\omega_s(0)$, will be equal to the electron beam divergence such that

$$x_s' \approx x_e'. \tag{13}$$

The condition for Eq. (13) to be true can be expressed as[4]

$$\frac{\varepsilon_{nx}^2}{2x_e^2} > \sqrt{\left(\frac{2\Delta\gamma}{\gamma}\right)^2 + \left(\frac{\Delta\omega_0}{\omega_0}\right)^2}, \tag{14}$$

where ε_{nx} is the rms normalized electron beam emittance, and $\Delta\gamma$ and $\Delta\omega_0$ refer to the electron beam energy spread and incident laser bandwidth respectively.

Finally, we make the additional assumption that the laser and electron bunches are short (i.e. $\alpha > 1$), such that the spot sizes of both beams will be roughly constant over the interaction. For a head on collision geometry, this will result in an x-ray pulse duration given by

$$\Delta t_s \approx \Delta t_e, \tag{15}$$

and a transverse source size given by

$$x_s^2 \approx \frac{x_e^2 x_L^2}{x_e^2 + x_L^2}. \tag{16}$$

Plugging Eqs. (11), (13), (15), and (16) into Eq. (12) results in the following expression for the peak x-ray brightness for the case of a head on collision,

$$B_x = \frac{K_{\lambda_s}\sigma_T}{(2\pi)^{7/2}} \frac{N_\gamma}{x_L^2} \gamma^2 \frac{N_e}{\Delta t_e \varepsilon_{nx}^2}, \tag{17}$$

where we used the relation $\varepsilon_{nx} = \gamma x_e x_e'$, assuming the electron beam is at a waist. Note that if Eqs. (14) and (10) are not satisfied, then the brightness will be less than that described above, and Eq. (17) can be considered an upper limit. In units of photons/s/mm^2/mrad^2/0.1%b.w., Eq. (17) can be expressed more conveniently as

$$B_x = 5.05 \times 10^{18} \gamma^2 \frac{\lambda(\mu m)\, Q_e(nC)\, W_\gamma(Joules)}{\Delta t_e(ps)\, \varepsilon_{nx}^2(mm-mrad)\, x_L^2(\mu m)}. \tag{18}$$

X-RAY BRIGHTNESS OPTIMIZATION

It is noteworthy that the final focus spot size of the electron beam does not appear in Eq. (17), implying that while improvements in the final focus optics of the electron beam may be beneficial in increasing the x-ray dose, the x-ray brightness will only be determined by the inherent quality, or brightness, of the electron beam provided it has been focused to a small enough spot for Eq. (14) to have been satisfied. In fact, for a given electron beam brightness and laser beam energy, the optimization of the peak x-ray brightness is essentially reduced to optimizing the laser spot size and duration. This optimization will involve finding the balance between minimizing the x-ray source size, while both avoiding spectral broadening due to the onset of non-linear dynamics[5][6], and maintaining the condition specified by Eq. (10) so as to maximize the x-ray dose.

The prior constraint requires that the normalized vector potential in the laser pulse, a_0, have a value much less than one, where

$$a_0^2 = \frac{4N_\gamma}{(2\pi)^{5/2}} \frac{\lambda r_0 \lambda_c}{x_L^2 c\Delta t_L}, \tag{19}$$

where r_0 is the classical electron radius, λ_c is the Compton wavelength, and λ is the wavelength of the incident laser pulse. To meet the latter constraint while minimizing a_0, the pulse duration should be set roughly equal to the Rayleigh length such that,

$$c\Delta t_L \sim z_0 = \frac{4\pi x_L^2}{\lambda}. \tag{20}$$

This insures that, for a given laser energy, the minimum acceptable laser spot size is obtained for a given value of a_0. Inserting Eq. (20) into Eq. (19) leads to

$$N_\gamma = \frac{(2\pi)^{7/2} x_L^4}{2\lambda^2 r_0 \lambda_c} a_0^2. \tag{21}$$

