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Abstract— Noise self-generated by a surface ship towing an array in search of a weak target presents a major problem for the signal processing especially if broadband techniques are being employed. In this paper we discuss the development and application of an adaptive noise cancelling processor capable of extracting the weak far-field acoustic target in a noisy ocean acoustic environment. The fundamental idea for this processor is to use a model-based approach incorporating both target and ship noise. Here we briefly describe the underlying theory and then demonstrate through simulation how effective the canceller and target enhancer perform. The adaptivity of the processor not only enables the "tracking" of the canceller coefficients, but also the estimation of target parameters for localization. This approach which is termed "joint" cancellation and enhancement produces the optimal estimate of both in a minimum (error) variance sense.

I. INTRODUCTION

One of the problems that arises in towed-array processing, along with the usual noise sources such as background ambients and transients, is that the platform itself is a source of interference. This is especially true when the platform is a surface ship, which generates broadband flow and cavitation noise as well as the usual narrowband spectral lines originating from the engine and propellers [1-2]. Attempts to reduce these noises and interferences usually assume that only narrowband processing is necessary for such tasks as detection, localization and tracking; therefore, much of this platform noise is inherently removed anyway and can therefore be ignored. Other approaches, recognizing the detrimental effects of the inherent noise, develop more complex conventional filters to mitigate it, but this approach can partially remove the weak signal being sought and therefore can actually decrease the effective signal-to-noise ratio (SNR) [3-4]. A more effective approach to solving the signal enhancement and noise cancellation problem is to use a reference sensor, close to the ship, to obtain a useful sampling of this interference and develop an optimal noise canceling processor [5]---this is the approach we pursue in this paper. We cast the problem into a model-based framework to develop a joint cancellation/signal enhancement solution. Given the reference measurement, a model-based processor is developed which provides a joint estimate of the signal *and* the noise, thereby allowing the signal to evolve as a separate component of this estimate from which the noise has effectively been subtracted. We start with the basic canceling problem and then investigate the structure of the processor in the model-based framework. It is shown that the joint processor can be designed under a wide set of operating conditions with the target known and unknown.

II. OPTIMAL NOISE CANCELLING

In this section we briefly develop the optimal noise canceller for stationary processes and then extend it to the non-stationary case by embedding it into a Gauss-Markov framework [6,7]. The basic structure of the noise canceller is shown in Fig. 1 where we see that the process is characterized by a space-time signal at the ℓ^{th}-sensor of an L-element array in additive white noise as

$$p(x_\ell;t) = s(x_\ell;t) + \eta(x_\ell;t); \ \ell = 1,\cdots,L\,, \tag{1}$$

for $p,\ s,\ \eta$, the respective measurement, signal and noise at position x and time t. We also assume that there exists a reference signal, $r(x_\ell;t)$, correlated to the noise which can be characterized by an invertible impulse response, $H_\eta(x_\ell;t)$, that is,

$$r(x_\ell;t) = H_\eta(x_\ell;t) * \eta(x_\ell;t) \tag{2}$$

Since it is assumed invertible, we can write the primary canceller result [5] that

$$\eta(x_\ell;t) = H(x_\ell;t) * r(x_\ell;t) \tag{3}$$

for $H(x_\ell;t) := H_\eta^{-1}(x_\ell;t)$. The optimal noise canceling problem (in terms of this model) is:

> GIVEN the set of discrete space-time sensor measurements, $\{p(x_\ell;t)\}$ in additive noise, $\eta(x_\ell;t)$, and reference measurements, $\{r(x_\ell;t)\}$ correlated to the noise $\eta(x_\ell;t)$ for $t = 1,\cdots,N_t$; FIND the best (minimum error variance) estimate of the noise, $\hat{\eta}(x_\ell;t)$, (or equivalently $\hat{H}(x_\ell;t)$) such that the cancelled output, $z(x_\ell;t)$, is optimal.

The solution to this problem is well-known [6-9] and leads to the optimal canceling (Wiener) filter given by

$$\mathbf{H}_{\text{opt}} = \mathrm{R}_{rr}^{-1}\mathbf{r}_{yr} \tag{4}$$

in the stationary case or the adaptive least-mean squared (LMS) solution in the non-stationary case [5,6]. Note that the purpose of the canceling filter is to "shape" the reference signal such that it best approximates $\eta(x;t)$, the contaminating noise for removal. Thus, we have that the cancelled output is

$$
\begin{aligned}
z(x_\ell;t) &= p(x_\ell;t) - \hat{\eta}(x_\ell;t) = s(x_\ell;t) + \left[\eta(x_\ell;t) - \hat{\eta}(x_\ell;t)\right] \\
&= s(x_\ell;t) + \left[\eta(x_\ell;t) - \hat{H}(x_\ell;t) * r(x_\ell;t)\right] \approx s(x_\ell;t)
\end{aligned} \tag{5}
$$

Clearly, when $\hat{\eta} \to \eta$, $z \to s$, the desired result is obtained.

