UCRL-PROC-212901

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Comparison of Modern Methods for Shock Hydrodynamics

A. W. Cook

June 14, 2005

Joint Russian-American Five-Laboratory Conference on
Computational Mathematics/Physics
Vienna, Austria
June 18, 2005 through June 23, 2005

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A.W. Cook

Lawrence Livermore National Laboratory, Livermore California 94551

The accuracy and efficiency of several methods are compared for simulating multi-fluid compressible flows. The methods include a Godunov scheme (Colella, 1985), a Weighted Essentially Non-Oscillatory method (Jiang and Shu, 1996), an Arbitrary Lagrangian Eulerian algorithm (Marinak et al., 2001) and a compact scheme (Cook and Cabot, 2005). Test problems include a compressible breaking wave, the Shu-Osher problem, the Taylor-Green vortex and decaying turbulence. The compact method employs an artificial bulk viscosity for treating shocks and an artificial shear viscosity for modeling turbulence. The compact method is demonstrated to capture shocks as well as the other schemes, while providing superior resolution of post-shock features.

Introduction

Flows involving shocks are usually treated with low-order methods, which either take advantage of the eigenstructure of the equations (Godunov) and/or adjust the local differencing stencil (ENO) to capture discontinuities with minimal oscillations. In many applications, ranging from Inertial Confinement Fusion (ICF) to supernovae, shocks deposit vorticity at material interfaces, which subsequently evolve into turbulent mixing zones; secondary shocks may then pass back through the turbulent regions. Such problems present a formidable challenge to traditional Computational Fluid Dynamics (CFD) algorithms, where high order of accuracy for turbulent mixing is usually sacrificed in favor of robustness and monotonicity for the shocks. Recently, the need for accurate predictions of mixing at shock-accelerated interfaces has driven the development of high-order shock-capturing schemes.

Spectral (Fourier, Chebyshev or Legendre polynomials) and spectral-like (compact/Pade) methods are much better at resolving a wide range of scales than lower-order finite-difference (FD) or finite-volume (FV) schemes. However, these methods suffer from Gibbs phenomenon when applied to shocks. Recently, is has been shown that Gibbs oscillations around shocks can be eliminated by applying a hyperviscosity in combination with a de-aliasing filter (Cook and Cabot, 2005). The purpose of this paper is to compare a high-order compact shock-capturing method with several standard low order methods. The particular schemes utilized for comparison are a Godunov method (Collela, 1985), a Weighted Essentially Non-Oscillatory (WENO) method (Jiang and Shu, 1996) and an Arbitrary Lagrangian Eulerian method (Marinak, 2001).

Test problems

Compressible Breaking Wave

The first test case is a compressible breaking wave. The problem is described in detail in Cook and Cabot (2004). It consists of sinusoidal initial conditions for density, pressure and velocity. As time progresses, the sinusoid steepens into a shock. An exact solution is available until the shock forms (Landau and Lifshitz, 1959). Figure 1 depicts the evolution of the density field in the moving frame of reference.

