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Abstract

Analytical simulations of fast-electron currents induced by high-density laser-plasma interactions require estimation of various plasma and beam parameters, including temperatures, densities, and collision rates. This note describes a technique used to estimate or calculate these parameters for the case of contemporary multi-terawatt experiments using foil targets as well as for anticipated fast-ignition-scale experiments.

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I. INTRODUCTION

Fast ignition is a technique which uses high-intensity short-pulse lasers to heat a fuel target and ignite a thermonuclear reaction [1]. The success of this technique relies on transporting energy through ever-denser plasma to a point of high pressure inside a target.

A 1 micron laser penetrates fully ionized plasma only to the critical density at about $3\,\mathrm{mg/cm^3}$ (in DT) where it can transfer energy to a beam of high-energy electrons, which in fast ignition would penetrate to the peak fuel density of about $300\,\mathrm{g/cm^3}$ and heat the target to an ignition temperature of about 10 keV. Alternatively, these electrons could be used to accelerate a beam of protons from an intermediary target which (with ballistic focusing) could heat the fuel [2].

In either case, the transport of these 'hot' electrons is not well understood. A number of phenomena could attenuate the electron beam sufficiently to preclude the transport of sufficient energy. Ohmic potential due to the return current and a two-stream electromagnetic instability known as the beam-Weibel instability (analogous to the well-known Weibel instability [3]) could both significantly impede electron transport.

The growth rate of this Weibel-like beam instability is dependent upon collision rates and plasma densities and temperatures. Estimation of collision rates and other relevant plasma parameters is therefore important and is the subject of this note. Two cases will be considered: contemporary laser-plasma experiments using aluminum as the target material [4], and full scale fast-ignition experiments using compressed deuterium-tritium targets.

II. PLASMA PARAMETERS OF INTEREST

A. Plasma temperature

The temperature of a laser-induced plasma as a function of density must be estimated. Temperatures in electron-heated aluminum foil targets on the order of 100 eV have been observed in experiment [5]. At lower densities, down to the critical density, there is an approximate pressure equilibrium resulting in a roughly $1/\rho$ temperature dependence [6].

In ICF targets (compressed DT) the temperatures at the highest density must reach 10 keV for ignition to occur. A time-averaged temperature of 3 keV is assumed. At lower densities, the $1/\rho$ dependence is used to estimate the temperature.

B. Electron beam temperatures

The characteristic parameters of the hot electron beam in near-term metal foil experiments can be estimated from previous experiments. Beam temperatures of 600 keV are typical [7], and the 20° half-angle spread of the beam as it propagates through the target observed in experiment geometrically corresponds to a transverse beam temperature of about 30 keV.

In the fast ignition scenario, the beam energies must be limited to about 1 MeV in order for the electrons to be absorbed within the target. A 20° half-angle spread of this beam would correspond to a transverse beam temperature of about 35 keV.

C. Hot electron density

In the case of the near-term experiments, the hot electron density is estimated from previous experiments. On the Vulcan laser at the Rutherford-Appleton Laboratory for example, a typical laser shot delivers 100 Joules in one picosecond, with about one-third of the energy concentrated in a radius of five microns; this corresponds to a laser intensity of 4.2×10^{19} W/cm^2. The laser energy is converted to hot electron energy with an efficiency of approximately 30% [8], resulting in an electron beam with an intensity of 1.3×10^{19} W/cm^2. Abrupt widening of the electron beam to about 35 microns in diameter is observed in experiment; this lowers the intensity to 2.6×10^{17} W/cm^2 or, alternatively, 1.6×10^{30} MeV/s $\cdot$ cm^2. If the average electron energy is assumed to be 600 keV, the current density is 4.3×10^{11} A/cm^2; with a relativistic beta of .89, the electron density of this expanded beam is estimated to be approximately 10^{20} cm^{-3}.

In the case of the fast ignition experiment, the hot electron density is determined to be the minimum required for 1 MeV electrons to carry sufficient energy to ignite the target [9]. The ignition threshold beam intensity is calculated to be

$$I_{ig} = 2.4 \times 10^{19} \left(\frac{\rho}{100\ g/cm^3} \right)^{.95} \left[\frac{W}{cm^2} \right]$$

and the appropriate beam radius has been estimated to be 17 microns. At 300 g/cm^3, this is $\mathrm{I_{ig}} = 6.8 \times 10^{19}\,\mathrm{W/cm^2} = 4.3 \times 10^{32}\,\mathrm{MeV/s \cdot cm^2}$. At 1 MeV per electron, this gives a current density of 6.8×10^{13} A/cm^2. At a relativistic beta of .94, this corresponds to a density of $1.5 \times 10^{22}\text{cm}^{-3}$. The electrons are excited from the background by the laser interaction; the resulting space charge prevents anything more than a fraction of the available electrons to be accelerated. For this analysis, a 10% excitation limit is applied; the hot electron density can never be more than one-tenth of the background cold electron density.