With this motivation in mind, we construct a Gauss-Markov representation of the canceller that will be used in solving the joint problem. Note that this approach is equivalent to compensating for colored noise [6-8]. Expanding over the L-elements and using the state-space representation, it is easy to show that the noise canceller can be represented (in general) by the *Gauss-Markov ship noise model* as (see Fig. 1)

$$
\begin{aligned}
&\boldsymbol{\xi}(t) = A_\xi(t-1)\boldsymbol{\xi}(t-1) + B_\xi(t-1)r(t-1) + \mathbf{w}_\xi(t-1) \\
&\boldsymbol{\eta}(t) = C_\xi(t)\boldsymbol{\xi}(t) + \mathbf{v}_\xi(t) \\
&\mathbf{p}(t) = \mathbf{s}(t) + \boldsymbol{\eta}(t) + \mathbf{v}(t)
\end{aligned} \quad , \tag{6}
$$

with $\boldsymbol{\xi} \in \mathrm{R}^{N_\xi \times 1}$ the colored noise state vector and r the known scalar reference noise (input) where the additive zero-mean, white gaussian noise sources have respective covariances, $R_{\mathrm{w}_\xi \mathrm{w}_\xi}$ and $R_{\mathrm{v}_\xi \mathrm{v}_\xi}$. Here $\mathbf{p}, \mathbf{s}, \boldsymbol{\eta}, \mathbf{v} \in \mathbf{C}^{L \times 1}$ are the respective pressure-field measurement, signal, colored and broadband measurement noise with $v \sim N(0, R_{vv}(t))$.

$A_\xi \in R^{N_\xi \times N_\xi}$, $B_\xi \in R^{N_\xi \times 1}$, $C_\xi \in R^{L \times N_\xi}$ are the system, input and measurement matrices corresponding to the ship noise model parameters. Note also that the spatial dimension is now incorporated in the dimensions of the vector-matrices in this model. That is, we have expanded over the L-elements in the sensor array, $x \to x_\ell;\quad \ell = 1, \cdots, L$ which gives

$\eta(x_\ell;t) \to \boldsymbol{\eta}(t);\ \xi(x_\ell;t) \to \boldsymbol{\xi}(t)$. Recall that the impulse response of the state-space model is

$H_\xi(t,k) = C_\xi(t)\Phi_\xi(t,k)B_\xi(k)$ for $\Phi_\xi(t,k) = A_\xi(t-k)$ (7)

which reduces to

$$
H_\xi(t,k) = C_\xi A_\xi^{t-k} B_\xi \quad \text{for} \quad t > k , \tag{8}
$$

in the time invariant case. So we see that ship noise can be completely captured by a Gauss-Markov representation in both stationary and nonstationary cases.

We assume that the *signal* can be characterized by a weak *target* in the far-field of the array given by

$$
s(t) = \alpha_o e^{i(\omega_o t - \mathbf{k}_o \cdot \mathbf{x})} = \alpha_o e^{i(\omega_o t - k_o \sin\theta_o (x_o + vt))} , \tag{9}
$$

for the source parameters: $\alpha_o,\ \omega_o,\ k_o,\ \theta_o,\ x_o$ that are the respective amplitude, temporal frequency, wavenumber, bearing angle and initial sensor location. Since the array is being towed, we include the tow speed, v, as well. We can simplify this model by defining the following terms,

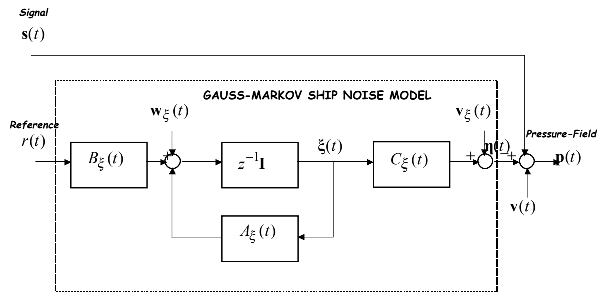

Fig. 1. Gauss-Markov ship noise model.

$$
s(t) = \alpha_o(t) e^{-i\beta_o(t)\sin\theta_o} , \tag{10}
$$

for $\alpha_o(t) := \alpha_o e^{i\omega_o t}$ and $\beta_o(t) := k_o\left(x_o + vt\right)$. Note that the statistics are not restricted to be stationary, so we can accommodate the nonstationarities (transients, etc.) that occur naturally in the ocean environment [9].