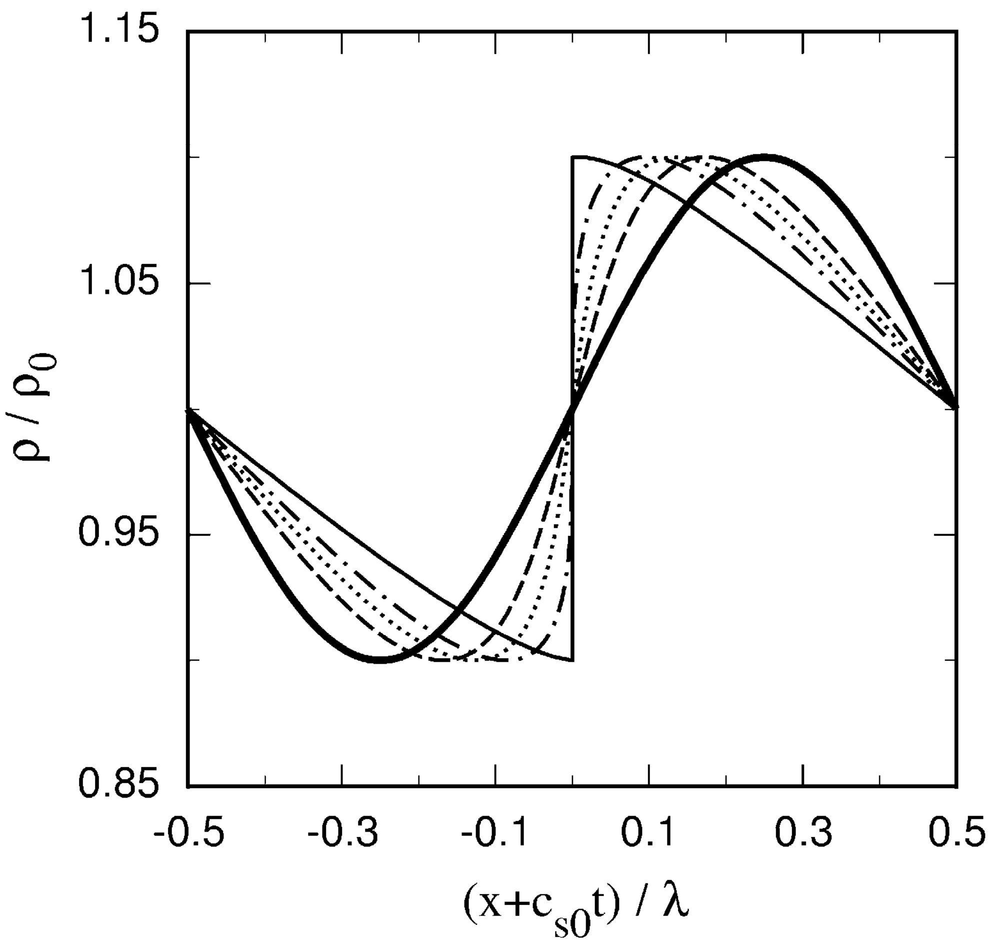

Figure 1. Normalized density for a compressible breaking wave in moving frame of reference. The heavy solid line is the initial condition. The profile steepens into a shock as time progresses.

Figure 2 shows how global errors progress for the WENO, Godunov and compact schemes as the flow develops.

