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On Rapidly Rotating Magnetic Core-Collapse Supernovae

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Abstract

We have analyzed magnetic effects which may occur in rapidly rotating core collapse supernovae. We consider effects from both magnetic turbulence and the formation of magnetic bubbles. For magnetic turbulence we have made a perturbative analysis for our spherically symmetric core-collapse supernova model that incorporates the build up of magnetic field energy in the matter accreting onto the proto-neutron star shortly after collapse and bounce. This significantly modifies the pressure profile and increases the heating of the material above the proto-neutron star resulting in an explosion even in rotating stars which would not explode otherwise. Regarding magnetic bubbles it shown that a model with an initial uniform magnetic field ($\sim 10^8$) gauss and uniform angular velocity of (~ 0.1 rad sec^{-1}) can form magnetic bubbles due to the very non homologous nature of the collapse. It is estimated that the buoyancy of the bubbles causes matter in the proto-neutron star to rise, carrying neutrino-rich material to the neutron-star surface. This increases the neutrino luminosity sufficiently at early times to achieve a successful neutrino-driven explosion. Both magnetic mechanisms thus provide new means for initiating a Type II core-collapse supernova.

Gravitation - Hydrodynamics - Instabilities - Stars: Magnetic Field - Stars: rotation - Stars: Supernovae: general

1 Introduction

In most supernova models with pure spherical symmetry, after a massive star collapses due to the exhaustion of it's nuclear fuel, the neutrino luminosity from the proto-neutron star is too low to heat the in-falling material sufficiently to expel matter from the star (e.g. Bruenn 1993, Burrows, Hayes, & Fryxell 1995; Fryer et al. 1999; Fryer & Heger 2000; Rampp & Janka 2000, 2002; Liebendvrfer, Mezzacappa et al. 2001; Liebendörfer et al. 2001; Mezzacappa et al 2001; Akiyama et al. 2003 Buras et al. 2003; Thompson, Burrows, & Pinto 2003; Burrows 2004; Cardall 2004). However, the Livermore supernova model (cf. Wilson & Mayle 1988; 1993; Wilson & Mathews 2003) avoids this problem and is able to explode in spherical symmetry by inducing a larger amount of neutrino heating soon after the core bounce. For this reason it is important to examine any possible means to induce additional heating above the proto-neutron star.

One important mechanism for such heating, for example, is that the proto-neutron star can become hydrodynamically unstable a few hundred milliseconds after the core bounce due to the so-called "neutron-finger instability" (Wilson & Mayle 1988; Wilson & Mathews 2003). This instability results from the build up of high density material with a large neutron-to-proton ratio near the surface of the proto-neutron star. Sufficiently neutron-rich material can overcome the buoyancy caused by the high entropy near the surface. As surface material sinks downward neutrino-rich material rises to the surface. This enhances the neutrino luminosity and produces enough heating of material behind the shock to produce an explosion (cf. Wilson & Mathews 2003).

The fact that other models of Type II supernovae do not exhibit this instability can be attributed to a number of possibilities (e.g. Bruenn et al. 2004) such as differences in the equation of state employed, the detailed way in which convection is treated, and/or the treatment of neutrino flow. Hence, this mechanism is controversial (Bruenn et al. 2004) as a means to induce core-collapse supernovae. In this paper, therefore, we describe first schematic calculations of some plausible, and perhaps more compelling, alternatives to the neutron-finger instability to overcome the lack of sufficient neutrino luminosity at early times in the explosion. We have investigated two magneto-hydrodynamic (MHD) effects both above and below the rotating proto-star surface that may be strong enough to enhance the neutrino luminosity and produce an explosion. These processes could either increase the neutron-finger instability or replace it in models that have no surface convection.

2 Models

The purpose of the present paper is to make a schematic study of the possible roles of magnetohydrodynamics in rotating core-collapse supernovae. While the models we utilize are adequate to illustrate the order of magnitude of these effects, we emphasize that it will be necessary to do this calculation in two or three spatial dimensional magneto-hydrodynamics (MHD) to prove that the ideas presented here are correct. Such calculations, however, present an exceedingly difficult computational challenge which a number of efforts (including our own) are attempting to achieve. For the purposes of the present schematic investigation, however, a much simpler model is employed which involves the average spherical effects of the inherently multidimensional rotation and magnetic effects as a perturbation on one-dimensional hydrodynamics. This is a reasonable approximation as a means of exploring the parameter space and extracting the essential physics as long as we are considering small rotation rates and relatively weak but realistic magnetic fields.