Using the Gauss-Markov representation of the noise, we can re-define the optimal *cancellation* problem as:

> GIVEN a set of discrete noisy pressure-field and reference measurements, $\{\mathbf{p}(t),\ r(t)\},\ t = 1, 2, \cdots, N_t$ in additive noise and the Gauss-Markov model of Eq. (6), FIND the best (minimum variance) estimate of the ship noise, $\hat{\boldsymbol{\eta}}(t \mid t)$, such that the canceller output, $\boldsymbol{\varepsilon}_p(t) = \mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t \mid t) \approx \mathbf{s}(t)$ is optimal.

The recursive solution to this problem is given by the MBP (Kalman filter) and shown in Table I (see [6] for details). Under the gaussian assumptions, this provides an optimal estimator for the noise cancellation problem with known signal; however, we must account for the more realistic case of an unknown far-field signal. Next we formulate the underlying joint estimation problem.

III. ADAPTIVE MODEL-BASED NOISE CANCELLING

In section we use the models developed in the previous section to develop the adaptive model-based processor (AMBP) for solving the joint cancellation/signal enhancement problem. We show that by augmenting the cancelling filter into the pressure-field representation that the cancelling operation *inherently* performs the noise cancellation as part of the usual filtering operation. Adaptivity follows by jointly estimating the target and cancelling filter parameters.

TABLE I
OPTIMUM NOISE CANCELLATION

NOISE ESTIMATOR	
$\hat{\xi}(t\mid t-1) = A_\xi(t-1)\hat{\xi}(t-1) + B_\xi(t-1)r(t-1)$	[Prediction]
$\hat{\boldsymbol{\eta}}(t\mid t-1) = C_\xi(t)\hat{\xi}(t\mid t-1)$	[Predicted Noise]
$\mathbf{e}_\eta(t) = \boldsymbol{\eta}(t) - \hat{\boldsymbol{\eta}}(t\mid t-1) = C_\xi(t)\tilde{\xi}(t\mid t-1) + \mathbf{v}_\xi(t)$	[Innov.]
$R_{e_\eta e_\eta}(t) = C_\xi(t)\tilde{P}_{\xi\xi}(t\mid t-1)C'_\xi(t) + R_{v_\xi v_\xi}(t)$	[Innov. Covar.]
$\hat{\xi}(t\mid t) = \hat{\xi}(t\mid t-1) + K_\xi(t)\mathbf{e}_\eta(t)$	[Correction]
$K_\xi(t) = \tilde{P}_{\xi\xi}(t\mid t-1)C'_\xi(t)R^{-1}_{e_\eta e_\eta}(t)$	[Gain]
$\tilde{\xi}(t\mid t-1) = \xi(\mathbf{t}) - \hat{\xi}(t\mid t-1)$	[State Est. Error]
CANCELLER	
$\hat{\boldsymbol{\eta}}(t\mid t) = C_\xi(t)\hat{\xi}(t\mid t)$	[Filtered Noise]
$\boldsymbol{\varepsilon}_p(t) = \mathbf{p}(t) - \hat{\mathbf{p}}(t\mid t) = \mathbf{p}(t) - \hat{\boldsymbol{\eta}}(t\mid t) \approx \mathbf{s}(t)$	[Cancelled Out.]
where $\tilde{\xi}(t\mid t-1)$, $\tilde{P}_{\xi\xi}(t\mid t-1)$ are state error and covariance.	

Since $\mathbf{s}(t)$ is assumed to be a far-field source, we have that at the ℓ^{th}-sensor, $s_\ell(t) = \alpha_\ell(t)e^{-i\beta_\ell(t)\sin\theta}$. Now expanding over the L-sensor array, we obtain the signal vector

$$\mathbf{s}(t) = \begin{bmatrix} \alpha_1(t)e^{-ik\beta_1(t)\sin\theta} \\ \vdots \\ \alpha_L(t)e^{-ik\beta_L(t)\sin\theta} \end{bmatrix} = \begin{bmatrix} \alpha e^{i\omega t}e^{-ik(x_1+vt)\sin\theta} \\ \vdots \\ \alpha e^{i\omega t}e^{-ik(x_L+vt)\sin\theta} \end{bmatrix}, \quad (11)$$

