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Symplectic Symmetry and the *Ab Initio* No-Core Shell Model

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The symplectic symmetry of eigenstates for the 0^+_{gs} in ^{16}O and the 0^+_{gs} and lowest 2^+ and 4^+ configurations of ^{12}C that are well-converged within the framework of the no-core shell model with the JISP16 realistic interaction is examined. These states are found to project at the $85-90\%$ level onto very few symplectic representations including the most deformed configuration, which confirms the importance of a symplectic no-core shell model and reaffirms the relevance of the Elliott SU(3) model upon which the symplectic scheme is built.

I. INTRODUCTION

With the development of high-precision nucleon-nucleon (*NN*) interactions derived from meson exchange theory and the latest advances in chiral effective field theory based on QCD that provide components of nuclear forces that can be matched to the underlying theory of quarks and gluons, *ab-initio* nuclear theoretical models play a crucial role towards a deeper understanding of fundamental aspects of nuclear physics. *Ab-initio* calculations target reproducing nuclear structure features while employing realistic *NN* (or many-nucleon) interactions and hence bridge from nuclear structure considerations to the nucleon constituent degrees of freedom and, in turn, to astrophysical phenomena, including nucleosynthesis and neutron stars, as well as towards a further exploration of $3N$ nuclear forces and exotic physics of rare isotopes.

The *ab-initio* No-Core Shell Model (NCSM) [1] with modern realistic interactions yields a good description of the low-lying states in few-nucleon systems [2] as well as in more complex nuclei like ^{12}C [1]. In addition to advancing our understanding of the propagation of the nucleon-nucleon force in nuclear matter and clustering phenomena [3, 4], modeling the structure of ^{12}C, ^{16}O and similar nuclei is also important for gaining a better understanding of other physical processes such as parity-violating electron scattering from light nuclei [5] and results gained through neutrino studies [6] as well as for making better predictions for capture reaction rates that figure prominently, for example, in the burning of He in massive stars [7].

Our investigations show that the 0^+_{gs} and the lowest 2^+ and 4^+ states in ^{12}C and the ^{16}O ground state reflect the presence of an underlying symplectic $\mathfrak{sp}(3,\mathbb{R})$ algebraic structure [17]. This is achieved through the projection of realistic NCSM eigenstates onto $\mathrm{Sp}(3,\mathbb{R})$-symmetric basis states of the symplectic shell model that are free of spurious center-of-mass motion. Typically, eigenstates of the NCSM are reasonably well converged in the $N_{max} = 6$ (or $6\hbar\Omega$) basis space with an effective interaction based on the JISP16 realistic interaction [8]. In particular, calculated binding energies as well as other observables for ^{12}C such as $\mathrm{B(E2;}2^+_1 \rightarrow 0^+_{gs})$, $\mathrm{B(M1;}1^+_1 \rightarrow 0^+_{gs})$, ground-state proton rms radii and the 2^+_1 quadrupole moment all lie reasonably close to the measured values. The symplectic shell model itself [9, 10] is a microscopic realization of the successful Bohr-Mottelson collective model. It is also a multiple oscillator shell generalization of Elliott's SU(3) model. Hence, this analysis provides, for the first time, a close examination of symmetries in nuclei as unveiled through *ab-initio* calculations of the NCSM type with realistic interactions.

The rotational nature of the ground-state band in ^{12}C nucleus has long been recognized, and early-on group-theoretical methods with SU(3) serving as the underpinning (sub-)symmetry were used in its description. Symplectic algebraic approaches have achieved a very good reproduction of low-lying energies and B(E2) values in light nuclei [11, 12] and specifically in ^{12}C using phenomenological interactions [13] or truncated symplectic basis with simplistic (semi-) microscopic interactions [14, 15]. Here, we establish the dominance of the symplectic $\mathrm{Sp}(3,\mathbb{R})$ symmetry in the *ab-initio* NCSM wavefunctions. This in turn opens up a new and exciting possibility for representing significant high-$\hbar\Omega$ collective modes by extending the NCSM basis space beyond its current limits through $\mathrm{Sp}(3,\mathbb{R})$ basis states, which yields a dramatically smaller basis space to achieve convergence of higher-lying collective modes.

