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Evidence For Departure in Self-Similarity: A New Spectral Ratio Method Using Narrowband Coda Envelopes

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Abstract

This study is motivated by renewed interest within the seismic source community to resolve the long-standing question on energy scaling of earthquakes, specifically, "Do earthquakes scale self-similarly or are large earthquakes dynamically different than small ones?" This question is important from a seismic hazard prediction point of view, as well as for understanding basic rupture dynamics for earthquakes. Estimating the total radiated energy (E_R) from earthquakes requires significant broadband corrections for path and site effects. Moreover, source radiation pattern and directivity corrections can be equally significant and also must be accounted for. Regional studies have used a number of different methods, each with their own advantages and disadvantages. These methods include: integration of squared shear wave moment-rate spectra, direct integration of broadband velocity-squared waveforms, empirical Green's function deconvolution, and spectral ratio techniques. The later two approaches have gained popularity because adjacent or co-located events recorded at common stations have shared path and site effects, which therefore cancel. In spite of this, a number of such studies find very large amplitude variance across a network of stations. In this paper we test the extent to which narrowband coda envelopes can improve upon the traditional spectral ratio using direct phases, allowing a better comparison with theoretical models to investigate similarity. The motivation for using the coda is its stability relative to direct waves and its unique property of spatially homogenizing its energy. The local and regional coda is virtually insensitive to lateral crustal heterogeneity and source radiation pattern, and the use of the coda might allow for more stable amplitude ratios to better constrain source differences between event pairs. We first compared amplitude ratio performance between local and near-regional *S* and coda waves in the San Francisco Bay region for moderate-sized events, then applied the coda spectral ratio method to the 1999 Hector Mine mainshock and some of its larger aftershocks. We find: (1) Average amplitude ratio standard deviations using coda are ~0.05 to 0.12, roughly a factor of 3 smaller than direct *S*-waves for $0.2 < f < 15.0$ Hz; (2) Coda spectral ratios for the M_w 7.1 Hector Mine earthquake and its aftershocks show a clear departure from self-similarity, consistent with other studies using the same datasets; (3) Event-pairs (Green's function and target events) can be separated by as much as ~25 km for coda amplitudes without any appreciable degradation, in sharp contrast to direct waves.

Introduction

Over the past decade, there has been renewed interest in apparent stress scaling for earthquakes [e.g., 2005 AGU Chapman Conference in Portland, Maine and associated AGU Monograph] and despite the plethora of recent studies using local, regional, and teleseismic waveforms, no clear consensus has been reached. The answer to this question is fundamental to our understanding of the earthquake process and addresses whether or not small earthquakes can be used to predict the source effects of larger, more damaging earthquakes. The main observational impediment has been properly accounting for the broadband attenuation, which includes both the path and site response for events that span a wide magnitude range. In addition to the double-couple earthquake source, even small events may suffer the effects of directivity [*D. Dreger*, pers. comm., 2007]. All of these effects make the isolation of the moment-rate spectrum difficult and usually fraught with large errors [e.g., *Prieto et al.,* 2006]. In an effort to eliminate the effects of path and site, a number of studies have used empirical Green's function deconvolution or spectral ratios [e.g., *Hough*, 1997; *Abercrombie and Rice*, 2005; *Venkataraman et al.,* 2002; *Boatwright et al*, 2002; *Izutani and Kanamori,* 2001; *Izutani*, 2005]. In an ideal scenario a co-located event pair would be chosen with identical source mechanism so that path, site, and radiation pattern were common. In practice, the choice of the Green's function event(s) is critical and much effort goes into the selection of event pairs. Unfortunately, even with very careful selection, event pairs are often few in number and their amplitude ratios vary considerably as a function of azimuth and distance [e.g., see Figure 3 of *Izutani*, 2005]. This high variance is likely due to the Green's function event not being exactly co-located and/or of the same source mechanism. Recently, a number of studies have focused on micro-earthquake pairs, or so-called repeating earthquakes [e.g., *Imanishi et al.,* 2004]. Even for these small repeating events, there is no guarantee that they will have the same source-time functions or rupture velocity. Moreover, the mere fact that these are repeating events nucleating from the same patch of fault suggests that these events might be unusual, and not necessarily indicative of rupture dynamics of larger, damaging earthquakes. Nonetheless, the idea of using event pairs is attractive because of its simplicity since path and site corrections are unnecessary. If the variance could be reduced, then an amplitude ratio at a given station would simply reflect the relative source differences and source models could be used to fit these ratios.