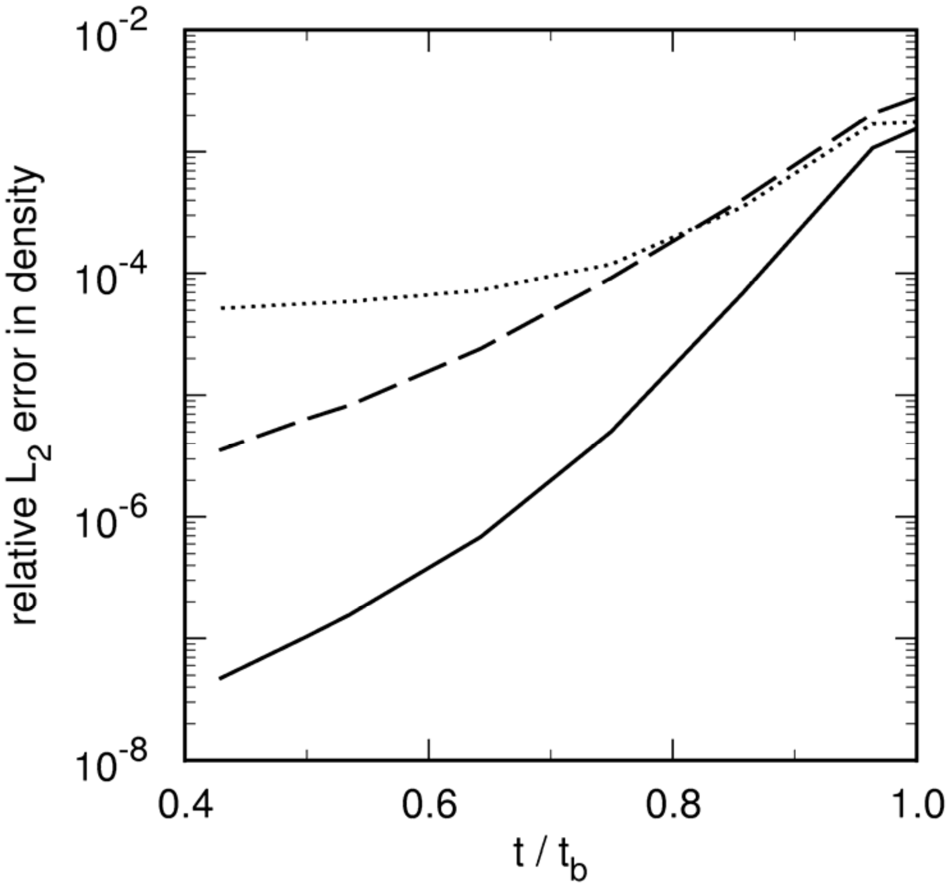

Figure 2. Time-evolution of errors for the compact (solid), WENO (dashed) and Godunov (dotted) schemes for the compressible breaking wave, using 128 points per wavelength. t_b is the time when the shock forms.

At early times, when the flow is smooth, The compact scheme is much more accurate than the WENO or Godunov methods. However, once the shock forms, all methods become first order with similar errors. A scale-dependent measure of error was obtained by taking the Fourier transform of the density field just prior to shock formation. Figure 3 displays the density spectrum for each of the schemes when the flow is still smooth. For low wavenumbers (large scales) all schemes have small errors; however, for high wavenumbers, the errors for the compact scheme are much smaller than those of the low-order methods.