For signal enhancement we begin by defining the signal vector in terms of its unknown parameters, $\mathbf{s}(t;\Theta)$, (for a single target), $\boldsymbol{\Theta} := [\alpha \mid \omega \mid \theta]'$. In this case we assume that the unknown parameters in the signal model, $\boldsymbol{\Theta}$, are characterized as piecewise constant ($\dot{\boldsymbol{\Theta}} = \mathbf{0}$) with a discrete Gauss-Markov model given by

$$\boldsymbol{\Theta}(t) = \boldsymbol{\Theta}(t-1) + \Delta t\mathbf{w}_\Theta(t-1), \quad (12)$$

where $w_\Theta \sim N(0, R_{w_\Theta w_\Theta})$ and Δt is the sampling interval. This parameter vector is then augmented with the cancelling filter by defining the new state vector as $\mathbf{x}(t) := [\xi(t) \mid \boldsymbol{\Theta}(t)]' \in \mathrm{R}^{(N_\xi + N_\Theta)\times 1}$. The augmented model requires more analysis before we develop the MBP solution. Consider the augmented state-space model first as:

$$\begin{bmatrix} \xi(t) \\ \boldsymbol{\Theta}(t) \end{bmatrix} = \begin{bmatrix} A_\xi(t-1) & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \xi(t-1) \\ \boldsymbol{\Theta}(t-1) \end{bmatrix} + \begin{bmatrix} B_\xi(t-1) \\ 0 \end{bmatrix} r(t-1) + \begin{bmatrix} \mathbf{w}_\xi(t-1) \\ \Delta t\mathbf{w}_\Theta(t-1) \end{bmatrix}. \quad (13)$$

Here we note that the cancelling filter and parameters are decoupled in the state-space and can therefore be written directly as

$$\begin{aligned} \xi(t) &= A_\xi(t-1)\xi(t-1) + B_\xi(t-1)r(t-1) + \mathbf{w}_\xi(t-1) \\ \boldsymbol{\Theta}(t) &= \boldsymbol{\Theta}(t-1) + \Delta t\mathbf{w}_\Theta(t-1) \end{aligned} \quad (14)$$

Next we note that the pressure-field measurement is the superposition of three distinct components: far-field signal, ship generated noise and instrumentation noise given by

$$\mathbf{p}(t) = \underbrace{\mathbf{s}(t;\boldsymbol{\Theta})}_{\text{signal}} + \underbrace{\boldsymbol{\eta}(t)}_{\text{ship noise}} + \underbrace{\mathbf{v}(t)}_{\text{measurement noise}} \quad (15)$$

First we note from Eq. (6) that the output of the decoupled cancelling filter remains (see Eq. (6))

$$\boldsymbol{\eta}(t) = C_\xi(t)\xi(t) + \mathbf{v}_\xi(t).$$

Therefore, substituting into Eq. (15) and accounting for the augmented state vector, we obtain

$$\mathbf{p}(t) = \begin{bmatrix} C_\xi(t) \mid 0 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \boldsymbol{\Theta}(t) \end{bmatrix} + \mathbf{s}(t;\boldsymbol{\Theta}) + \mathbf{v}_\xi(t) + \mathbf{v}(t) \quad (16)$$

Since the far-field signal is a nonlinear function of the parameters (augmented states), that is, at the ℓ^{th} sensor, $s_\ell(t;\boldsymbol{\Theta}) = \alpha_\ell(t)e^{-i\beta_\ell(t)} = \Theta_1 e^{i(\Theta_2 t - k(x_\ell+vt)\sin\Theta_3}$ for the single target case, then the pressure-field across the array is also a nonlinear function, that is,

$$\begin{aligned} \mathbf{p}(t) &= \mathbf{c}[\xi(t), \boldsymbol{\Theta}(t)] + \mathbf{v}(t) = [\mathbf{s}(t;\boldsymbol{\Theta}(t)) + \boldsymbol{\eta}(t)] + \mathbf{v}(t) \\ &= \mathbf{s}(t;\boldsymbol{\Theta}(t)) + C_\xi(t)\xi(t) + \mathbf{v}_\xi(t) + \mathbf{v}(t) \end{aligned} \quad (17)$$

Therefore, we have the following approximate model given by the underlying *augmented* Gauss-Markov representation as:

$$
\begin{aligned}
\mathbf{x}(t) &= \mathbf{A}_{\xi\Theta}(t-1)\mathbf{x}(t-1)+\mathbf{B}_{\xi\Theta}(t-1)r(t-1)+\mathbf{w}_{\xi\Theta}(t-1)\\
\mathbf{p}(t) &= \mathbf{c}[\boldsymbol{\xi}(t),\boldsymbol{\Theta}(t)]+\mathbf{v}(t)=\left[\mathbf{s}\left(t;\boldsymbol{\Theta}(t)\right)+\boldsymbol{\eta}(t)\right]+\mathbf{v}(t)\\
\boldsymbol{\eta}(t) &= C_{\xi}(t)\boldsymbol{\xi}(t) + \mathbf{v}_{\xi}(t)
\end{aligned}
$$