As we shall see, the main effects in our perturbation analysis are build up of magnetic turbulence and field energy above the proto-neutron star shortly after collapse and bounce due to the accretion of magnetized matter, and the

formation of magnetic bubbles and magnetic-driven convection below the surface of the PNS. We demonstrate that both of these effects can significantly impact the explosion mechanism.

2.1 Supernova Model

Details of the current version of Livermore supernova model have been described in Wilson & Mathews (2003). For completeness we here summarize the basic physics and the way in which the effects of rotation and MHD implemented. To begin with the metric for a spherical neutron star is written in Lagrangian coordinates,

$$\begin{aligned} ds^2 &= -a^2\left[1-\left(\frac{U}{\Gamma}\right)^2\right]dt^2 - \frac{2aU}{\Gamma^2}drdt + \frac{dr^2}{\Gamma^2} \\ &+ r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad , \end{aligned} \tag{1}$$

where a is the inverse of the time component of four velocity $a \equiv 1/U^t$. It is, thus, related to the gravitational red shift. The quantity r is a distance coordinate with proper distance is given by

$$\text{Proper Distance} = \int \frac{dr}{\Gamma} \quad . \tag{2}$$

where,

$$\Gamma \equiv \left(1 + U^2 - \frac{2M}{r}\right)^{1/2} \quad . \tag{3}$$

The quantity M is the gravitational mass interior to r as defined below, and $U \equiv U^r$ is the radial component of the four velocity.

$$U = \frac{1}{a}\frac{\partial r}{\partial t} \quad . \tag{4}$$

For the metric coefficient a the vanishing of the covariant derivative $T^{mj}_{\ ;j} = 0$ implies,

$$\begin{aligned} a &= exp\left[\int_m^{m_{max}} \frac{dm}{\rho h}\left(\frac{\partial P_{eff}}{\partial m} + \frac{br^2\rho}{a}\frac{\partial}{\partial t}\left\{\frac{\Phi_\nu}{r^2\rho^2}\right\}\right.\right. \\ &- \left.\left.\frac{2b\Gamma}{r}W_\nu\right)\right] \quad , \end{aligned} \tag{5}$$

where m_{max} is the mass coordinate at the boundary of the numerical grid. The quantity $h \equiv 1 + \epsilon + P/\rho$ is the relativistic enthalpy, and

$$b \equiv \frac{1}{4\pi r^2 \rho} \quad . \tag{6}$$

The quantities Φ_ν and W_ν refer to the angle integrated neutrino flux and a nonthermal neutrino pressure correction factor, respectively, as described in Wilson & Mathews (2003).

2.2 Matter Equations

For the present application we write the radial four acceleration as,

$$\frac{1}{a}\frac{\partial U}{\partial t} = -\frac{\Gamma}{\rho h}\left[\frac{1}{b}\frac{\partial P_{eff}}{\partial m} + \frac{r^2\rho^2}{a}\frac{\partial}{\partial t}\left\{\frac{\Phi_\nu}{r^2\rho^2}\right\} - \frac{2\Gamma}{r}W_\nu\Big)\right] - \frac{M}{r^2} - 4\pi r P_{eff} \quad . \tag{7}$$

As discussed below, effects of rotation and magnetic field energy are absorbed into an effective pressure in the acceleration equation, i.e. we write:

$$P_{eff} = P_M + P_\nu + P_{rot} + P_{mag} \quad , \tag{8}$$

where P_M and P_ν are the usual contributions from matter in thermal equilibrium and neutrinos which are nonthermal and must be transported explicitly. The effective pressure perturbations from rotation and magnetic field energy, P_{rot} and P_{mag}, are defined below in section **??**.