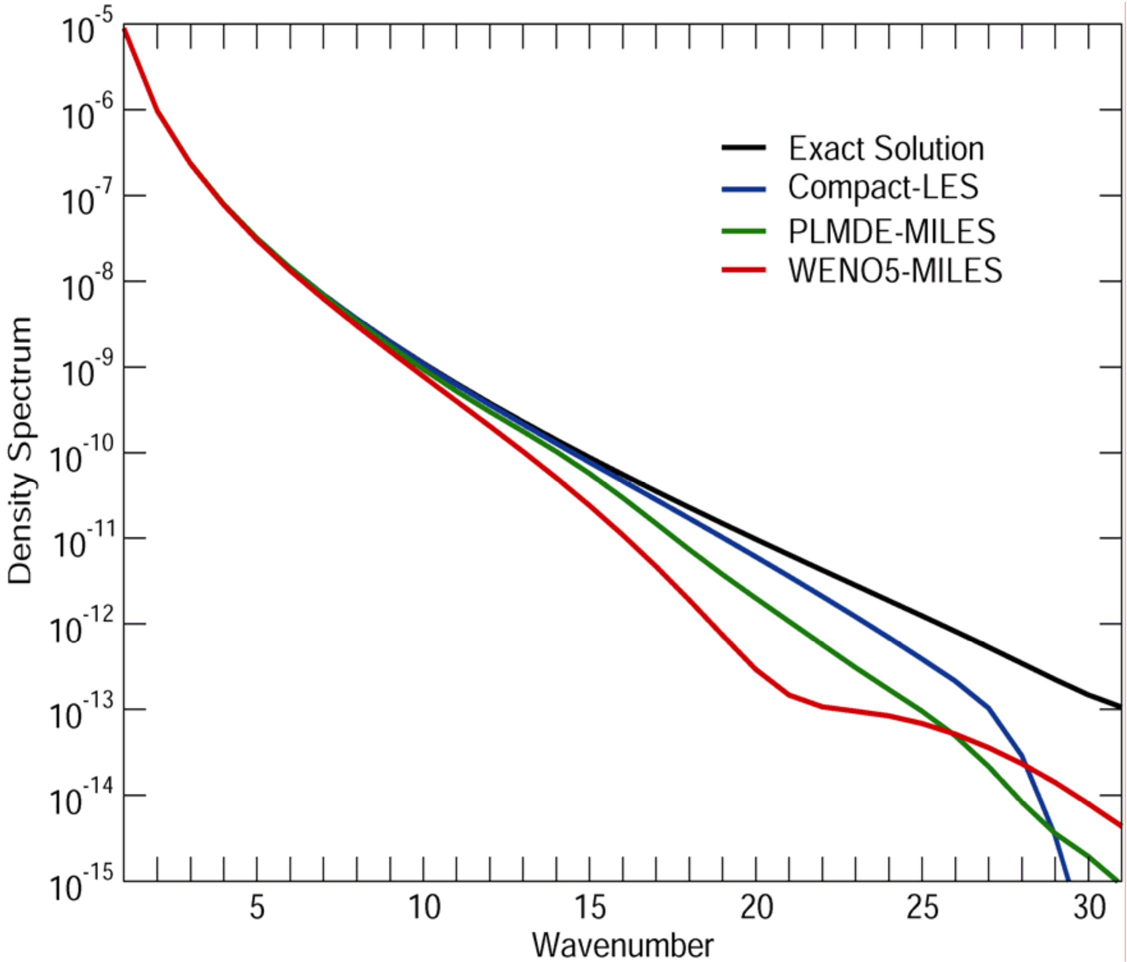

Figure 3. Density spectrum for the breaking wave at $t=3t_b/4$.

Figure 4 shows the results for each of the schemes after shock formation. All of the methods capture the shock in a monotonic fashion and each of the schemes spreads the shock over 3-4 grid points.

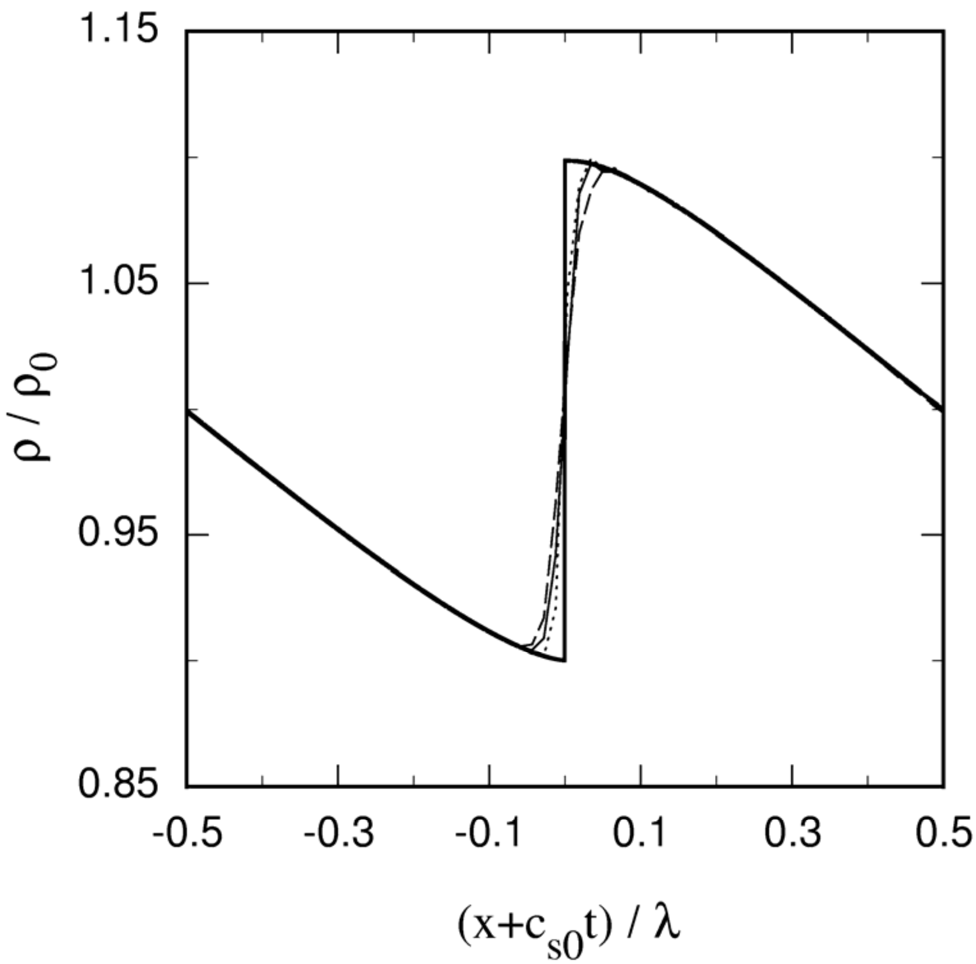

Figure 4. Normalized density for the breaking wave computed with Godunov (dotted line), WENO (dashed line) and compact (thin solid line) schemes. The resolution is 64 points per wavelength in each case. The thick solid line is the converged solution.