Application to the San Francisco Bay Area:

For the first part of our amplitude ratio study, we chose 10 stations operated by the Berkeley Digital Seismic Network (BDSN) along with 62 events between 3.5 < M_L < 5.2 spanning ~1 to 220 km epicentral distance (Figure 1). All stations are three-component high-frequency (80 sps) channels and the events are all located to within 2 km in absolute location by the Northern California Data Center (NCDC) in conjunction with the University of California, Berkeley Seismological Laboratory (BSL).

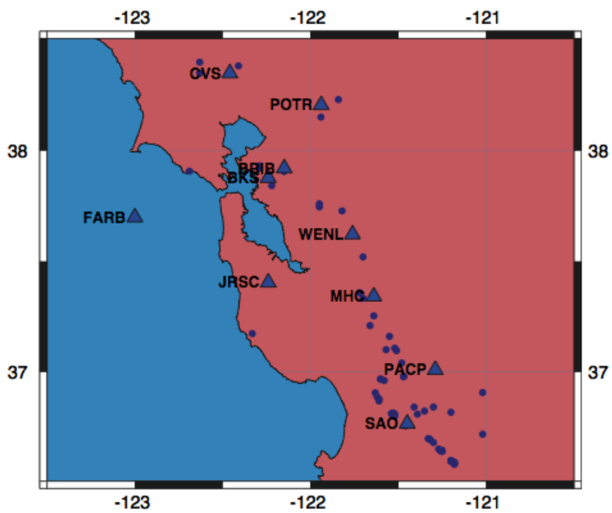

Figure 1. Map of 62 earthquakes and the 10 BDSN stations used in this study.

Each three component waveform was corrected for instrument response, then visually inspected and clipped data were not processed. Since the aim of this part of the study was to look at amplitude ratio variability with event separation and azimuth, we selected event pairs without consideration of magnitude. In total, we found 259 possible event pairs that ranged between ~0 and 25 km distance in absolute separation. For both the coda and direct waves we deconvolved the waveforms to velocity, and chose 14 narrow frequency bands ranging between 0.2 and 25.0 Hz. For the coda, we formed averaged narrowband time-domain envelopes using the two horizontal components following the method of *Mayeda et al.* [2003]. For the direct *S*-waves, log RMS amplitudes from the amplitude spectra were obtained for a 5 second window starting at the *S*-wave onset for each of the three components of ground motion. To further screen the data, we required a minimum of 5 stations recording the event pairs per frequency band, and ended up with as many as 184 pairs at 4.0-6.0 Hz and as few as 35 pairs at 15.0-20.0 Hz. Virtually all events have source mechanisms derived from moment-tensor inversion and, as expected, strikes and dips are consistent with the orientation of the regional faults in the greater San Francisco Bay region.

Amplitude Ratios:

Based on prior work on local and regional coda, we hypothesize that amplitude ratios of the same event-pair will be much more stable for coda than for direct *S*-waves. We tested this hypothesis by forming narrowband amplitude ratios for both wave types and compared their standard deviations for many event pairs. In practice, direct wave empirical Green's function studies have limited their data to co-located events with the same source mechanism [e.g., *Mori et al.,* 2003; *Izutani and Kanamori*, 2001; *Venkataraman et al.,* 2002]. This however, severely limits the useable amount of data, and if proven feasible, the coda's stability and minimal move-out will allow inclusion of more events that are separated in distance and not necessarily of the same focal mechanism. This will allow for redundancy and lower variance estimates, a point thoroughly discussed recently by *Prieto et al*. [2006].

For both the coda and direct *S*-waves, we formed amplitude ratios for event pairs by simply subtracting the $\log_{10}$ amplitudes for each station that recorded the event pair. Since the site and path are the same for both events, the ratio should reflect the source differences in the frequency band. Figure 2 shows an example of direct wave amplitude ratios (top) and coda wave ratios (bottom) for an event pair that has the same depth of 7 km, with an epicentral location difference of 15.7 km. In general, for a magnitude 4 event, the measurement window for the coda envelope was ~300 seconds for 0.2-0.3 Hz, and ~100 seconds at 2-3 Hz. We note that for the direct waves, there was no difference if we used horizontal or vertical elements. Both events are roughly the same size and, as expected, the $\log_{10}$ average of the ratios is close to 0, however the direct wave results are significantly more scattered.