where $\mathbf{A}_{\xi\Theta}=\begin{bmatrix} A_{\xi}(t-1) & 0 \\ 0 & \mathbf{I}\end{bmatrix}$, $\mathbf{B}_{\xi\Theta}=\begin{bmatrix} B_{\xi}(t-1) \\ 0\end{bmatrix}$,

$$
\mathbf{C}_{\xi\Theta}=\left[C_{\xi}(t)\,|\,0\right], \qquad \mathbf{w}_{\xi\Theta}=\begin{bmatrix}\mathbf{w}_{\xi}(t-1)\\ \mathbf{w}_{\Theta}(t-1)\end{bmatrix}
$$

and
$$
\mathbf{s}(t;\Theta)=\begin{bmatrix}\Theta_1 e^{i\Theta_2 t}e^{-ik(x_1+vt)\sin\Theta_3}\\ \vdots \\ \Theta_1 e^{i\Theta_2 t}e^{-ik(x_L+vt)\sin\Theta_3}\end{bmatrix} \tag{18}
$$

The basic joint cancelling/signal enhancement problem can now be stated in terms of this augmented Gauss-Markov representation as:

> GIVEN a set of discrete noisy pressure-field and reference measurements, $\{\mathbf{p}(t),\ r(t)\},\ t=1,2,\cdots,N_t$ and the Gauss-Markov model of Eq. (18), FIND the best (minimum variance) estimate of the augmented state (ship noise+signal), $\hat{\mathbf{x}}(t\,|\,t)$, or equivalently, $\hat{\mathbf{s}}(t;\Theta)$ and $\hat{\boldsymbol{\eta}}(t\,|\,t)$, such that the canceller output, $\boldsymbol{\varepsilon}_p(t)=\mathbf{p}(t)-\hat{\boldsymbol{\eta}}(t\,|\,t)$ is optimal.

We have a linear decoupled state-space, but (unfortunately) a nonlinear measurement system requiring a nonlinear processor. This problem can be solved by a parametrically adaptive MBP using the recursive extended Kalman filter (EKF) given in Table II for the augmented system algorithm.

If we decompose the state vector and perform the partitioned operations, then we see immediately that the canceling filter and signal parameters are estimated "jointly" along with the enhanced signal and noise estimates as shown in Table III.

To formalize the processor further in terms of our ocean acoustic problem, let us first investigate the predicted measurement in more detail to focus on the actual operations performed. We start with the augmented representation, which is a nonlinear function due to the augmentation of the parameters, that is,

$$
\begin{aligned}
\hat{\mathbf{p}}(t\,|\,t-1) &= \mathbf{c}\left[\hat{\mathbf{x}}(t\,|\,t-1)\right]=\mathbf{s}\left(t;\hat{\boldsymbol{\Theta}}(t\,|\,t-1)\right)+\hat{\boldsymbol{\eta}}(t\,|\,t-1)\\
&= \mathbf{s}(t;\hat{\boldsymbol{\Theta}})+C_{\xi}(t)\hat{\boldsymbol{\xi}}(t\,|\,t-1)
\end{aligned},
\tag{19}
$$