II. SYMPLECTIC SHELL MODEL

The symplectic shell model is based on the noncompact symplectic $\mathfrak{sp}(3,\mathbb{R})$ algebra that with its subalgebraic structure unveils the underlying physics of a microscopic description of collective modes in nuclei [9, 10]. The latter follows from the fact that the mass quadrupole and monopole moment operators, the many-particle ki-

netic energy, the angular and vibrational momenta are all elements of the $\mathfrak{sp}(3,\mathbb{R}) \supset \mathfrak{su}(3) \supset \mathfrak{so}(3)$ algebraic structure. Hence, collective states of a nucleus with well-developed quadrupole and monopole vibrations as well as collective rotations are described naturally in terms of irreducible representations (irreps) of $\mathrm{Sp}(3,\mathbb{R})$. Furthermore, the elements of the $\mathfrak{sp}(3,\mathbb{R})$ algebra are constructed as bilinear products in the harmonic oscillator (HO) raising and lowering operators that in turn are expressed through particle coordinates and linear momenta. This means the basis states of a $\mathrm{Sp}(3,\mathbb{R})$ irrep can be expanded in a HO (m-scheme) basis, the same basis used in the NCSM, thereby facilitating symmetry identification.

The symplectic basis states are labeled (in standard notation [9, 10]) according to the reduction chain

$$\begin{array}{ccccc} \mathrm{Sp}(3,\mathbb{R}) & \supset & \mathrm{U}(3) & \supset & \mathrm{SO}(3) \\ \Gamma_\sigma & \Gamma_n\rho & \Gamma_\omega & \kappa & L \end{array} \tag{1}$$

and are constructed by acting with polynomials $\mathcal{P}$ in the symplectic raising operator, $A^{(2\,0)}$, on a set of basis states of the symplectic bandhead, $|\Gamma_\sigma\rangle$, which is a $\mathrm{Sp}(3,\mathbb{R})$ lowest-weight state ($B^{(0\,2)}|\Gamma_\sigma\rangle = 0$, where the symplectic lowering operator $B^{(0\,2)}$ is the adjoint of $A^{(2\,0)}$); that is,

$$|\Gamma_\sigma \Gamma_n \rho \Gamma_\omega \kappa (LS) J M_J\rangle = \left[\mathcal{P}^{\Gamma_n}(A^{(2\,0)}) \times |\Gamma_\sigma\rangle\right]^{\rho\Gamma_\omega}_{\kappa(LS)JM_J}, \tag{2}$$

where $\Gamma_\sigma \equiv N_\sigma\,(\lambda_\sigma\,\mu_\sigma)$ labels $\mathrm{Sp}(3,\mathbb{R})$ irreps with $(\lambda_\sigma\,\mu_\sigma)$ denoting a SU(3) lowest-weight state, $\Gamma_n \equiv n\,(\lambda_n\,\mu_n)$, and $\Gamma_\omega \equiv N_\omega\,(\lambda_\omega\,\mu_\omega)$. The $(\lambda_n\,\mu_n)$ set gives the overall SU(3) symmetry of $\frac{n}{2}$ coupled raising operators in $\mathcal{P}$, $(\lambda_\omega\,\mu_\omega)$ specifies the SU(3) symmetry of the symplectic state, and $N_\omega = N_\sigma + n$ is the total number of oscillator quanta related to the eigenvalue, $N_\omega\hbar\Omega$, of a HO Hamiltonian that is free of spurious modes. Consequently, the symplectic basis states bring forward important information about the nuclear shape deformation in terms of the SU(3) labels, $(\lambda_\omega\ \mu_\omega)$, for example, $(0\ 0)$, $(\lambda\ 0)$ and $(0\ \mu)$ describe spherical, prolate and oblate shapes, respectively.