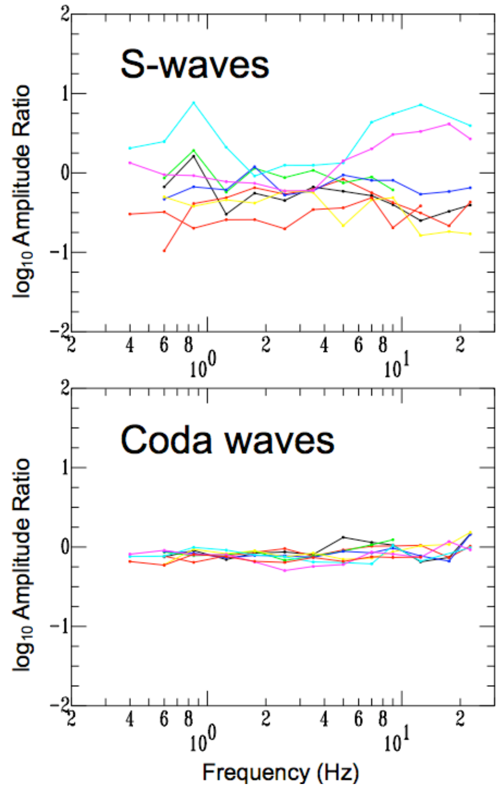

Figure 2. An example of amplitude ratios for a pair of events (M_w 3.63 and 3.74) separated by 15.7 km in epicentral distance and identical depths of 7 km. Direct *S*-wave ratios (top) for 8 color-coded stations show significant scatter over the entire frequency range whereas the coda wave amplitude ratios (bottom) are very stable from station to station.

Using all available ratios, such as the example shown in Figure 2, we plot the amplitude ratio standard deviation as a function of event-pair offset for each frequency band (Figure 3). From the figure, we see that the coda amplitude ratios are roughly a factor of 3 smaller and do not show any appreciable increase with event separation, in sharp contrast to the direct waves. In general, the average standard deviation for the coda amplitude ratios are less than 0.1, roughly a factor of 3 smaller than the direct waves.

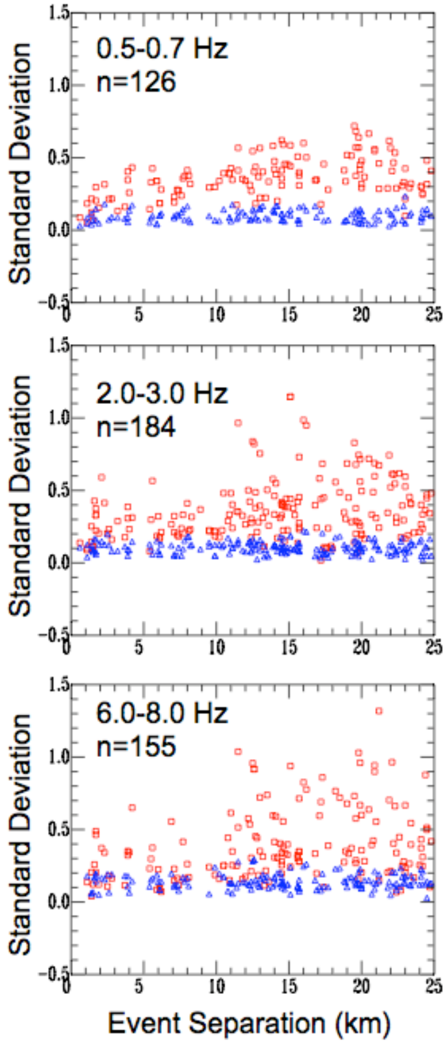

Figure 3. For event ratios that had at least 5 stations recording, we plot the amplitude ratio standard deviation versus event separation for coda (blue triangles) and direct waves (red squares) for three frequency bands, 0.5-0.7 Hz (top), 2.0-3.0 Hz (middle), and 6.0-8.0 Hz (bottom). Note that the coda scatter shows almost no dependence upon distance, in sharp contrast with the direct waves and is roughly a factor of 3 smaller.