TABLE II
JOINT MODEL-BASED PROCESSOR

EXTENDED KALMAN FILTER	
$\hat{\mathbf{x}}(t\,|\,t-1) = \mathbf{A}_{\xi\Theta}(t-1)\hat{\mathbf{x}}(t-1\,|\,t-1)+\mathbf{B}_{\xi\Theta}(t-1)r(t-1)$	[Predict]
$\tilde{P}(t\,|\,t-1)= \bar{\mathbf{A}}_{\xi\Theta}[\hat{\mathbf{x}}]\tilde{P}(t-1\,|\,t-1)\bar{\mathbf{A}}'_{\xi\Theta}[\hat{\mathbf{x}}]+R_{\mathbf{w}_{\xi\Theta}\mathbf{w}_{\xi\Theta}}(t-1)$	
$\mathbf{e}_{\mathrm{p}}(t) = \mathbf{p}(t)-\hat{\mathbf{p}}(t\,|\,t-1)$	[Innov.]
$\hat{\mathbf{p}}(t\,|\,t-1) = \mathbf{c}[\hat{x}(t\,|\,t-1)]$	
$R_{\mathbf{e}_{\mathrm{p}}\mathbf{e}_{\mathrm{p}}}(t)= \bar{\mathbf{C}}_{\xi\Theta}[\hat{\mathbf{x}}]\tilde{P}(t\,|\,t-1)\bar{\mathbf{C}}'_{\xi\Theta}[\hat{\mathbf{x}}]+R_{v_{\xi}v_{\xi}}(t)+R_{vv}(t)$	
$\mathbf{K}_{\xi\Theta}(t) = \tilde{P}(t\,|\,t-1)\bar{\mathbf{C}}'_{\xi\Theta}[\hat{\mathbf{x}}]R^{-1}_{\mathbf{e}_{\mathrm{p}}\mathbf{e}_{\mathrm{p}}}(t)$	[Gain]
$\hat{\mathbf{x}}(t\,|\,t) = \hat{\mathbf{x}}(t\,|\,t-1)+\mathbf{K}_{\xi\Theta}(t)\mathbf{e}_{\mathrm{p}}(t)$	[Correct]
$\tilde{P}(t\,|\,t) = \left(\mathbf{I}\text{-}\mathbf{K}_{\xi\Theta}(t)\bar{\mathbf{C}}_{\xi\Theta}[\hat{\mathbf{x}}]\right)\tilde{P}(t\,|\,t-1)$	
with jacobians: $\bar{\mathbf{A}}_{\xi\Theta}[\mathbf{x}]:=\left.\frac{\partial \mathbf{A}_{\xi\Theta}}{\partial \mathbf{x}}\right\|_{\mathbf{x}=\hat{\mathbf{x}}}$; $\bar{\mathbf{C}}_{\xi\Theta}[\hat{\mathbf{x}}]:=\left.\frac{\partial \mathbf{c}[\mathbf{x}]}{\partial \mathbf{x}}\right\|_{\mathbf{x}=\hat{\mathbf{x}}}$	

for

$$
\mathbf{s}(t;\hat{\boldsymbol{\Theta}}) = \begin{bmatrix}\hat{\Theta}_1(t\,|\,t-1)e^{i\hat{\Theta}_2(t|t-1)t}e^{-ik(x_1+vt)\sin\hat{\Theta}_3(t|t-1)}\\ \vdots \\ \hat{\Theta}_1(t\,|\,t-1)e^{i\hat{\Theta}_2(t|t-1)t}e^{-ik(x_L+vt)\sin\hat{\Theta}_3(t|t-1)}\end{bmatrix}. \tag{20}
$$

The corresponding innovations for the adaptive processor can also be written in terms of its components as

$$
\begin{aligned}
\mathbf{e}_{\mathrm{p}}(t) &= \mathbf{p}(t)-\hat{\mathbf{p}}(t\,|\,t-1)=\mathbf{p}(t)-\mathbf{s}\left(t;\hat{\boldsymbol{\Theta}}\right)-\hat{\boldsymbol{\eta}}(t\,|\,t-1)\\
&= \left(\mathbf{s}(t)-\mathbf{s}\left(t;\hat{\boldsymbol{\Theta}}\right)\right)+\left(\boldsymbol{\eta}(t)-\hat{\boldsymbol{\eta}}(t\,|\,t-1)\right)+\mathbf{v}(t)\\
&= \tilde{\mathbf{s}}\left(t;\hat{\boldsymbol{\Theta}}\right)+\tilde{\boldsymbol{\eta}}(t\,|\,t-1)+\mathbf{v}(t)\\
&= \tilde{\mathbf{s}}\left(t;\hat{\boldsymbol{\Theta}}\right)+C_{\xi}(t)\tilde{\boldsymbol{\xi}}(t\,|\,t-1)+\mathbf{v}_{\xi}(t)+\mathbf{v}(t)
\end{aligned}
\tag{21}
$$

So we see that the joint parametrically adaptive processor is capable of not only providing the optimal cancelling solution $\left(\hat{\boldsymbol{\eta}}(t\,|\,t-1)\rightarrow\boldsymbol{\eta}(t)\right)$, but also capable of estimating the far-field target signal parameters for optimal enhancement $\left(\hat{\mathbf{s}}(t;\hat{\boldsymbol{\Theta}})\rightarrow\mathbf{s}(t)\right)$.