Thus, for a given laser energy and maximum value of a_0, the optimum laser spot size and duration can be expressed as

$$x_L|_{opt} = \frac{3.24\mu m}{\sqrt{a_0}} \times [\lambda\,(\mu m)]^{3/4} \times \left[W_\gamma\,(Joules)\right]^{1/4}, \tag{22}$$

and

$$\Delta t_L|_{opt} = \frac{0.44ps}{a_0} \times \sqrt{\lambda\,(\mu m)} \times \sqrt{W_\gamma\,(Joules)}. \tag{23}$$

Plugging these into Eq. (18), the following expression is obtained for the optimum peak x-ray brightness obtainable from linear Thomson scattering for the case of a 180^0 interaction geometry:

$$B_x\,(\omega_s)\,|_{opt} \equiv 4.81 \times 10^{17} a_0 \gamma^2 \frac{Q_e\,(nC)\,\sqrt{W_\gamma\,(Joules)}}{\Delta t_e\,(ps)\,\varepsilon_{nx}^2\,(mm-mrad)\,\sqrt{\lambda\,(\mu m)}}, \tag{24}$$

where it has been assumed that Δt_e is shorter than and β_e greater than the optimized laser pulse duration, $\Delta t_L|_{opt}$.

TABLE 1. Typical laser and electron beam parameters for PLEIADES

Electron Beam		Laser Beam	
Energy	50 MeV	Wavelength	800 nm
Charge	0.25 nC	Energy	500 mJ
x_e	30 μm	x_l	18 μm
Δt_e	$3ps$	Δt_l	$50fs$
ε_{nx}	$\approx 10mm-mrad$	z_0	$\approx 5mm$

COMPARISON TO MEASUREMENTS AND 3D THEORY

Equations (22) and (23) provide a very simple formalism for optimizing a Thomson backscattered x-ray source for a fixed incident laser energy and electron beam brightness. As an illustrative example, we discuss this optimization in the context of the PLEIADES Thomson x-ray source[2][3]. PLEIADES produces Thomson backscattered x-rays through the head-on collision of 50-80 MeV electron bunches with a 50 fs, 800 nm laser pulse. Measurements of the x-ray dose have resulted in up to 10^7 photons per pulse, with maximum photon energies ranging from 40-140 keV depending on the electron beam energy. The estimated peak brightness of the x-ray pulse produced from this experiment is about 10^{16} photons/s/mm^2/mrad^2/0.1%b.w.

Typical electron and laser beam parameters are listed in Table 1. The experimentally determined brightness of the x-ray source, estimated from measurements of the x-ray dose and 3D simulations of the x-ray spectrum based on measurements of the electron beam phase space, agrees well with the theoretical value of 5×10^{16} photons/s/mm^2/mrad^2/0.1%b.w. derived from Eq. (18). However, using Eq. (24) along with the parameters in Table 1, and specifying that $a_0 \leq 0.1$, we find that $B_x|_{opt} = 3\times10^{17}$ photons/s/mm^{2}mrad^2/0.1%b.w. with $\Delta t_L|_{opt} = 2.8$ ps and $x_L|_{opt} = 7\mu m$. Thus, a roughly order of magnitude improvement in the peak brightness of the source should be possible.

To verify this scaling, we compare in Fig. 1 the peak brightness predicted by Eq. (18) to that calculated through three dimensional time and frequency domain simulations of the Thomson interaction[4]. The laser energy is kept constant at 500 mJ, while the spot size and pulse duration are varied while maintaining $a_0 = 0.1$. The electron beam is simulated by constructing a series of macro-particles with an angular and energy distribution consistent with measurements of the electron beam energy spread and emittance. The simulated brightness is plotted vs. x_L, with $B_x|_{opt}$ and $x_L|_{opt}$ (as previously specified) indicated by the horizontal and vertical lines on the plot. In addition, the calculated brightness determined by Eq. (18) is plotted for reference. While the maximum simulated x-ray brightness is roughly coincident with the predicted optimum parameters, the actual optimum does occur at a somewhat smaller value of x_L, with a slightly larger value of $B_x|_{opt}$. This is not surprising since the condition specified by Eq. (20) is only a rough guideline, with the exact optimum also depending on the electron beam spot size. Also plotted in Fig. 1 is the simulated brightness optimization for the case of an ideal, high brightness electron beam. In this case, we assume $\varepsilon_{nx} = 1.0$ mm-mrad, $\Delta t_e = 0.3$