The condition of baryon number conservation leads to auxiliary equations for the matter evolution:

$$\rho = \frac{1}{b}\frac{\partial r}{\partial m} \quad . \tag{9}$$

$$\frac{1}{a}\frac{\partial \rho}{\partial t} = -\rho\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 U) + \frac{1}{2\Gamma}\rho r \Phi_\nu \quad . \tag{10}$$

The gravitational mass is given by

$$M = 4\pi\int_0^m dm\frac{\partial r}{\partial m}r^2\left[\rho(1+\epsilon) + \frac{U}{\Gamma}\Phi_\nu\right] \tag{11}$$

$$\frac{1}{a}\frac{\partial M}{\partial t} = 4\pi r^2(UP + \Gamma\Phi_\nu) \quad . \tag{12}$$

$$\frac{1}{b}\frac{\partial M}{\partial m} = 4\pi r^2\left[\Gamma\rho(1+\epsilon) + U\Phi_\nu\right] \quad . \tag{13}$$

The baryon rest mass of the star is then simply given by the integral over the proper volume, $d(Vol) = 4\pi r^2 dr/\Gamma$,

$$M_0 = 4\pi\int r^2 dr\frac{\rho}{\Gamma} \quad . \tag{14}$$

The matter internal energy evolves according to

$$\frac{1}{a}\frac{\partial \epsilon_M}{\partial t} = -P_M\frac{1}{a}\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) - \frac{1}{\rho}\sum_{i=1}^{6}\int \Lambda_i dE d\Omega_\nu \quad , \tag{15}$$

where P_M is the matter pressure and the Λ_i are various neutrino scattering and absorption source terms (Wilson & Mathews 2003). The neutrino transport is treated with appropriate flux-limited diffusion.

The condition of lepton number conservation leads to an expression for the change in the average electron fraction (or charge per baryon) Y_e due to weak interactions,

$$\frac{\rho}{m_b}\frac{1}{a}\frac{\partial Y_e}{\partial t} = -\sum_i (\Lambda_i - \bar{\Lambda}_i)\frac{dq}{q}d\Omega_\nu \quad , \tag{16}$$

where $q \equiv a\epsilon_\nu$, ϵ_ν is the neutrino energy, and q is the energy a neutrino would have if it was removed to infinity.

2.3 Initial Conditions

The initial MHD model assumes that the star is rotating with a uniform angular velocity in the inner 5 $M_\odot$ of the star. It is also threaded by a uniform magnetic field in the direction of the axis of rotation before the start of the dynamic core-collapse phase. Various initial models have been explored. For the models reported on here, the strength of the initial magnetic field is chosen such that the final neutron star would have a surface magnetic field of $H \sim 10^{12}$ gauss.

This initial rotational velocity in these models leads to a post collapse rotation period ($P \sim 1.4$ ms). This is close to the shortest possible Keplerian neutron-star rotation period ($P \lesssim 1$ ms, Burgio, Schulze & Weber 2003), and is comparable to the minimum values in the observed period distribution for pulsars (Phinney & Kulkarni 1994; Weber 1999; Manchester 2004). Furthermore, observed young pulsars (Manchester 2004) in supernova remnants have much longer periods (~ 1 sec) than that obtained in these calculations. However they also have large spin down rates. Indeed, the stars modeled here should also have a large spin-down rate. Since the poloidal field threads the rapidly rotating neutron star and we adopt a massive outer envelope of very low angular velocity, a very high torque should develop between the spinning neutron star and the outer star plus magnetic field. These torques are sufficient to slow the neutron star down to well within the observed range by the time it becomes an observable pulsar.

2.4 Magnetic Turbulence above the PNS

Above the proto-neutron star for a few tenths of a second after bounce is a region below the bounce shock front and above the almost static proto-neutron star radius where matter is slowly accreting. At post bounce times typically $t_{pb} \sim 200$ ms, the proto-neutron star radius is ~ 40 km and the shock radius is ~ 170 km. Later, at $t_{pb} \approx 300$ ms, the proto-neutron star radius is 32 km and the shock radius has contracted to 140 km. The Mach number in the sub-shock region is < 0.1.

In the sub-shock region magnetic field generation is possible for a rapidly rotation magnetized collapsing star. To model the evolution of the magnetic field, we follow the general principles given in Balbus and Hawley (1998) [hereafter BH98].