Using the EKF algorithm it is necessary to provide the jacobians for implementation, that is,

TABLE III
JOINT MODEL-BASED CANCELLER/ENHANCER

PREDICTOR
$\hat{\xi}(t\mid t-1) \;= A_\xi(t-1)\hat{\xi}(t-1\mid t-1)+B_\xi(t-1)r(t-1)$
$\hat{\mathbf{\Theta}}(t\mid t-1) \;= \hat{\mathbf{\Theta}}(t-1\mid t-1)$
INNOVATIONS
$\mathbf{e}_\mathrm{p}(t) \;= \mathbf{p}(t)\text{-}\hat{\mathbf{p}}(t\mid t-1)$
$\hat{\mathbf{\eta}}(t\mid t-1) \;= C_\xi(t)\hat{\xi}(t\mid t-1)$
$\hat{\mathbf{p}}(t\mid t-1) \;= \mathbf{s}\left(t;\hat{\mathbf{\Theta}}(t\mid t-1)\right)+\hat{\mathbf{\eta}}(t\mid t-1)$
CORRECTOR
$\hat{\xi}(t\mid t) \;= \hat{\xi}(t\mid t-1)+\mathrm{K}_\xi(t)\mathbf{e}_\mathrm{p}(t)$
$\hat{\mathbf{\Theta}}(t\mid t) \;= \hat{\mathbf{\Theta}}(t\mid t-1)+\mathrm{K}_\Theta(t)\mathbf{e}_\mathrm{p}(t)$
CANCELLER
$\hat{\mathbf{\eta}}(t\mid t) \;= C_\xi(t)\hat{\xi}(t\mid t)$
$\mathbf{\varepsilon}_\mathrm{p}(t) \;= \mathbf{p}(t)-\hat{\mathbf{\eta}}(t\mid t)$
ENHANCER
$\hat{\mathbf{s}}(t;\mathbf{\Theta}) \;= \mathbf{s}\left(t;\hat{\mathbf{\Theta}}(t\mid t)\right)$
for $\mathbf{K}_{\xi\Theta}(t):=\begin{bmatrix}\mathrm{K}_\xi(t)\\ \mathrm{K}_\Theta(t)\end{bmatrix}$ with $\mathbf{K}_{\xi\Theta}\in \mathrm{R}^{(N_\xi+N_\Theta)\times N_\mathrm{p}}$
and $\mathrm{K}_\xi \in \mathrm{R}^{N_\xi\times N_\mathrm{p}};\mathrm{K}_\Theta\in \mathrm{R}^{N_\Theta\times N_\mathrm{p}}$

$$\frac{\partial \mathrm{a}[\xi,\mathbf{\Theta}]}{\partial \xi}=A_\xi(t),\qquad \frac{\partial a[\xi,\mathbf{\Theta}]}{\partial \mathbf{\Theta}}=\mathrm{I}$$
$$\frac{\partial \mathrm{c}[\xi,\mathbf{\Theta}]}{\partial \xi}=C_\xi(t),\quad \frac{\partial \mathrm{c}[\xi,\mathbf{\Theta}]}{\partial \theta}=i\alpha(t)\beta_\ell(t)\cos\theta e^{i\beta_\ell(t)\sin\theta}$$
$$\frac{\partial \mathrm{c}[\xi,\mathbf{\Theta}]}{\partial \omega}=it\alpha(t)e^{i\beta_\ell(t)\sin\theta},\quad \frac{\partial \mathrm{c}[\xi,\mathbf{\Theta}]}{\partial \alpha}=e^{i(\omega t-\beta_\ell(t)\sin\theta)}$$
$$\ell=1,\cdots,L \tag{22}$$

completing the development of the parametrically adaptive solution to the joint cancellation/signal enhancement problem. Next we summarize our results and discuss future efforts.

IV. RESULTS

The most dominant type of ship noise evolves from the noisy engine, propellers and gears resulting in spectral lines (sinusoids) at low frequency along with their harmonics. Therefore, we will concentrate on these dominant noise and interference sources that directly couple to the towed array to demonstrate the capability of the optimum canceller coupled to the signal enhancer. We assume the target bearing is unknown and construct the adaptive signal enhancer while jointly removing the interferences and noise.

In our problem we will assume that the ship noise is dominated by the engine spectral lines at 10, 20 and 30 Hz; therefore, the optimal canceling filter can be captured nicely by a 6th-order autoregressive (AR) model [6], [8]. The AR model is simply

$$\eta(t)=\sum_{i=1}^{N_a}a_i\eta(t-1)+\varepsilon(t) \tag{23}$$

This model can easily be converted to a Gauss-Markov representation as a special case of the observer canonical form [6], that is,

$$\xi(t)=A_\xi\xi(t-1)+\mathbf{w}_\xi(t-1)$$
$$\mathbf{\eta}(t)=C_\xi\xi(t)+\mathbf{v}_\xi(t)$$

for

$$A_\xi=\begin{bmatrix}0&0&\cdots&0&-a_{N_a}\\1&0&\cdots&0&-a_{N_a-1}\\0&1&0&0&-a_{N_a-2}\\ \vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&1&-a_1\end{bmatrix};\quad C_\xi=[0\;0\;\cdots\;0\;1]; \tag{24}$$

$$\mathbf{w}_\xi\sim N(0,R_{w_\xi w_\xi}),\mathbf{v}_\xi\sim N(0,R_{v_\xi v_\xi})$$