D. Collision rates

The average electron relaxation time of an electron in a plasma, as determined by Lee and More [10], is

$$\tau = \frac{3\sqrt{m}\,(kT)^{\frac{3}{2}}}{2\sqrt{2}\pi\,(Z^*)^2\, n_i e^4 \ln\Lambda}\left[1+\exp\left(-\frac{\mu}{kT}\right)\right] F_{\frac{1}{2}}\left\{\frac{\mu}{kT}\right\}$$

where m is the electron mass, k is the Boltzmann constant, T is the plasma temperature, Z^* is the average ionization state, n_i is the ion number density, e is the electron charge, $\ln\Lambda$ is the Coulomb logarithm, μ is the chemical potential, and $F_{\frac{1}{2}}\left\{\frac{\mu}{kT}\right\}$ is the Fermi-Dirac integral of order $\frac{1}{2}$ of the fugacity. The collision rate is the reciprocal of the relaxation time. These collision rate calculations were benchmarked against Figure 3 of Lee and More.

For numerical analyses requiring the determination of collision rates, the ion number density is used as the independent variable. At each density, Z^*, $\ln\Lambda$, μ, and $F_{\frac{1}{2}}\left\{\frac{\mu}{kT}\right\}$ must be estimated or calculated.

E. Ionization state

At low temperatures, the effective ionization state of a material is the same as its number of conduction-band electrons. For aluminum, this number is three; for hydrogen, it is zero. As the temperature increases, the ions are thermally ionized to higher states. Using a Thomas-Fermi model, the average thermal ionization can be calculated as a function of pressure and temperature [11]. (This model may overestimate the ionization, as it assumes that the material in question is in equilibrium.) The plasmas involved in this research are not in equilibrium; the conditions used to make the calculations in this note are 'snapshots' of a transient process, especially at the lower plasma densities.

F. Coulomb logarithm

The Coulomb logarithm is defined as the logarithm of the plasma parameter Λ, defined as $\Lambda \equiv \sqrt{1 + \frac{b_{max}^2}{b_{max}^2}}$ where $\mathrm{b_{max}}$ and $\mathrm{b_{min}}$ are the upper and lower cutoffs respectively of the Coulomb scattering impact parameter.

The maximum impact parameter is determined by screening effects. At high temperatures and low densities, where the Debye-Hückel theory is applicable, this parameter can be set to the Debye-Hückel screening length λ_{DH}:

$$\frac{1}{\lambda_{DH}^2} = \frac{4\pi n_e e^2}{kT} + \frac{4\pi n_i \left(Z^* e\right)^2}{kT_i}$$

where T_i is the ion temperature and n_e is the electron number density. At higher densities or lower temperatures, this equation must be modified to account for degeneracy:

$$\frac{1}{\lambda_{DH}^2} = \frac{4\pi n_e e^2}{kT} \frac{F'_{\frac{1}{2}}\left\{\frac{\mu}{kT}\right\}}{F_{\frac{1}{2}}\left\{\frac{\mu}{kT}\right\}} + \frac{4\pi n_i \left(Z^* e\right)^2}{kT_i} \approx \frac{4\pi n_e e^2}{k\sqrt{T^2 + T_F^2}} + \frac{4\pi n_i \left(Z^* e\right)^2}{kT_i}$$

where T_F is the Fermi temperature. On the short time scale of the laser pulse, the ions are considered to be immobile (that is, $T_i = 0$) and so in each case the second term is neglected.

As the plasma density increases, the Debye-Hückel model becomes invalid when the calculated screening length drops below the inter-atomic distance $\frac{1}{n^3}$. The maximum impact parameter is then $\mathrm{b_{min}} = \max\left[\lambda_{DH}, \frac{1}{n_i^3}\right]$.