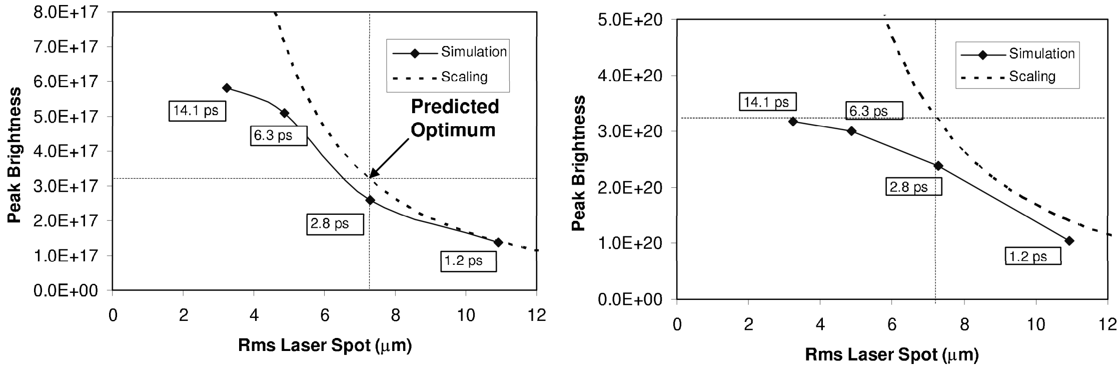

FIGURE 1. Simulated peak x-ray brightness in units of photons/s/mm^{2}mrad^2/0.1%b.w. (solid line) for the case of experimentally determined electron beam parameters (left), and idealized electron beam parameters (right). The laser pulse duration (boxes) is varied to keep the maximum normalized vector potential ,a_0, constant. Dotted line: calculated britghtness based on Eq. 18

ps, $x_e = 10\ \mu m$, and $Q_e = 0.25$ nC. In this case as well, the scaling relations provide a good guideline for optimization. The much higher optimized brightness of this case also highlights the importance of high brightness electron beam production.

CONCLUSIONS

In this paper we have presented scaling relations illustrating the dependence of the x-ray beam peak brightness obtained from linear Thomson scattering in a 180 degree interaction geometry on the electron beam and laser beam parameters. These relations provide an easy way to optimize a source design based on available electron beam brightness and laser energy. In addition, it is apparent that there is a direct relationship between the electron beam peak brightness and the x-ray source peak brightness, highlighting the importance of ultra-low emittance, femtosecond electron bunch production for the advancement of high brightness x-ray source development.

REFERENCES

1. Schoenlein, W. P. L. R. W., Volfbeyn, P., Chin, A. H., Glover, T. E., Balling, P., Zolotorev, M., Kim, K. J., Chattopadhyay, S., and Shank, C. V., *IEEE J. Quantum Electron.*, **33**, 1925 (1997).
2. Gibson, D. J., Anderson, S. G., Barty, C. P. J., Betts, S. M., Booth, R., Brown, W. J., Crane, J. K., Cross, R. R., Fittinghoff, D. N., Hartemann, F. V., Kuba, J., Sage, G. P. L., Tremaine, A. M., and Springer, P. T., *Phys. Plasmas*, **11**, 2857–2864 (2004).
3. Brown, W. J., Anderson, S. G., Barty, C. P. J., Betts, S. M., Booth, R., Crane, J., Cross, R. R., Fittinghoff, D., Gibson, D., Hartemann, F., Hartouni, E. P., Kuba, J., Sage, G. P. L., Slaughter, D. R., Tremaine, A., Wootton, A., and Springer, P., *Phys. Rev. Special Topics: Accelerators and Beams*, **7**, 060702 (2004).
4. Brown, W. J., and Hartemann, F. V., *Phys. Rev. Special Topics: Accelerators and Beams*, **7**, 060703 (2004).
5. Hartemann, F. V., and Kerman, A. K., *Physical Review Letters*, **76**, 624 (1996).
6. Esarey, E., Ride, S. K., and Sprangle, P., *Phys. Rev. E*, **48**, 3003 (1993).