Shu-Osher Problem

Given that each of the schemes under question performs roughly the same for shocks, the question arises as to whether high-order methods provide any benefit for the flow behind a shock. To address this issue, we examine the Shu-Osher problem (Shu and Osher, 1989) as a second test case. This problem is a canonical model of a 1-D shock-turbulence interaction. It consists of a shock propagating into a sinusoidal density field, leaving a steeply oscillating flow in its wake. The density solution for the various schemes is displayed in Figure 5, where the Godunov method is denoted PLMDE (Piecewise Linear MUSCL Direct Eulerian).

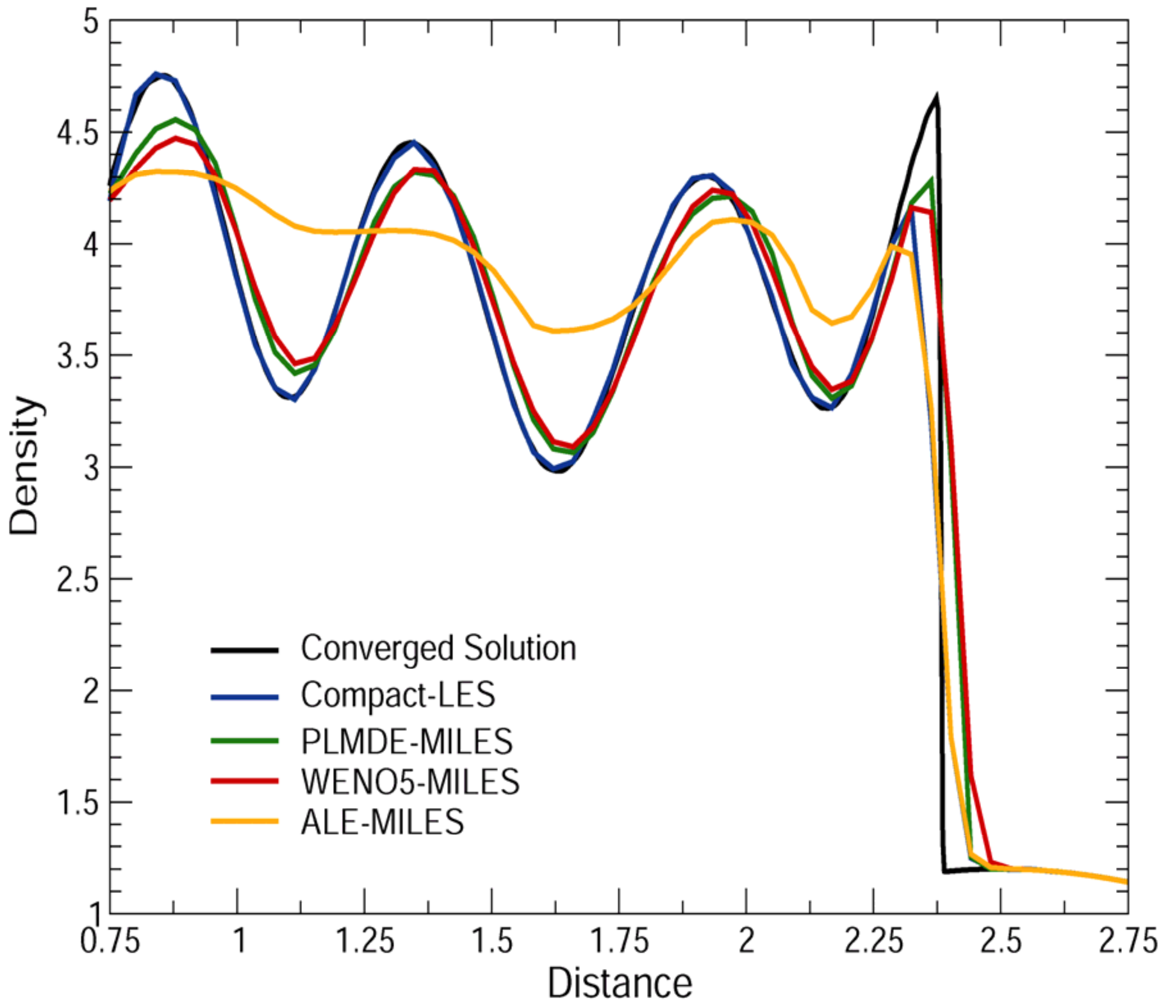

Figure 5. Density field for the Shu-Osher problem at t=1.8.