For this case we perform a Gauss-Markov simulation using the following measurement model (as in Eq. (15))

$$\mathbf{p}(t)=\mathbf{s}(t;\mathbf{\Theta})\quad+\quad\mathbf{\eta}(t)\quad+\quad\mathbf{v}(t).$$

We simulated two planar sources of unit amplitude, 50Hz frequency, 7.5m wavelength impinging on an 8-element towed array with a sampling interval of dt=0.005 sec for measurement noise with a variance of $R_{vv}=4$. The arrival angles were $\{\theta_1,\theta_2\}=45^o,-10^o$ and the canceling filter obtained from the canceling algorithm with coefficients:

$$\{a_1,a_2,a_3,a_4,a_5,a_6\}=\{-1,5.7809,-14.1358,18.7096,\\-14.1358,5.7809\}.$$

The simulation results are shown in Figs. 2-4 below. In Fig. 2 we show the true value of the 45^0 on the top left and its estimate on the top right Below are the true (left) and estimated (right) values of the signal from hydrophone No. 1. Figs. 3 and 4 show the innovations for the first four measurements. Figure 3 is for the case with the noise model included and fig. 4 is the case for the noise model removed. The degradation in the innovations is clearly seen here.

This completes the implementation of the canceller demonstrating that it can effectively incorporated into the MBP framework while removing its effect and enhancing the signal.

V. SUMMARY

In this paper we have developed a solution to the joint cancellation/signal enhancement problem using a model-based approach [6]. Starting with the optimal noise canceller solution we developed the corresponding model-based solution demonstrating their equivalence for the case where the signal is known *a priori*. Next we developed the solution to the joint problem with the signal unknown, but parameterized as a far-field target. The solution to this problem lead to the parametrically adaptive model-based processor implemented with the (nonlinear) extended Kalman filter (EKF) algorithm. It was shown how to design the processor for this problem.

Future efforts will be aimed at applying this technique to both simulated and measured hydrophone data. We plan to use the discrete implementation of the EKF available in MATLAB [10] with the toolbox SSPACK_PC [11].

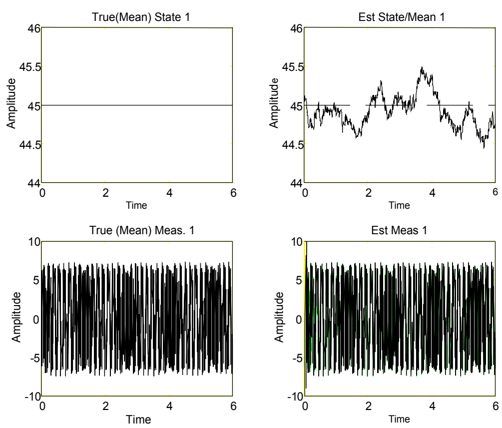

Figure 2. True and estimated values of the first bearing angle and the signal on the first hydrophone.

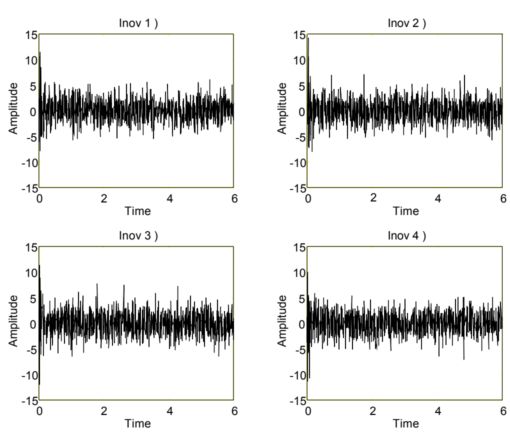

Figure 3. Innovations for the first four signals

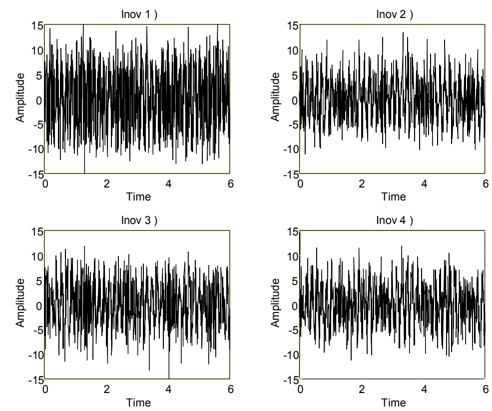

Figure 4. Innovations for the first four signals With the noise model removed.

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