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SPATIALLY CONTINUOUS MIXED SIMPLIFIED P_2-P_1 SOLUTIONS FOR MULTIDIMENSIONAL GEOMETRIES

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1 Introduction

The simplified P_N (SP_N) angular approximation to the neutron transport equation has received renewed interest in recent years [1–3]. The even-order simplified P_2 angular approximation has been shown to yield improved accuracy over the P_1 (diffusion) angular approximation for problems in which the P_1 approximation is reasonably accurate [1]. In particular, integral quantities such as reaction rates and eigenvalues computed by the SP_2 approximation are typically more accurate. The SP_2 scalar flux spatial distributions are also often more accurate than the P_1 distributions away from material interfaces and source discontinuities. However, the SP_2 scalar flux distributions are spatially discontinuous at material interfaces and source discontinuities. These spatial discontinuities are unphysical, aesthetically displeasing, and can lead to large inaccuracies near interfaces.

Brantley [4] has recently investigated combining the P_1 and P_2 angular approximations in planar geometry to eliminate the discontinuities in the P_2 scalar flux distributions near material interfaces and source discontinuities. In essence, the P_1 angular approximation is used at material interfaces and source discontinuities to obtain spatially continuous scalar flux distributions; the P_2 angular approximation is utilized elsewhere to take advantage of its improved accuracy. The combined angular approximation smoothly transitions between the P_1 and P_2 approximations.

In this paper, we extend this planar geometry mixed P_2-P_1 angular approximation approach to the multidimensional SP_2 angular approximation, i.e. we develop a spatially continuous mixed SP_2-P_1 angular approximation. We present numerical results from a two-dimensional test problem to demonstrate the accuracy of this spatially-continuous mixed SP_2-P_1 angular approximation.

2 The Mixed SP_2–P_1 Angular Approximation

The simplified P_2 angular approximation to the monoenergetic general geometry neutron transport equation in a spatial domain $\underline{r} \in V$ is given by [1]

$$-\underline{\nabla} \cdot \frac{1}{3\sigma_{a1}(\underline{r})} \underline{\nabla} \widehat{\phi}(\underline{r}) + \widehat{\sigma}_{a0}(\underline{r}) \widehat{\phi}(\underline{r}) = \widehat{Q}(\underline{r}) \quad , \tag{1}$$

where

$$\widehat{\sigma}_{a0}(\underline{r}) = \frac{\sigma_{a0}(\underline{r})}{1 + \frac{4}{5}\rho(\underline{r})} \quad , \tag{2}$$

$$\widehat{Q}(\underline{r}) = \frac{Q(\underline{r})}{1+\frac{4}{5}\rho(\underline{r})} \quad , \tag{3}$$

$$\widehat{\phi}(\underline{r}) = \left[1+\frac{4}{5}\rho(\underline{r})\right]\phi_0(\underline{r}) - \frac{4}{5}\rho(\underline{r})\frac{Q(\underline{r})}{\sigma_{a0}(\underline{r})} \quad , \tag{4}$$

and

$$\rho(\underline{r}) = \frac{\sigma_{a0}(\underline{r})}{\sigma_{a2}(\underline{r})} \quad . \tag{5}$$

Here $Q(\underline{r})$ is a neutron source, $\sigma_{an}(\underline{r}) = \sigma_t(\underline{r}) - \sigma_{sn}(\underline{r})$, $n = 0,1,2$, are the "absorption" cross sections obtained by subtracting the Legendre angular moments of the differential scattering cross section from the total cross section, and $\phi_0(\underline{r})$ is the neutron scalar flux. Eq. (1) is a diffusion-like equation (with a modified absorption cross section and source) for the modified scalar flux unknown $\widehat{\phi}(\underline{r})$. The neutron scalar flux $\phi_0(\underline{r})$ is readily obtained from the unknown $\widehat{\phi}(\underline{r})$ using Eq. (4). Marshak vacuum boundary conditions for the SP_2 approximation are given by [1]

$$\frac{1}{4}\left(\frac{1+\frac{1}{2}\rho(\underline{r_b})}{1+\frac{4}{5}\rho(\underline{r_b})}\right)\widehat{\phi}(\underline{r_b}) + \frac{1}{6\sigma_{a1}(\underline{r_b})}\underline{n}\cdot\underline{\nabla}\widehat{\phi}(\underline{r_b}) = -\frac{3}{40}\rho(\underline{r_b})\frac{\widehat{Q}(\underline{r_b})}{\sigma_{a0}(\underline{r_b})} \quad , \quad \underline{r_b}\in\partial V \quad . \tag{6}$$

Defining the function $\rho(\underline{r})$ by Eq. (5) gives the SP_2 approximation and setting $\rho(\underline{r}) = 0$ results in the standard P_1 approximation.