There are two things to note in Figure 4. First, the compact scheme is best able to capture the post-shock oscillations, and second, there is an apparent phase shift in the oscillations for the low-order schemes at the nominal resolution. At higher resolutions, all schemes converge to the black line and the phase shift disappears (results not shown). Figure 5 suggests that there is indeed benefit to using high-order schemes to resolve post-shock turbulence.

Taylor-Green Vortex

To further elucidate the relative merits of each scheme for turbulence simulations, we examine their behavior on the Taylor-Green vortex (Taylor and Green, 1937). This is a three-dimensional problem with triply periodic boundary conditions. Each of the methods was run on a 64^3 grid. Initially, the flow consists of a single two-dimensional vortex with all of the energy in the bottom wavenumber. As the flow develops, the vortex stretches and bends and energy cascades to higher wavenumbers, thus filling out the spectrum. A semi-analytic solution is available for enstrophy out to a time of about 3. Figure 6 shows normalized total enstrophy for each of the methods applied to this flow.

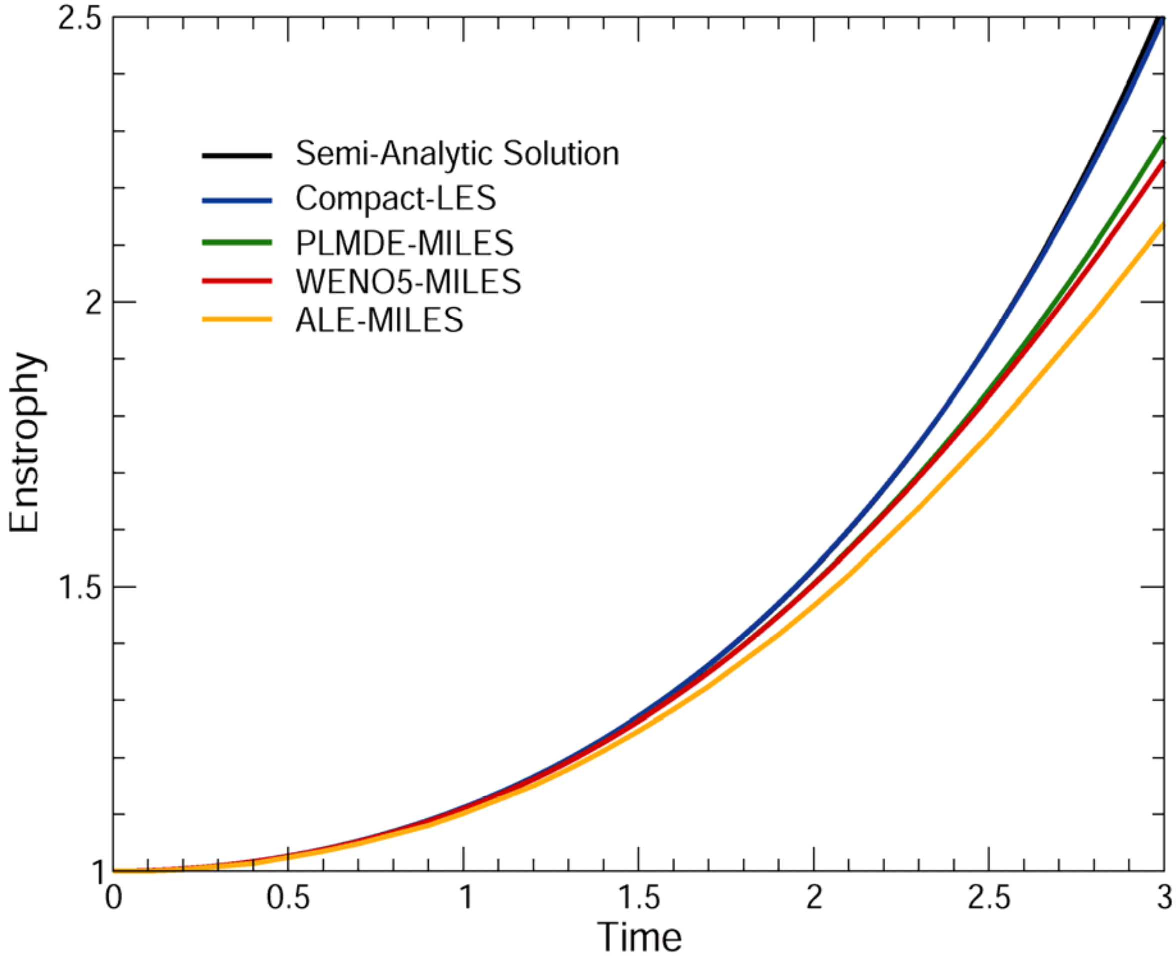

Figure 6. Time-evolution of normalized enstrophy for the Taylor-Green vortex in a 64^3 computational domain.