The work of BH98 has led us to examine the stability of the accretion flow of matter in the waist region shortly after core bounce. After bounce a shock moves out above the proto-neutron star. This produces a region of slowly moving (Mach number < 0.1) matter (see Figure **??**). For an initially uniformly rotating iron core this subshock region has an angular velocity profile of $\omega \propto r^{-1.8}$. While this velocity profile is different than the Keplerian profile ($\omega \propto r^{-3/2}$) studied in BH98, it is still unstable to magnetohydrodynamic flow. BH98 give a maximum growth rate of $\lambda = (r/2)(d\omega/dr) \approx 0.9\omega$. We adopt this growth rate. Hence we write: $\dot{H} = \lambda H$,

BH98 treat accretion disks that are almost static and the orbital velocities are close to Keplerian. However, in our supernova model the accretion is rapid and the angular velocity is not Keplerian.

We assume that the turbulent magnetic field amplitude then grows as

$$\tilde{H}(t) = H_0(r)e^I \quad , \tag{17}$$

where I is the integrated growth rate

$$I = \int \lambda dt = 0.9 \int \omega dt = 0.9 \int_r^{r_{sh}} \left| \frac{\omega}{v} \right| dr \quad , \tag{18}$$

where r_{sh} is the shock radius.

In the above, H_0 is the ordered in H_Z, H_R, and H_Φ that arises from the spherical inflow of the magnetized matter. The transition to a turbulent magnetic field is assumed to be rapid on the problem time scale. For the lowest initial angular velocity that produced an explosion, $\omega = 0.071$ s^{-1}, the integrated growth rate was $I \approx 30$ at a post bounce time of $t_{pb} = 0.14$ sec and increased to $I \approx 90$ by $t_{pb} = 0.25$ sec

The initial field was selected so that the final neutron star field will be $\approx 10^{12}$ Gauss. The ordered field is thus take to be $H_0 = 10^{12}(10 \text{ km}/r)^2$ Gauss. As demonstrated in BH98, the field is taken to grow until near equipartition,

$$\frac{H_{max}^2}{8\pi\rho} \equiv \frac{\omega^2 r^2}{4} \quad . \tag{19}$$

Energy is deposited in matter after the field surpasses the H_{max}. Since the field still tries to grow after $\tilde{H} = H_{max}$ we let,

$$\dot{\epsilon}_{matter} = 2\omega H_{max}^2/8\pi\rho \quad . \tag{20}$$

The thermal matter pressure $P_M(\epsilon, \rho)$ above the PNS is thus augmented by Eq. **??**.

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Figure 1: The lowest curve is the radius of the proto-neutron star as a function of time post bounce. The upper curves give the radius of the accretion shock for models with an initial angular velocity of $\omega = 0.071$ s^{-1} with and without magnetic turbulence as labeled. The upper curve clearly shows the effects of magnetic field amplification on the shock front. The lower curve shows that rotation alone has little effect for this particular model.

In addition, however, there are contributions from rotation and magnetic effects. In cylindrical coordinates the rotational energy density is just

$$E_{rot} = \frac{1}{2}\rho\omega^2 R^2 \quad . \tag{21}$$

We note that $r = R\sin(\theta)$ and deduce an effective isotropic pressure due to rotational energy from an angular average of the rotational energy,

$$P_{rot} = \frac{E_{rot}}{3} = \frac{1}{6}\rho\omega^2 r^2 \quad . \tag{22}$$

Similarly, the energy density due to the isotropic turbulent magnetic field $\tilde{H}$ is

$$\tilde{E}_{mag} = \frac{\tilde{H}^2}{8\pi} \quad , \tag{23}$$

The average pressure is then is then a third of this as is usual for an isotropic massless field. However, we then reduce this by another factor of two due to the solid angle of the directional flow of the accreting material. Hence we write

$$P_{rot} = \frac{\tilde{H}^2}{48\pi} \quad . \tag{24}$$

Calculations were made with different initial angular velocities to find out how much rotation was needed to result in an explosion. These conditions are summarized in Table **??**.

lr 1 0pt Explosion times and angular velocities ω_0 (sec^{-1}) t_{pb} ms

0.20 200 0.10 250 0.0707 340 0.05 ∞

Figure **??** illustrates the effects on a rotating collapse simulation with and without effects of the magnetic turbulent pressure contribution. For this example, the rejuvenation of the shock due to magnetic field amplification above the PNS is clearly demonstrated.