The symplectic raising operator $A^{(2\,0)}_{lm}$, which is a SU(3) tensor with $(\lambda\,\mu) = (2\,0)$ character, can be expressed as a bilinear product of the HO raising operators,

$$A^{(20)}_{lm} = \tfrac{1}{\sqrt{2}} \textstyle\sum_i \left[b^\dagger_i \times b^\dagger_i\right]^{(20)}_{lm} - \tfrac{1}{\sqrt{2A}} \textstyle\sum_{s,t} \left[b^\dagger_s \times b^\dagger_t\right]^{(20)}_{lm}, \tag{3}$$

where the sums are over all A particles of the system. The first term in (3) describes $2\hbar\Omega$ one-particle-one-hole (1p-1h) excitations (one particle raised by two shells) and the second term eliminates the spurious center-of-mass excitations in the construction (2). For the purpose of comparison to NCSM results, the basis states of the $|\Gamma_\sigma\rangle$ bandhead in (2) are constructed in a m-scheme basis,

$$|\Gamma_\sigma \kappa (L_0 S_0) J_0 M_0\rangle = \left[\mathcal{P}^{(\lambda_\pi\,\mu_\pi)}_{S_\pi}(a^\dagger_\pi) \times \mathcal{P}^{(\lambda_\nu\,\mu_\nu)}_{S_\nu}(a^\dagger_\nu)\right]^{(\lambda_\sigma\,\mu_\sigma)}_{\kappa(L_0S_0)J_0M_0} |0\rangle, \tag{4}$$

where $|0\rangle$ is a vacuum state, $\mathcal{P}^{(\lambda_\pi\,\mu_\pi)}_{S_\pi}$ and $\mathcal{P}^{(\lambda_\nu\,\mu_\nu)}_{S_\nu}$ denote polynomials of proton ($a^\dagger_\pi$) and neutron ($a^\dagger_\nu$) creation operators coupled to good SU(3)×SU(2) symmetry.

III. RESULTS AND DISCUSSIONS

The lowest-lying eigenstates of ^{12}C and ^{16}O were calculated using the NCSM as implemented through the Many Fermion Dynamics (MFD) code [16] with an effective interaction derived from the realistic JISP16 NN potential [8] for different $\hbar\Omega$ oscillator strengths. We are particularly interested in the ^{16}O ground state and the $J=0^+_{gs}$ and the lowest $J=2^+(\equiv 2^+_1)$ and $J=4^+(\equiv 4^+_1)$ states of the ground-state (gs) rotational band in ^{12}C.

TABLE I: Probability distribution of NCSM eigenstates for ^{12}C across the dominant 0p-0h and $2\hbar\Omega$ 2p-2h $\mathrm{Sp}(3,\mathbb{R})$ irreps, $\hbar\Omega$=15 MeV.

		$0\hbar\Omega$	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	Total
		$J=0$				
$\mathrm{Sp}(3,\mathbb{R})$	$(0\,4)S=0$	46.26	12.58	4.76	1.24	64.84
	$(1\,2)S=1$	4.80	2.02	0.92	0.38	8.12
	$(1\,2)S=1$	4.72	1.99	0.91	0.37	7.99
	$2\hbar\Omega$ 2p-2h		3.46	1.02	0.35	4.83
	Total	55.78	20.05	7.61	2.34	85.78
NCSM		56.18	22.40	12.81	7.00	98.38
		$J=2$				
$\mathrm{Sp}(3,\mathbb{R})$	$(0\,4)S=0$	46.80	12.41	4.55	1.19	64.95
	$(1\,2)S=1$	4.84	1.77	0.78	0.30	7.69
	$(1\,2)S=1$	4.69	1.72	0.76	0.30	7.47
	$2\hbar\Omega$ 2p-2h		3.28	1.04	0.38	4.70
	Total	56.33	19.18	7.13	2.17	84.81
NCSM		56.18	21.79	12.73	7.28	98.43
		$J=4$				
$\mathrm{Sp}(3,\mathbb{R})$	$(0\,4)S=0$	51.45	12.11	4.18	1.04	68.78
	$(1\,2)S=1$	3.04	0.95	0.40	0.15	4.54
	$(1\,2)S=1$	3.01	0.94	0.39	0.15	4.49
	$2\hbar\Omega$ 2p-2h		3.23	1.16	0.39	4.78
	Total	57.50	17.23	6.13	1.73	82.59
NCSM		57.64	20.34	12.59	7.66	98.23