Clearly, the compact scheme is much better at resolving small scale vorticity than the Godunov, WENO or ALE methods.

Decaying Turbulence

Finally, we turn to decaying turbulence, a standard test case for both Reynolds-averaged and subgrid-scale turbulence models. The motivation for looking at this case is to see if the hyperviscosity introduced in the compact method acts as a proper subgrid-scale model. The recent experiment of Kang et. al. (2003) provides excellent high Reynolds number data for testing numerical methods and turbulence models. In their experiment, air is blown past an active grid in the Corrsin wind tunnel, generating near-isotropic turbulence at a Taylor microscale Reynolds number of about 720. An array of four X-wire probes is used to measure velocity at four downstream stations. We compare two numerical simulations of this experiment, one with the hyperviscosity model turned on and one with it turned off. The initial conditions for the simulations consist of a triply-periodic velocity field in a 192^3 box, with a kinetic energy spectrum matched to first 64 wavenumbers of the experimental spectrum at the first station. Wavenumbers above 64 are truncated in each simulation in order to match the resolution of the simulations in Kang et.al. (2003). Pressure is obtained by solving a Poisson equation with a divergence-free velocity field. The kinematic viscosity in the simulations matches that of the experiment. Simulation time is related to distance downstream using the mean experimental flow velocity. Figure 7 depicts the evolution of the 3-D kinetic energy spectrum, as well as the decay of turbulent kinetic energy (KE) for the experiment and simulations.