In this paper, we propose to define the function $\rho(\underline{r})$ by the equation

$$\rho(\underline{r}) = \frac{\sigma_{a0}(\underline{r})}{\sigma_{a2}(\underline{r})}\left[1 - exp\left(-p\sigma_t(\underline{r})\,d(\underline{r})\right)\right] \quad , \tag{7}$$

where p is a user-specified parameter and $d(\underline{r})$ is a distance function that provides a measure of the distance from a given spatial point $\underline{r}$ to a material interface or source discontinuity. The distance function is zero on the boundary of the material region and positive in the interior of the material region. For $p = 0$, the pure P_1 approximation is recovered for all $\underline{r}$; for $p \to \infty$, the pure SP_2 approximation is obtained for all $\underline{r}$. With this representation for $\rho(\underline{r})$ and $1 \le p \ll \infty$ (e.g. $p \approx 2.5$), the P_1 approximation is used at material interfaces and source discontinuities and the angular approximation transitions smoothly to the SP_2 approximation within approximately one mean free path from the material interface or source discontinuity.

3 Numerical Results

We have implemented the one-group simplified P_2 and the spatially-continuous mixed SP_2-P_1 angular approximations in the SMPLTN code [2] that solves the multigroup simplified P_N equations of odd-order in two-dimensional Cartesian geometries using a finite volume spatial discretization. The numerical solution of the SP_2 and the mixed SP_2-P_1 equations requires essentially the same computational expense as the solution of the standard P_1 equations. For a two-dimensional material region defined by the Cartesian corner coordinates (x_l, y_b) and (x_r, y_t), we have utilized a simple distance function $d(x,y)$ of the form

$$d(x,y) = min\left[(x-x_l),(x_r-x),(y-y_b),(y_t-y)\right] \quad , \tag{8}$$

which gives the minimum distance to a material interface.

In this section, we compare the accuracy of the mixed SP_2-P_1 angular approximation to the pure P_1 and SP_2 angular approximations for a two-dimensional one-group fixed source test problem originally proposed by Azmy [5]. The problem geometry is shown in Fig. 1, and the material parameters are given in Table 1. This problem was originally designed to test nodal transport methods and consists of a source region (I) with scattering ratio $c = 0.5$ surrounded by strongly absorbing regions (II-IV) with $c = 0.05$. We solved this

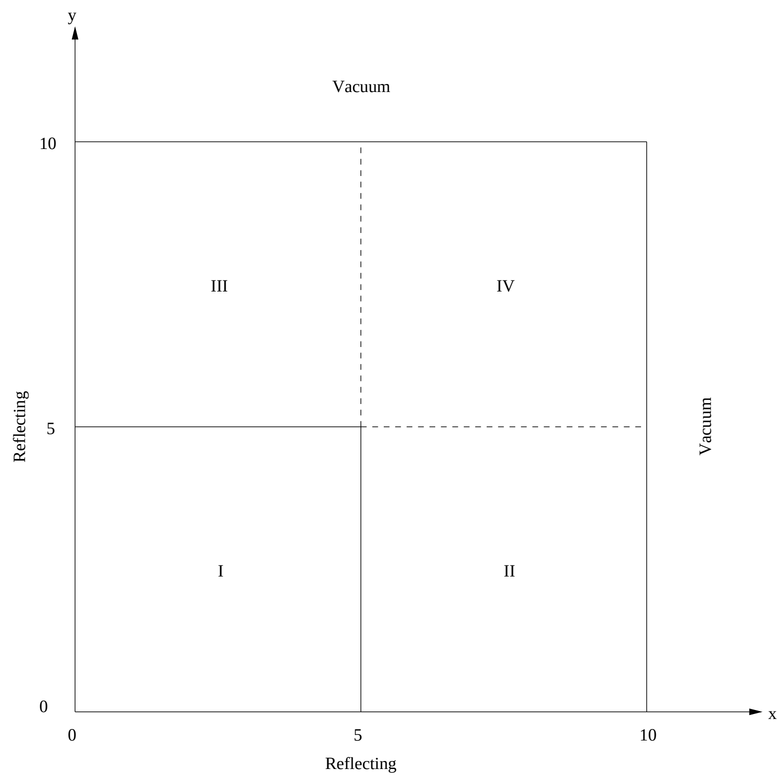

Figure 1: Problem geometry.

problem using the P_1, SP_2, and mixed SP_2-P_1 (with $p = 2.5$) angular approximations with a spatial mesh such that $\Delta x = \Delta y < 0.03$ diffusion lengths in all regions. Reference transport solutions computed using the discrete ordinates method were obtained using the ARDRA code [6] with an S_{16} level symmetric quadrature set and a spatial mesh such that $\Delta x = \Delta y \approx 0.012$ mfp in all regions. We compare the computed region average scalar flux values to the benchmark values [5] as well as details of the scalar flux distributions.