2.5 Magnetic Bubble Driven Explosion below the PNS Surface

The nonhomologous collapse of a uniformly rotating iron core leads to differential rotation and the build up of a toroidial magnetic field (see Wheeler, Meier &

Wilson 2001 for a general discussion and references). The toroidial field builds up to large values and produces a region unstable to magnetic bouyancy as we now describe.

The magnetic bubble model assumes that the magnetic fields evolve in matter which can be treated as having perfect conductivity. Also, the magnetic braking back reaction of the field on the fluid is not explicitly included, though we estimate its effect. We start with the 18 $M_\odot$ stellar progenitor model of Woosley & Weaver (1985) at the time at which the iron core has just become unstable to infall. While the hydrodynamics is taken to evolve spherically, the magnetic field is assumed to be axially symmetric and is evolved in cylindrical coordinates. In what follows, therefore, we will need to simultaneously consider quantities in both cylindrical and spherical coordinates. Hence, we use capital R, Z, Φ to distinguish cylindrical coordinates, while r, θ, Φ is used to denote for spherical coordinates. We assign all the matter with an angular rotational velocity ω_0 about the Z axis. A uniform magnetic field in the Z direction is assumed. Each mass shell is then zoned in the θ direction for the calculation of the magnetic fields and rotational motion.

The magnetic flux in the Z direction is taken as constant in Φ for each (r, θ) zone. Each mass shell rotates rigidly and preserves its angular momentum. These assumptions lead to the following equation for the evolution of the the toroidal field.

$$\begin{aligned} \dot{H}_\Phi &= H_Z^0 \omega_0 \left(\frac{r_0}{r}\right)^4 \sin(\theta)\cos(\theta)\left(\frac{dr_0}{dr}\frac{r}{r_0} - 1\right) \\ &\quad - \frac{H_\Phi}{r}\frac{\partial(rv)}{\partial r} \quad . \end{aligned} \tag{25}$$

The poloidal field components then follow from flux conservation,

$$\dot{H}_Z = H_Z^0 \left(\frac{r_0}{r}\right)^2 \left(\sin^2(\theta)\frac{dr_0}{dr}\frac{r}{r_0} + \cos^2(\theta)\right) \quad . \tag{26}$$

$$\dot{H}_R = H_Z^0 \left(\frac{r_0}{r}\right)^2 \sin(\theta)\cos(\theta)\left(\frac{dr_0}{dr}\frac{r}{r_0} - 1\right) \quad , \tag{27}$$

where r_0 is the initial radius of a mass shell and r is the shell radius at a later time. H_Z^0 is the initial uniform magnetic field and ω_0 is the initial angular velocity.

The initial angular rotational velocity was chosen to be large enough that a toroidal field will build up to a size such that the buoyancy will be large enough to over come the stabilizing outward increasing entropy gradient of the matter. Such buoyancy will then cause matter to turnover by the quasi-Ledoux convection (Wilson & Mayle 1993; Wilson & Mathews 2003). This turnover then brings ν-rich material to the surface. This enhances the neutrino luminosity enough at early times to achieve a successful explosion.

3 Results

Here, we present results for a plausible model in which the initial precollapse magnetic field, H_Z, was chosen to be 3.16×10^7 gauss. This field magnitude was chosen because it was estimated that the resulting neutron star would have a magnetic field of the order of 10^{12} gauss. Several initial angular velocities were tried in order to find the minimum amount of rotation required to produce an explosion. The minimum angular velocity was found to be 0.3 rad sec^{-1} for this magnetic field strength.

This angular velocity is rather high. Nevertheless preliminary axially-symmetric hydrodynamics calculations (Tipton 2004) of the collapse of a star with $\omega = 0.3$ s^{-1} have found only a small distortion from sphericity for the first 0.2 s after bounce.