This means that the use of the coda will allow for the inclusion of many more events in spectral ratios studies, whereas in direct wave studies, only those events that are virtually co-located are used. Equally important, the coda spectral ratios are significantly less scattered and thus source parameters, such as corner frequency, will be better constrained when we fit the observed data with theoretical source models, such as the commonly used omega-square model [*Aki*, 1967; *Brune*, 1970].

Application to the M_w 7.1 Hector Mine sequence:

Next, we turn our attention to local and regional recordings of the M_w 7.1 Hector Mine mainshock and 6 aftershocks ranging between M_w 3.7 and 5.4. In this case we consider 6 broadband stations ranging between ~60 and 700 km: GSC, PFO, MNV, CMB, TUC, and ELK. All the events have independent regional seismic moment estimates from full waveform inversion by *G. Ichinose* [*pers. comm.,* 2006] using the methodology outlined in *Ichinose et al.,* [2003]. These results are in excellent agreement with those obtained previously by *Mayeda et al.* [2005] using regional coda envelopes. For consistency, we used the regional estimates of M_w to avoid any biases since they were all computed using the same method, velocity model, and station distribution. For the mainshock, we use an M_w of 7.0 following the results of *G. Ichinose* [*pers. comm.,* 2006].

Assuming a simple single corner frequency source model [*Aki*, 1967; *Brune*, 1970], the ratio of the moment-rate functions for two events (1 and *2*) is given by,

$$\frac{\dot{M}_1(\omega)}{\dot{M}_2(\omega)} = \frac{M_{0_1}\left[1+\left(\omega/\omega_{c_2}\right)^2\right]^{p/2}}{M_{0_2}\left[1+\left(\omega/\omega_{c_1}\right)^2\right]^{p/2}} \qquad (1)$$

where M_0 is the seismic moment and ω_c is the angular corner frequency ($2\pi f_c$) and p is the high frequency decay rate. At the low frequency limit the source ratio shown in equation 1 is proportional to the ratio of the seismic moments, whereas at the high frequency limit, equation 1 is asymptotic to $\left[M_{0_1}/M_{0_2}\right]^{\left(1-\frac{p}{3}\right)}$ under self-similarity. If we follow the usual *Brune* [1970] omega-squared model and set p=2, the exponent of the high-frequency ratio becomes 1/3. However, it has been proposed by *Kanamori and Rivera* [2004] that the scaling between moment and corner frequency could take on the form,

$$M_o \sim \omega_c^{-(3+\varepsilon)} \qquad (2)$$

where ε (epsilon) represents the deviation from self-similarity and is usually thought to be a small positive number. For example, *Walter et al.* [2006] and *Mayeda et al.* [2005] found ε to be close to 0.5 for the Hector Mine mainshock and its aftershocks using independent spectral methods. For the current study we use the source spectrum portion of the Magnitude Distance Amplitude Correction (MDAC) methodology of *Walter and Taylor* [2001], which allows for the variation of the corner frequency that does not have to be self-similar. For example,

$$\omega_c = \left(\frac{k\sigma_a}{M_0}\right)^{1/3} \text{ and } \sigma_a = \sigma_a'\left(\frac{M_0}{M_0'}\right)^{\psi} \text{ and } \psi = \frac{\varepsilon}{\varepsilon + 3} \qquad (3)$$

where σ_a is the apparent stress [*Wyss*, 1970], σ_a' and M_0' are the apparent stress and seismic moment of the reference event, and ψ is a scaling parameter. For constant apparent stress, $\psi = 0$ and ε=0, however, *Mayeda and Walter* [1996] found ψ=0.25 for moderate to large earthquakes in the western United States. By using the corner frequency defined in (3) into equation 1, we can apply a grid search to find the parameters that best fit the spectral ratio data.

Using all 6 stations, we formed the average spectral ratio between the mainshock and each of the aftershocks, then grid-searched using equation 1 and 3 assuming that the reference moment corresponded to an M_w 5.0 event and the reference apparent stress was varied between 0.5 and 10 bars. As observed for San Francisco Bay Area events, the coda spectral ratios for Hector Mine events were very stable, with average standard deviations less than 0.1 for all frequencies.