The relative errors in the region average scalar flux values as computed using the various angular approximations are shown in Table 2. The errors in the average SP_2 scalar flux values in regions I-III are larger than the P_1 errors, while the error SP_2 in region IV is smaller. The spatially continuous SP_2-P_1 approximation (with $p = 2.5$) is the most accurate overall.

The computed scalar flux distributions along $y = 2.5$ cm near the interface between region regions I and II are shown in Fig. 2. The spatial discontinuity in the SP_2 scalar flux distribution near the interface is evident. The mixed SP_2-P_1 scalar flux distribution is spatially continuous near the interface and is the most accurate. The scalar flux distributions along $y = 4.95$ cm (close to the region interfaces) near the corner of the four regions are shown in Fig. 3. The SP_2 scalar flux is inaccurate because of the proximity to the material interfaces. The mixed SP_2-P_1 scalar flux distribution is spatially continuous and is generally more accurate than the P_1 and the SP_2 distributions.

Additional numerical results from an eigenvalue problem will be presented in the full paper.

4 Conclusions

In this paper, we showed that the simplified P_2 angular approximation could be modified to give spatially continuous scalar flux solutions at material interfaces and source discontinuities. The proposed modification

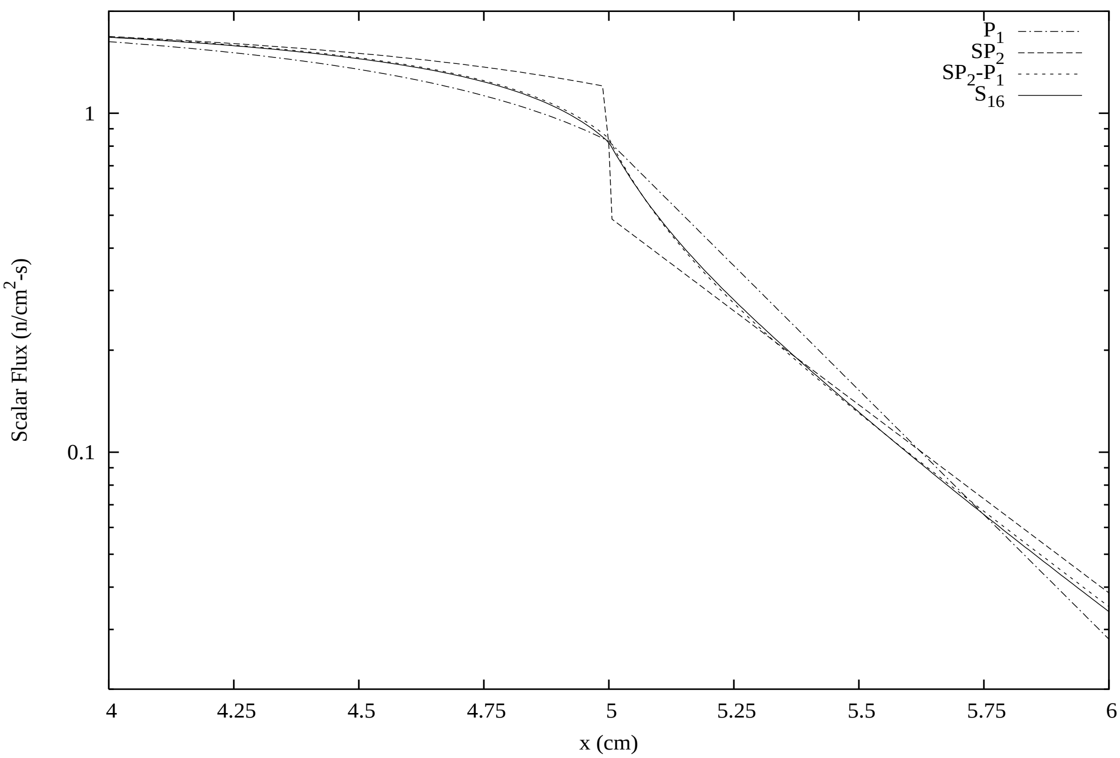

Figure 2: Scalar flux near interface along y = 2.5 cm.