The minimum impact parameter is the classical closest approach distance $\frac{Z^* e^2}{mv^2}$. This distance decreases with temperature and thus electron energy, but is limited by the Heisenberg uncertainty to be greater than half of the deBroglie wavelength $\frac{h}{mv}$. The minimum impact parameter is then $\mathrm{b_{min}} = \max\left[\frac{Z^* e^2}{mv^2}, \frac{h}{mv}\right]$. Evaluated at the average thermal electron velocity $\overline{v} = \sqrt{\frac{3kT}{m}}$, $\mathrm{b_{min}} = \max\left[\frac{Z^* e^2}{3kT}, \frac{h}{2\sqrt{3mkT}}\right]$

G. Chemical potential and Fermi-Dirac integral

The probability of a fermion being found in a state of given energy E is given by the Fermi-Dirac distribution $f\left\{E\right\} = \frac{1}{\exp\left(\frac{E-\mu}{kT}\right)+1} = \frac{1}{\exp\left(\frac{E}{kT}\right)\exp\left(\frac{-\mu}{kT}\right)+1}$ where μ is the chemical potential (the energy required to add another particle to the system). In principle it is possible to measure both the energies of the electron states and the chemical potential directly, but in practice the energy levels are proposed by a model of the electron states and the corresponding chemical potential is determined implicitly.

The number of states in a given energy interval dE is given by $g\{E\}\,dE$ where $g\{E\}$ is the state density function. In a system with a constant potential (the free electron gas, for example) the density function is

$$g\{E\} = g_D\left(\frac{2\pi V}{h^3}\right)(2m)^{\frac{3}{2}}\sqrt{E}$$

where V is volume, m is the particle mass, and g_d is the spin degeneracy 2S+1, which is 2 for the spin-$\frac{1}{2}$ electron.

The chemical potential can be determined implicitly from the integral for the number of particles:

$$\begin{aligned} N &= \int_0^\infty g_D\left(\frac{2\pi V}{h^3}\right)(2m)^{\frac{3}{2}}\sqrt{E}\frac{1}{\exp\left(\frac{E}{kT}\right)\exp\left(\frac{-\mu}{kT}\right)+1}dE \\ &= g_d\left(\frac{2\pi V}{h^3}\right)(2m)^{\frac{3}{2}}\int_0^\infty \frac{\sqrt{E}dE}{\exp\left(\frac{E}{kT}\right)\exp\left(\frac{-\mu}{kT}\right)+1} \end{aligned}$$

With the substitutions $t=\frac{E}{kT}$, $dt=\frac{dE}{kT}$, and $\alpha=\frac{\mu}{kT}$, this becomes:

$$\begin{aligned} N &= g_D\left(\frac{2\pi V}{h^3}\right)(2m)^{\frac{3}{2}}\int_0^\infty (kT)^{\frac{3}{2}}\sqrt{E}\frac{\sqrt{t}dt}{e^t e^{-\alpha}+1} \\ &= g_D\left(\frac{2\pi V}{h^3}\right)(2mkT)^{\frac{3}{2}}\int_0^\infty \sqrt{E}\frac{\sqrt{t}dt}{e^t e^{-\alpha}+1} \end{aligned}$$

The integral $F_{\frac{1}{2}}\{\alpha\} = \int_0^\infty \frac{\sqrt{t}dt}{e^t e^{-\alpha}+1}$ is the Fermi-Dirac integral of order $\frac{1}{2}$ of the fugacity $\alpha=\frac{\mu}{kT}$. The correct chemical potential μ is the solution of $N = g_D\left(\frac{2\pi V}{h^3}\right)(2m)^{\frac{3}{2}}F_{\frac{1}{2}}\left\{\frac{\mu}{kT}\right\}$.

In the degenerate limit ($T\rightarrow 0$) the chemical potential of plasma tends towards the Fermi energy and the fugacity tends towards infinity. The behavior of the chemical potential in the nondegenerate limit is not as simple; as the temperature increases, the chemical potential first rises, then precipitously falls faster than T increases. In the nondegenerate limit ($T\rightarrow\infty$), both the chemical potential and the fugacity tend towards negative infinity.

III. RESULTS

For a range of densities along the path of the electron beam into the aluminum and DT targets, the electron temperatures and hot electron densities have been estimated and the remaining relevant parameters have been calculated and are given in Tables I and II respectively.

These parameters are currently being used in modeling of the beam-Weibel instability to predict the effect of the instability upon electron propagation in both near-term and full-scale Fast Ignition experiments.