TABLE II: Probability distribution of the NCSM eigenstate for the $J=0$ ground state in ^{16}O across the 0p-0h and dominant $2\hbar\Omega$ 2p-2h $\mathrm{Sp}(3,\mathbb{R})$ irreps, $\hbar\Omega$=15 MeV.

		$0\hbar\Omega$	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	Total
$\mathrm{Sp}(3,\mathbb{R})$	$(0\,0)S=0$	50.53	15.87	6.32	2.30	75.02
	$2\hbar\Omega$ 2p-2h		5.99	2.52	1.32	9.83
	Total	50.53	21.86	8.84	3.62	84.85
NCSM		50.53	22.58	14.91	10.81	98.83

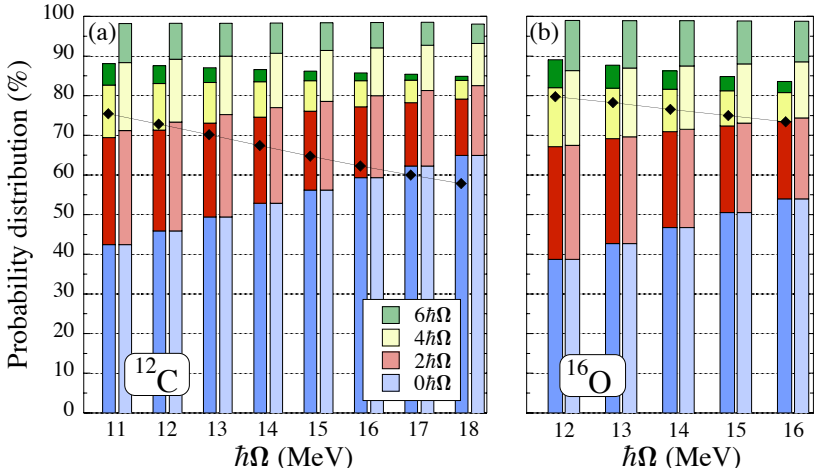

FIG. 1: Ground 0^+ state probability distribution over $0\hbar\Omega$ (blue, lowest) to $6\hbar\Omega$ (green, highest) subspaces for the most dominant 0p-0h + $2\hbar\Omega$ 2p-2h Sp$(3,\mathbb{R})$ irrep case (left) and NCSM (right) together with the leading irrep contribution (black diamonds), (0 4) for ^{12}C (a) and (0 0) for ^{16}O (b), as a function of the $\hbar\Omega$ oscillator strength, $N_{max} = 6$.

For both nuclei we constructed all of the 0p-0h and $2\hbar\Omega$ 2p-2h (2 particles raised by one shell each) symplectic bandheads and generated their Sp$(3,\mathbb{R})$ irreps up to $N_{max} = 6$ ($6\hbar\Omega$ model space). The typical dimension of a symplectic irrep basis in the $N_{max} = 6$ space is on the order of 10^2 as compared to 10^7 for the full NCSM m-scheme basis space.