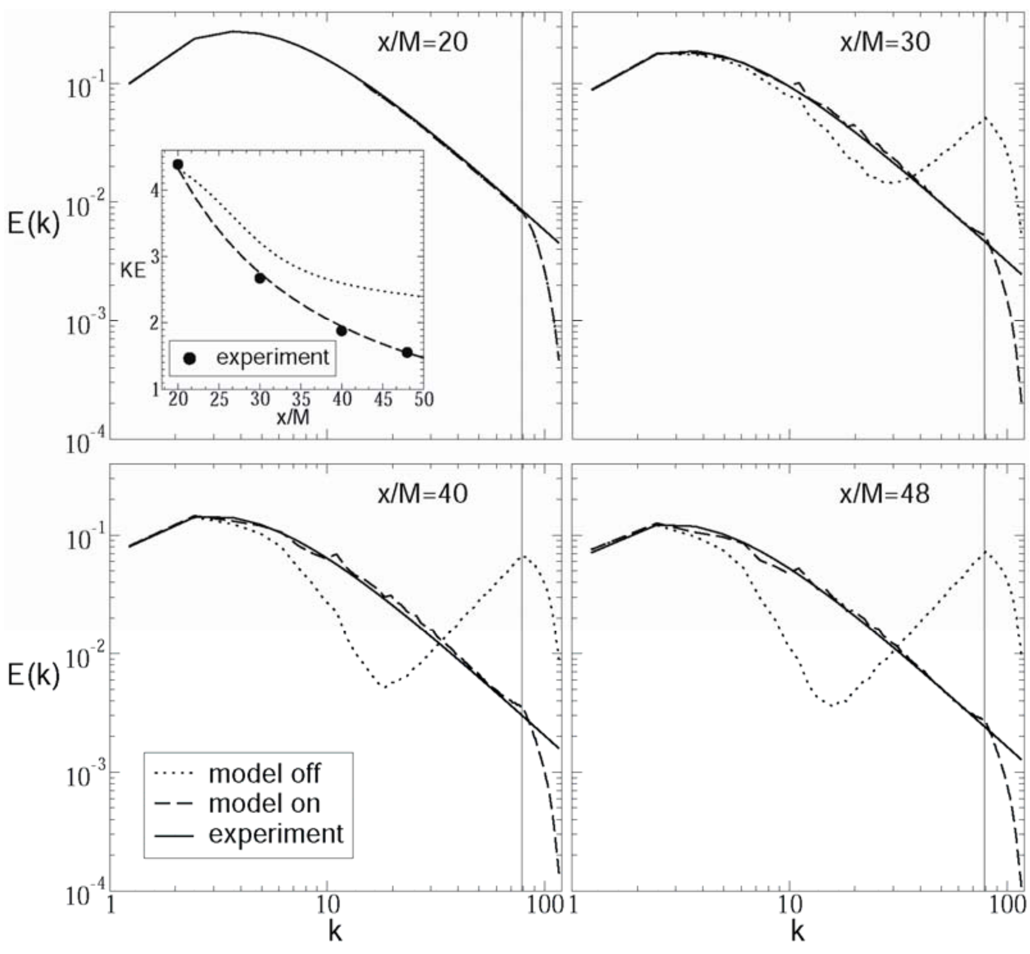

Figure 7. Evolution of 3D energy spectrum for wind tunnel experiment of Kang et. al. (2003). The inset in the first plot shows decay of turbulent kinetic energy. The vertical lines correspond to the 2/3-wavenumber truncation.

The figure shows that with the hyperviscosity model turned on, excellent agreement is obtained between the experimental and simulated spectrum. Furthermore, the overall rate of energy decay is correctly represented. Without the model, energy piles up near the filter cut-off and propagates back to lower wavenumbers, eventually corrupting the entire spectrum. Excess energy at high wavenumbers is sometimes referred to as the "bottleneck effect". It is caused by an incorrect rate of energy transfer between resolved and subgrid-scales. Implicit large-eddy simulations (sometimes referred to as MILES) typically exhibit this anomaly.

Conclusions

The compact scheme with hyperviscosity provides an attractive alternative to standard upwinded schemes when computing flows containing both shocks and turbulence. While the compact method does not provide any better representation of shocks themselves, it does allow for improved resolution of flow features away from shocks. Additionally, hyperviscosity can act as an effective subgrid-scale model, as demonstrated by the decaying turbulence problem.

Acknowledgement

This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

References

Colella, P., "A Direct Eulerian MUSCL Scheme for Gas Dynamics," *SIAM J. Sci. Stat. Comput.*, **6,** 104-117 (1985).

Cook, A.W. and Cabot, W.H., "A High-Wavenumber Viscosity for High-resolution Numerical Methods,"*J. Comput. Phys.*, **195**, 594-601 (2004).

Cook, A.W. and Cabot, W.H., "Hyperviscosity of Shock-Turbulence Interactions," *J. Comput. Phys.,* **203**, 379-385 (2005).

Jiang, G.-S., and Shu, C.-W., "Efficient Implementation of Weighted ENO Schemes," *J. Comput. Phys.,* **126**, 202-228 (1996).

Kang, H.S., Chester, S. and Meneveau, C., "Decaying Turbulence in an Active-Grid-Generated flow and Comparisons with Large-Eddy Simulation," Journal of Fluid Mech. **480**, 129-160 (2003).

Landau, L.D. and Lifshitz, E.M., "Fluid Mechanics," Pergamon Press, (1959).

Marinak, M.M., Kerbel, G.D., Gentile, N.A., Jones, O., Munro, D., Pollaine, S., Dittrich, T.R. and Haan, S. W., "Three-Dimensional HYDRA Simulations of NIF Targets," *Phys. Plasmas* **8**, 2275 (2001).

Marinak, M.M., Kerbel, G.D., Gentile, N.A., Jones, O., Munro, D., Pollaine, S., Dittrich, T.R. and Haan, S. W., "Three-Dimensional HYDRA Simulations of NIF Targets," *Phys. Plasmas* **8**, 2275 (2001).

Shu, C.-W. and Osher, S.J., "Efficient Implementation of Essentially Nonoscillatory Shock Capturing Schemes II," *J. Comput. Phys.,* **83**, 32-78 (1989).

Taylor, G.I. and Green, A.E., "Mechanism of the Production of Small Eddies from Large Ones", Proc, Roy. Soc. A, **158**, 499-521 (1937).