Cold Aluminum	.001 g/cm^3	.01 g/cm^3	.1 g/cm^3	1 g/cm^3	2.7 g/cm^3
Ion density (cm^{-3})	2.23×10^{19}	2.23×10^{20}	2.23×10^{21}	2.23×10^{22}	2.23×10^{23}
Cold electron density (cm^{-3})	2.90×10^{20}	2.90×10^{21}	2.89×10^{22}	2.50×10^{23}	4.72×10^{23}
Hot electron density (cm^{-3})	2.90×10^{19}	1.0×10^{20}	1.0×10^{20}	1.0×10^{20}	1.0×10^{20}
Cold electron temperature (keV)	108	10.8	1.08	.108	.04
Hot electron temperature (keV)	28.978	28.978	28.978	28.978	28.978
Beam energy (keV)	600	600	600	600	600
Average ion charge	13	12.999	12.966	11.188	7.833
Chemical potential (keV)	-2205.33	-158.364	-9.622	-.355	-.0426
Cold electron collision rate(s^{-1})	2.95×10^{9}	7.47×10^{11}	1.5×10^{14}	1.39×10^{16}	3.73×10^{16}
Cold electron collision rate ($\times \omega_p$)	3.07×10^{-6}	2.46×10^{-4}	1.56×10^{-2}	4.94×10^{-1}	9.64×10^{-1}
Hot electron collision rate(s^{-1})	1.98×10^{10}	1.8×10^{11}	1.17×10^{12}	7.13×10^{12}	8.95×10^{12}
Hot electron collision rate ($\times \omega_p$)	2.06×10^{-5}	5.93×10^{-5}	1.21×10^{-4}	2.53×10^{-4}	2.31×10^{-4}

Table I: Parameters for aluminum target using contemporary laser energies

Compressed DT	$.001\,\mathrm{g/cm^3}$	$.01\,\mathrm{g/cm^3}$	$.1\,\mathrm{g/cm^3}$	$1\,\mathrm{g/cm^3}$	$10\,\mathrm{g/cm^3}$	$100\,\mathrm{g/cm^3}$	$300\,\mathrm{g/cm^3}$
Ion density $(\mathrm{cm^{-3}})$	2.41×10^{20}	2.41×10^{21}	2.41×10^{22}	2.41×10^{23}	2.41×10^{24}	2.41×10^{25}	7.23×10^{25}
Cold electron density $(\mathrm{cm^{-3}})$	2.41×10^{20}	2.41×10^{21}	2.41×10^{22}	2.41×10^{23}	2.40×10^{24}	2.40×10^{25}	7.19×10^{25}
Hot electron density $(\mathrm{cm^{-3}})$	2.41×10^{19}	2.41×10^{20}	2.41×10^{21}	1.50×10^{22}	1.50×10^{22}	1.50×10^{22}	1.50×10^{22}
Cold electron temperature (keV)	3000	300	30	3	3	3	3
Hot electron temperature (keV)	32.9	32.9	32.9	32.9	32.9	32.9	32.9
Beam energy (keV)	1000	1000	1000	1000	1000	1000	1000
Average ion charge	.999	.999	.999	.999	.998	.996	.994
Chemical potential (keV)	-6641.07	-491.141	-31.873	-24.966	-18.060	-11.135	-7.793
Cold electron collision rate$(\mathrm{s^{-1}})$	3.96×10^{7}	9.94×10^{9}	2.32×10^{12}	2.00×10^{12}	1.66×10^{14}	1.32×10^{15}	3.38×10^{15}
Cold electron collision rate $(\times\omega_\mathrm{p})$	4.52×10^{-8}	3.59×10^{-6}	2.66×10^{-4}	7.21×10^{-5}	1.90×10^{-3}	4.77×10^{-3}	7.06×10^{-3}
Hot electron collision rate$(\mathrm{s^{-1}})$	9.6×10^{8}	8.71×10^{9}	6.37×10^{10}	5.42×10^{11}	4.50×10^{12}	$3.6. \times 10^{13}$	9.69×10^{13}
Hot electron collision rate $(\times\omega_\mathrm{p})$	1.10×10^{-6}	3.15×10^{-6}	7.28×10^{-6}	1.96×10^{-5}	5.15×10^{-5}	1.31×10^{-4}	2.03×10^{-4}

Table II: Parameters for DT target using fast-ignition-class laser

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