Analysis of overlaps of the symplectic states with the NCSM eigenstates for $2\hbar\Omega$, $4\hbar\Omega$, and $6\hbar\Omega$ model spaces ($N_{max} = 2, 4, 6$) reveals the dominance of the 0p-0h Sp$(3,\mathbb{R})$ irreps. For the 0^+_{gs} and the lowest 2^+ and 4^+ states in ^{12}C there are nonnegligible overlaps for only 3 of the 13 0p-0h Sp$(3,\mathbb{R})$ irreps, namely, the leading (most deformed) representation with $N_\sigma = 24.5$ and $(\lambda_\sigma\ \mu_\sigma) = (0\ 4)$, and carrying spin $S = 0$ together with two 24.5 (1 2) $S = 1$ irreps with different bandhead constructions for protons and neutrons. For the ground state of ^{16}O there is only one possible 0p-0h Sp$(3,\mathbb{R})$ irrep, 34.5 (0 0) $S = 0$. In addition, among the $2\hbar\Omega$ 2p-2h Sp$(3,\mathbb{R})$ irreps only a small fraction contributes significantly to the overlaps and it includes the most deformed symplectic bandhead configurations that correspond to oblate shapes in ^{12}C and prolate ones in ^{16}O.

The overlaps of the most dominant symplectic states with the ^{12}C and ^{16}O NCSM eigenstates under consideration in the 0, 2, 4 and $6\hbar\Omega$ subspaces are given in Table I and II. In order to speed up the calculations, we retained only the largest amplitudes of the NCSM states, those sufficient to account for at least 98% of the norm which is quoted also in the table. The results show that approximately 85% of the NCSM eigenstates for ^{12}C (^{16}O) fall within a subspace spanned by the few most significant 0p-0h and $2\hbar\Omega$ 2p-2h Sp$(3,\mathbb{R})$ irreps, with the $2\hbar\Omega$ 2p-2h Sp$(3,\mathbb{R})$ irreps accounting for 5% (10%) and with the leading irrep, (0 4) for ^{12}C and (0 0) for ^{16}O, carrying close to 70% (75%) of the NCSM wavefunction.

In addition, the projection of the NCSM wavefunctions onto the symplectic space slightly changes as one varies the oscillator strength $\hbar\Omega$ (see, e.g., Fig. 1 for the 0^+_{gs} state). The overall overlaps increase towards smaller $\hbar\Omega$ HO frequencies and, for example, for 0^+_{gs} it is 90% in the $N_{max} = 6$ and $\hbar\Omega = 11$MeV case. Clearly, the largest contribution comes from the leading, most deformed (0 4)$S = 0$ Sp$(3,\mathbb{R})$ irrep for ^{12}C and (0 0)$S = 0$ for ^{16}O, growing to $\approx 90\%$ of the total Sp$(3,\mathbb{R})$-symmetric part for $\hbar\Omega$ =11 MeV. As expected, Fig. 1 also confirms that with increasing $\hbar\Omega$ the higher $\hbar\Omega$ excitations contribute less while the lower $0\hbar\Omega$ configurations grow in importance.

Furthermore, the $S = 0$ part of all three NCSM eigenstates for ^{12}C is almost entirely projected (95%) onto only six $S = 0$ symplectic irreps included in Table I, with as much as 90% of the spin-zero NCSM states accounted solely by the leading (0 4). The $S = 1$ part is also remarkably well described by merely two Sp$(3,\mathbb{R})$ irreps. Similar results are observed for the ground state of ^{16}O. The outcome reveals an important property for the symplectic dynamical symmetry reflected within the spin projections of the converged NCSM states, namely, their Sp$(3,\mathbb{R})$ symmetry and hence the geometry of the nucleon system is independent of the $\hbar\Omega$ oscillator strength (Fig. 2). The symplectic symmetry is equally present in the spin parts of the NCSM wavefunctions for ^{12}C as well as ^{16}O regardless of whether the bare or the effective interactions are used. This suggests that the Lee-Suzuki transformation, which effectively compensates for the finite space truncation by renormalization of the bare interaction, does not affect the Sp$(3,\mathbb{R})$ symmetry structure of the spatial wavefunctions. Hence, the symplectic structure detected in the present analysis for $6\hbar\Omega$ model space is what would emerge in NSCM evaluations with a sufficiently large model space and bare interaction.