As noted above, this angular velocity leads to a rotation rate than is comparable to the shortest observed period pulsars and is near the maximum Keplerian limit on neutron-star spin and is somewhat larger than the rotation rate observed in young pulsars (Manchester 2004). We point out, however, that lower spins are obtained if a higher initial magnetic field is adopted. For example, a calculation was made with a smaller initial rotation rate of 0.1 rad sec^{-1} and a higher precollapse magnetic field of 10^8 gauss. The final magnetic field was correspondingly higher (3×10^{12} gauss). A good explosion resulted and the final neutron star period increased to a few ms. With this scaling a final field of 10^{13} gauss only requires an initial rotation rate of 0.03 s^{-1}. Field strengths of $\sim 10^{13}$ are comparable to that observed in a large number of pulsars (Manchester 2004), e.g the Crab pulsar for which $H \approx 8 \times 10^{12}$ Gauss. Indeed, it is by now well established that magnetars with fields as high as 10^{15} gauss exists. Such stars would require only rather small rotation rates.

As the inner core of the star collapses in our bench-mark model, the rotation and magnetic fields cause the collapse to become very non-homologous. Density in the inner region quickly rises from $\rho \approx\sim 4 \times 10^9$ to a bounce density of $\sim 5 \times 10^{14}$ g cm^{-3}. Above a mass cut of about 1.4 M$_\odot$ the density rises very slowly. This leads to large values of r_0/r and $(dr_0/dr)(r/r_0)$. Hence, a large H_Φ field is developed according to Eq. (**??**). H_R and H_Z rise as well but not nearly as much as H_Φ. The azimuthal H_Φ component thus becomes the dominant field component in the proto-neutron star.

A key result from these simulations is that the principle field energy density, $H_\Phi^2/8\pi$, forms two toroidal shaped regions. It is the effects of the buoyancy of these toroidal regions on the transport of matter and neutrinos that we wish to analyze in the explosion. For ease and consistency we model the magnetic convection using a diffusion algorithm to transport all matter, radiation, and neutrino properties. It is the same the algorithm as has been employed for neutron-finger convection in the Livermore supernova model (Wilson & Mathews 2003).

The effect of the magnetic instability is therefore modeled as follows. When

the buoyancy of the magnetic field is sufficient to overcome the positive entropy gradient then a magnetic diffusion algorithm is initiated. The effective neutrino diffusion coefficient is taken as proportional to the product of a mixing length times the magnetic buoyancy parameter

$$D = \frac{l}{30}\sqrt{\frac{H^2}{8\pi\rho}} \quad , \tag{28}$$

where H is the maximum of the magnitude of the H_Φ field and

$$\begin{aligned} l &= r_\nu - r \ , \quad r < r_H \\ &= r_\nu - r_H \ , \ r \geq r_H \ , \end{aligned} \tag{29}$$

where r_H is the radius at which the maximum of H occurs, and r_ν is the radius of the neutrino photosphere. The denominator of 30 was deliberately taken large so as to be conservative in our estimate of the size and behavior of the magnetic convective cells. A smaller cell size implies a slower convective lifetime.

As in the case of the neutron-finger instability (Wilson & Mathews 2003), the magnetic convection brings up proton-rich matter (compared to the deleptonized surface regions) as well as neutrinos towards the surface of the proto-neutron star and results in an enhanced neutrino luminosity soon after bounce.

Figures **??**-**??** show some of the details of the calculation with an initial field of 3.16×10^7 gauss and a rotation rate of 0.3 rad sec^{-1}. In Figures **??**-**??** these are compared to calculations with no convection and also those with neutron finger convection. As can be seen in Figures **??**-**??**, all three calculations give nearly the same behavior until a few tenths of seconds after the core bounce.

In Figure **??** the neutrino photospheric radius r_ν and the proto-neutron star average angular velocity $\bar{\omega}$ are presented. The final angular speed is very high, $\omega \approx 9 \times 10^3$ rad sec^{-1}, corresponding to a rotation period of $P \approx 1.4$ ms. The star, however will quickly spin down. The surface field H_Φ is very high and it is anchored in the massive non-rotating envelope. The magnetic torque should be of order,

$$\tau_H \approx \int \left(\frac{\partial H_\Phi}{\partial R} H_Z - \frac{\partial H_\Phi}{\partial Z} H_R \right) R r^2 dr \quad . \tag{30}$$

Putting in numbers for H_Φ, H_R, and H_Z, from the simulations, we obtain $\tau_H \approx$ a few $\times 10^{46}$ erg. Then for a rotational moment of inertia of the nascent neutron star of $I \approx 10^{45}$ g cm^2 one has,

$$\frac{\dot{\omega}}{\omega} = \frac{\tau_H}{I\omega} \approx \text{ a few } \times 10^{-3} \ \text{ s}^{-1} \quad . \tag{31}$$

Hence the magnetic torque should be able to slow the spin of the neutron star considerably within a several minutes. The maximum values of the magnetic fields H_Z, H_Φ as a function of time are shown in Fig. **??**. This figure shows that it only takes several tenths of seconds to get large magnetic fields.

