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Erratum: Bounds and Estimates for Elastic Constants of Random Polycrystals of Laminates

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The self-consistent estimates for bulk modulus are found from the bounds given by Equation (9) in the paper by taking $K_\pm \to K^*$, $\mu_\pm \to \mu^*$, and therefore $\zeta_\pm \to \zeta^*$. The resulting formula for bulk modulus is

$$K^* = K_V \frac{(G^r_{\text{eff}} + \zeta^*)}{(G^v_{\text{eff}} + \zeta^*)}.$$

This formula was published as (17) in the paper and is correct. However, the self-consistent formula for shear modulus requires a bit more work. The reason for this is that the formula given for the bounds in Equation (15) has already made use of a constraint equation that is only true along the bounding curve defining the upper and lower bounds on shear modulus. Since the self-consistent estimate normally lies off this curve, a more general result must be employed when deriving the self-consistent formula. When this inappropriate constraint is replaced by the correct general formula and then the formula for self-consistent bulk modulus (noted above) is substituted, we then find the self-consistent formula that replaces (19) for shear modulus in the published paper is given instead by

$$\frac{1}{\mu^* + \zeta^*} = \frac{1}{5}\left(\frac{1 - \alpha^*(K_V - K^*)}{G^v_{\text{eff}} + \zeta^*} + \frac{2}{c_{44} + \zeta^*} + \frac{2}{c_{66} + \zeta^*}\right),$$

the main difference being that the denominator of the first term on the right hand side is simpler than it is in the bounds for shear modulus.

Unfortunately, the error resulting from the incorrect formula (19) in the published paper does propagate into the examples and figures. However, the examples all use sufficiently small values of contrast in the composite's bulk moduli so that the numerical differences are not noticeable to graphical accuracy in any of these figures. But for other applications with higher contrasts, the resulting errors can be sufficiently large so that the values computed with the wrong formulas actually may fall outside of the rigorous bounds — which should never happen. So this correction should be made in all cases to avoid a rather obvious, potentially confusing, and apparently contradictory result. The corrected self-consistent formula shown above does not have this problem even at very high bulk modulus contrasts.

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