As N_{max} is increased the dimension of the $J = 0, 2,$ and 4 symplectic space built on the 0p-0h Sp$(3,\mathbb{R})$ ir-

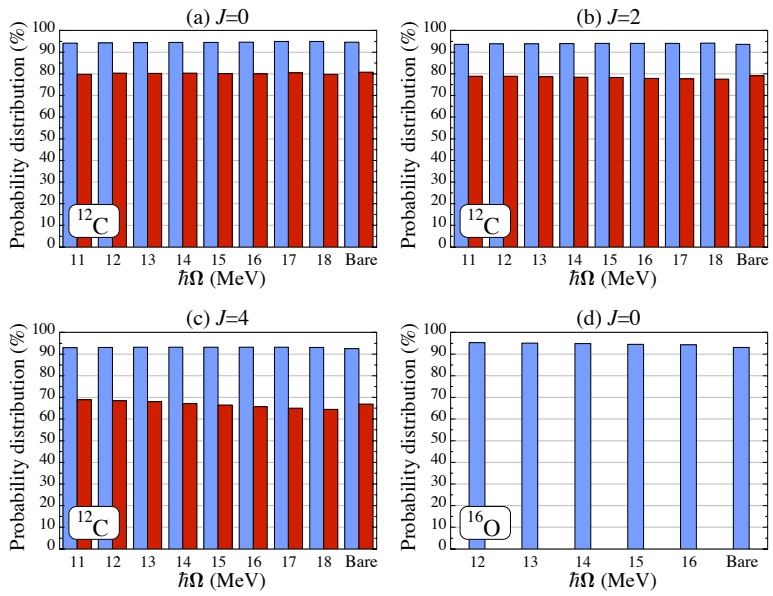

FIG. 2: Projection of the $S = 0$ (blue, left) [and $S = 1$ (red, right)] $\mathrm{Sp}(3,\mathbb{R})$ irreps onto the corresponding significant spin components of the NSCM wavefunctions for (a) 0^+_{gs}, (b) 2^+_1, and (c) 4^+_1 in ^{12}C and (d) 0^+_{gs} in ^{16}O, for effective interaction for different $\hbar\Omega$ oscillator strengths and bare interaction. (We present only the most significant spin values that account for more than 90% of a NCSM wavefunction).

reps grows very slowly compared to the NCSM space dimension (Fig. 3), which remains a small fraction of the NCSM basis space even when the most dominant $2\hbar\Omega$ 2p-2h $\mathrm{Sp}(3,\mathbb{R})$ irreps are included [it is 1.29% for the $2\hbar\Omega$ model space, $8.7 \times 10^{-2}\%$ ($4\hbar\Omega$), $8.5 \times 10^{-3}\%$ ($6\hbar\Omega$), $9.9 \times 10^{-4}\%$ ($8\hbar\Omega$), $1.3 \times 10^{-4}\%$ ($10\hbar\Omega$), and $1.6 \times 10^{-5}\%$ ($12\hbar\Omega$)]. The space reduction is even more dramatic in the case of ^{16}O for the most dominant 0p-0h and $2\hbar\Omega$ 2p-2h configurations [0.88% for the $2\hbar\Omega$ model space, $1.2\times10^{-2}\%$ ($4\hbar\Omega$), $4.4\times10^{-4}\%$ ($6\hbar\Omega$), $2.6\times10^{-5}\%$ ($8\hbar\Omega$), $2.1 \times 10^{-6}\%$ ($10\hbar\Omega$), and $2.1 \times 10^{-7}\%$ ($12\hbar\Omega$)]. This means that a space spanned by a set of symplectic basis states may be computationally manageable even when high-$\hbar\Omega$ configurations are included. It is important to note that $2\hbar\Omega$ 2p-2h (2 particles raised by one shell each) and higher rank np-nh excitations and allowed multiples thereof can be included by building them into an expanded set of lowest-weight $\mathrm{Sp}(3,\mathbb{R})$ starting state configurations. The same "build-up" logic, (2), holds because by construction these additional starting state configurations are also required to be lowest-weight $\mathrm{Sp}(3,\mathbb{R})$ states. Note that if one were to include all possible lowest-weight np-nh starting state configurations ($n \leq N_{max}$), and allowed multiples thereof, one would span the entire NCSM space.