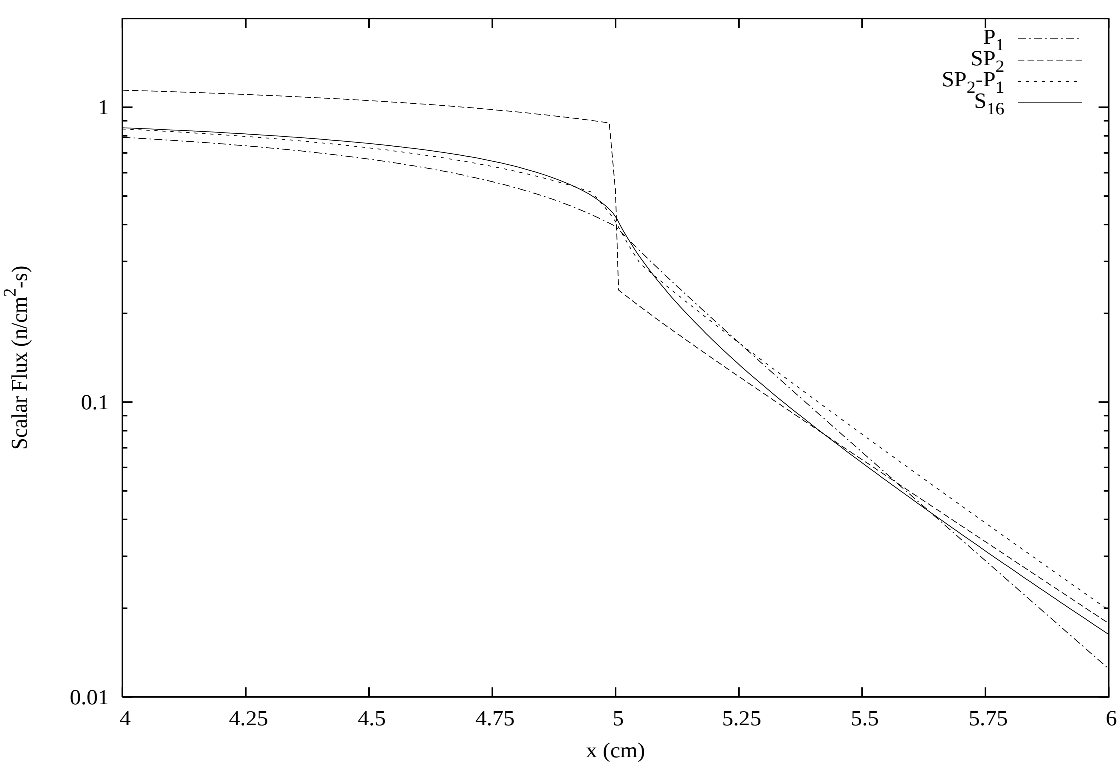

Figure 3: Scalar flux near corner along y = 4.95 cm.

Table 1: Material Properties

	Region	
	I	II–IV
σ_t	1.0	2.0
σ_{s0}	0.5	0.1
Q	1.0	0.0

Table 2: Relative Errors in Average Scalar Flux Values

Region	Exact Average Scalar Flux	Relative Error (%)		
		P_1	SP_2	SP_2–P_1
I	1.676	-1.68	2.52	0.63
II,III	4.159×10^{-2}	9.61	-12.90	-2.87
IV	1.992×10^{-3}	-23.95	-15.33	-15.84

uses the P_1 angular approximation at material interfaces and source discontinuities and smoothly transitions to the SP_2 approximation within approximately a mean free path from the interface. Neither the P_1 nor the SP_2 approximation is convincingly more accurate near material interfaces, so the P_1 approximation can be utilized at these locations to give spatially continuous results. Numerical results from a two-dimensional test problem demonstrate that the mixed SP_2-P_1 approximation can yield improvements in accuracy over both the pure P_1 and SP_2 approximations.

Motivated by the promising results presented in this paper, future work will be aimed at theoretically investigating the user-prescribed parameter p of Eq. (7) and exploring potential theoretically-inspired alternative forms for Eq. (7). We also plan to investigate the implementation of the spatially continuous SP_2-P_1 approximation in a nodal diffusion and SP_2 code.

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