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Figure 2: Curve marked r_ν is the neutrino photospheric radius in units of 10^2 km versus time as the neutron star relaxes to a radius of ~ 10 km. The curve labeled $\bar{\omega}$ is the angular speed (in units of 10^4 rad sec^{-1}) of the proto-neutron star averaged for matter inside r_ν as a function of time.

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Figure 3: Curve H_Z is the maximum of the Z-component of the magnetic field as a function of time in unite of 10^{12} gauss. Curve H_Φ is the maximum value of the Φ-component of the magnetic field as a function of time in units of 10^{16} gauss.

Figures **??** and **??**, show the radial velocity and entropy per baryon, respectively, at various times for the mass shell with the highest outward velocity in a model with magnetic convection and no neutron-finger instability. Here it is apparent from the expanding radii and increasing entropies that a good explosion has resulted. When the entropy rises to about 80 the heating by neutrino-electron scattering is equal to that of neutrino capture. Hence, even though the luminosities are falling the heating within the high-entropy bubble will remain substantial.

Figure **??** compares the electron neutrino luminosities as functions of time for three runs: no convection, neutron-finger convection, and magnetic-field driven convection. Here we see that in the case of the neutron-finger convection, luminosity comes on early but is eventually surpassed by the magnetic luminosity. The entropy profiles in radius at a time of 0.68 sec post-bounce seconds are shown in Figure **??**. The entropy in the magnetic case is only slightly less than of the neutron-finger case. From Fig. **??** we see that the outward velocity for the magnetic calculation is only slightly below the neutron-finger velocity.

4 Conclusions

Although the Livermore supernova model with neutron-finger convection is a viable description of core-collapse supernovae, the present calculations suggest an alternative to the neutron-finger instability for initiating an explosion. If

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Figure 4:
Radial velocity in units of 10^3 km sec^{-1} at various indicated times post bounce (from left to right) of 0.60, 0.63, 0.66, and 0.68 sec for the model with magnetic convection.

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Figure 5: Entropy per baryon S versus radius for the post-bounce times (from left to right) of 0.60, 0.63, 0.66, and 0.68 sec for the model with magnetic convection.

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Figure 6: Electron neutrino luminosities in units of 10^{52} erg sec^{-1} as a function of post-bounce time for three calculations as labeled: no convection; neutron-finger convection, and magnetic convection. Note, that the early short-duration shock break-out luminosity has been suppressed.

correct, this may provide the long sought after insight as to how core-collapse supernovae become sufficiently heated behind the shock to explode. An interesting possible side effect of the magnetic field generation that we studied is that an axial jet and/or a prolate bulge in the mass distribution should arise independently of how the convection is driven. Such features, for example, might be an explanation of the observation that most remnants emit polarized optical radiation.

It should be noted that both this magnetic turbulence effect and the magnetic bubble formation below the proto-neutron star surface will act together to induce an explosion. Clearly, more detailed work utilizing two and three dimensional MHD simulations is required to explore whether the magnetic buoyancy effect described herein is sufficient to induce an explosion. Efforts along this line are currently underway (D. Balsara, R. Tipton; priv. comm.). This is, however, a difficult and time-consuming calculation. It is hoped the present work will stimulate further effort to understand this possibly important contribution to the complex paradigm of core-collapse supernovae.

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Figure 7: Entropy per baryon at post-bounce time of 0.68 sec versus radius for the cases of no convection; neutron-finger convection, and magnetic convection.

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Figure 8: Velocity in units of 10^3 km sec^{-1} at a post-bounce time of 0.68 sec for the cases of no convection; neutron-finger convection, and magnetic convection.

which tool place at the Aspen Center for Physics.

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