Examination of the role of the model space truncation specified by $N_{max}\hbar\Omega$ reveals that the general features of all outcomes are retained as the space is expanded from $2\hbar\Omega$ to $6\hbar\Omega$ (see, e.g., Fig. 4 for 0^+_{gs} in ^{12}C). Specifically, the same three $\mathrm{Sp}(3,\mathbb{R})$ irreps in ^{12}C, $(0\ 4)S = 0$ and the two $(1\ 2)S = 1$, dominate for all N_{max} values with the large overlaps of the NCSM eigenstates with the leading symplectic irreps preserved, albeit distributed outward across higher $\hbar\Omega$ excitations as the number of active shells increases. In this regard, it may be interesting to understand the importance of the latter beyond the $6\hbar\Omega$ model space and their role in shaping other low-lying states in ^{12}C and ^{16}O such as the second 0^+. This task, albeit challenging, is feasible for the no-core shell model with symplectic $\mathrm{Sp}(3,\mathbb{R})$ extension and will be part of a follow-on study.

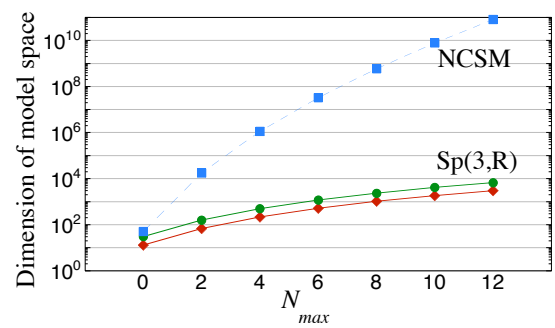

FIG. 3: Dimension of the NCSM (blue squares) and $J = 0, 2$, and 4 $\mathrm{Sp}(3,\mathbb{R})$ (red diamonds for the 3 most significant 0p-0h irrep case and green circles for when all 13 0p-0h irreps are included) model spaces as a function of maximum allowed $\hbar\Omega$ excitations, N_{max}, for ^{12}C.

The 0^+_{gs} and 2^+_1 states in ^{12}C, constructed in terms of the three $\mathrm{Sp}(3,\mathbb{R})$ irreps with probability amplitudes defined by the overlaps with the NCSM wavefunctions for $N_{max} = 6$ case, were also used to determine $B(E2\ :\ 2^+_1 \rightarrow 0^+_{gs})$ transition rates. The latter,

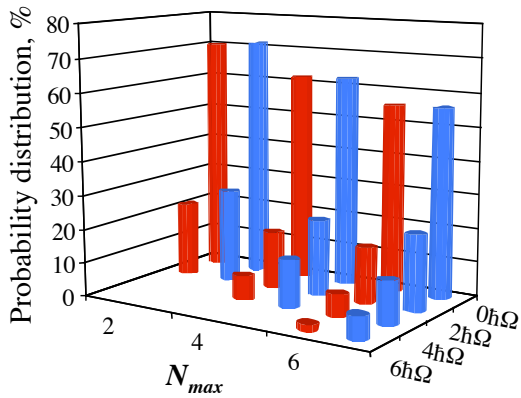

FIG. 4: NCSM (blue, right) and Sp(3, ℝ) (red, left) probability distribution over $0\hbar\Omega$ to N_{max} $\hbar\Omega$ subspaces for the 0^+_{gs} of ^{12}C for different model spaces, N_{max}, with $\hbar\Omega = 15$MeV.

slightly increasing from 101% to 107% of the corresponding NCSM numbers with increasing $\hbar\Omega$, clearly reproduce the NCSM results.

The focus here has been on demonstrating the existence of Sp(3, ℝ) symmetry in NCSM results for ^{12}C, and therefore a possible path forward for extending the NCSM to a Sp-NCSM (symplectic no-core shell model) scheme. This will allow one to account for even higher $\hbar\Omega$ configurations required to realize experimentally measured B(E2) values without an effective charge, and especially highly deformed spatial configurations required to reproduce α-cluster modes in heavier nuclei. In addition, the results can also be interpreted as a further strong confirmation of Elliott's SU(3) model since the projection of the NCSM states onto the $0\hbar\Omega$ space [Fig. 1, blue (right) bars] is a projection of the NCSM results onto the SU(3) shell model. For example, for ^{12}C the $0\hbar\Omega$ SU(3) symmetry ranges from just over 40% of the NCSM 0^+_{gs} for $\hbar\Omega = 11$ MeV to nearly 65% for $\hbar\Omega = 18$ MeV [Fig. 1, blue (left) bars] with 80%-90% of this symmetry governed by the leading (0 4) irrep. These numbers are consistent with what has been shown to be a dominance of the leading SU(3) symmetry for SU(3)-based shell-model studies with realistic interactions in $0\hbar\Omega$ model spaces. It seems the simplest of Elliott's collective states can be regarded as a good first-order approximation in the presence of realistic interactions, whether the latter is restricted to a $0\hbar\Omega$ model space or the richer multi-$\hbar\Omega$ NCSM model spaces.

IV. CONCLUSIONS

In summary, we demonstrated that *ab-initio* NCSM analysis starting with the JISP16 nucleon-nucleon interaction realize a collective nucleon motion with a clear symplectic symmetry structure, which moreover remains unaltered whether the bare or effective interactions for various $\hbar\Omega$ strengths are used. Specifically, NCSM wavefunctions for the lowest 0^+_{gs}, 2^+_1 and 4^+_1 states in ^{12}C and the ground state in ^{16}O project at the 85-90% level onto very few 0p-0h and $2\hbar\Omega$ 2p-2h spurious center-of-mass free symplectic irreps. While the total dimensionality of the latter is only $\approx 10^{-3}$% of the NCSM space, they also closely reproduce the NCSM $B(E2)$ estimates. The results confirm for the first time the validity of the Sp(3, ℝ) approach when realistic interactions are invoked and when the most deformed $2\hbar\Omega$ 2p-2h symplectic irreps, which clearly improved the overlaps, are included. This demonstrates the importance of the Sp(3, ℝ) symmetry, which simply matches the nuclear geometry to the many-nucleon dynamics, as well as reaffirm the value of the simpler SU(3) model upon which it is based.

The results further suggest that a Sp-NCSM extension of the NCSM may be a practical scheme for achieving convergence to measured B(E2) values without the need for introducing an effective charge and even for modeling cluster-like phenomena as these modes can be accommodated within the general framework of the Sp(3, ℝ) model if extended to large model spaces (high N_{max}). In addition, the symplectic extension of the ab-initio NCSM that is "structured" to take advantage of massively parallel computing capability holds promise to allow us to model heavier nuclei including neutron-deficient and $N \approx Z$ nuclei along the nucleosynthesis rp-path and unstable nuclei currently explored in radioactive beam experiments. Heavy nuclei are also feasible due to the natural extension of the Sp(3, ℝ) shell model to chiral-invariant pseudospin.

In short, the NCSM with a modern realistic NN potential supports the development of collective motion in nuclei as can be realized within the framework of the Sp-NCSM and as is apparent in its $0\hbar\Omega$ Elliott model limit. The Sp-NCSM is designed to model real nuclei starting with realistic interactions (such as the ones based on effective field theory), including, especially, momentum dependent forms.

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[17] We use lowercase (capital) letters for algebras (groups).