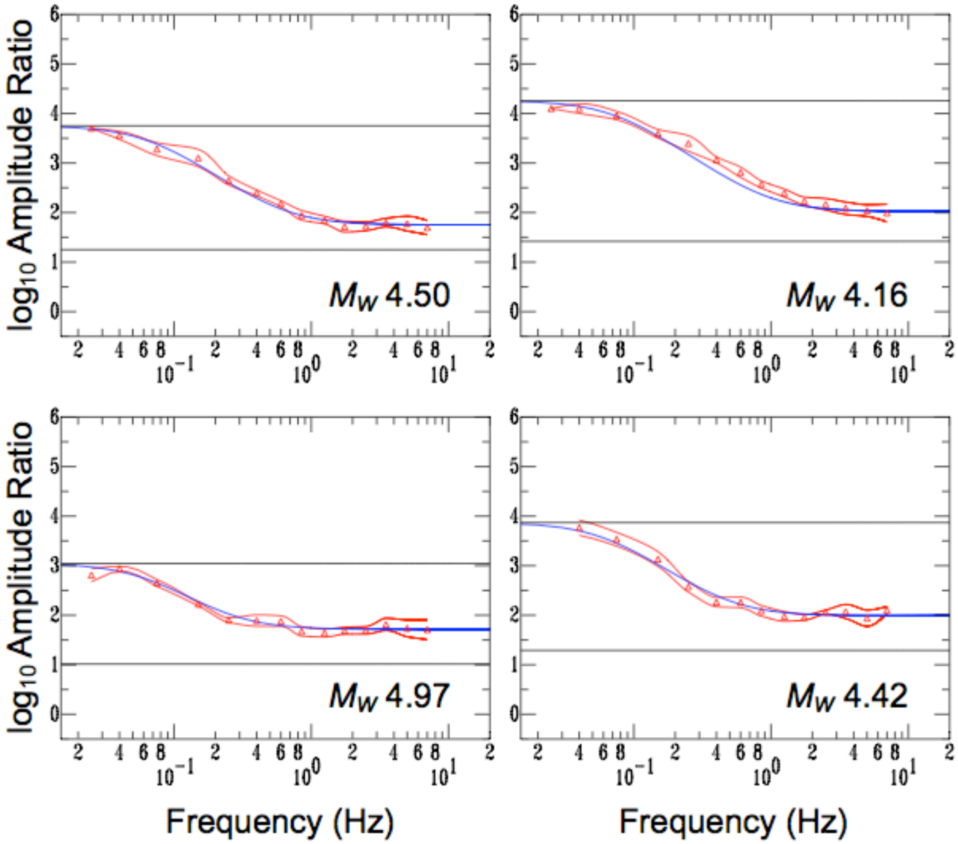

Figure 4. Example spectral ratios for the Hector Mine mainshock relative to 4 aftershocks. In each figure, we show the low and high frequency asymptotes from equation 1 as solid black horizontal lines. The average spectral ratio for each band is shown by red triangles and red lines represent +/- 1 σ and the best fitting MDAC spectral ratio is shown as the blue line. We find that the best fitting MDAC spectra are those with a reference moment corresponding to an M_w 5.0 event with an apparent stress of ~1 bar. The best fitting scaling parameter (ψ) between the mainshock and each of the aftershocks ranges between 0.15 and 0.25, in good agreement with previous studies using independent methods.

Figure 4 shows typical examples of spectral ratios for the 6 stations along with the high and low frequency asymptotes for equation 1. In all cases the high frequency asymptote is significantly above the theoretically predicted value. This is consistent with a break in self-similarity where epsilon is between 0.5 and 1.0, and it is inconsistent with a standard self-similar *Brune* [1970] style omega-squared model. The only way to keep the self-similar assumption and match the data is to have a less than omega-squared spectral decay at high frequencies. The decay parameter p must be greater than 1.5 to keep the energy finite [e.g., *Walter and Brune*, 1993]. If the high-frequency decay value p were close to 1.5 it would be possible to nearly match the spectral ratios shown here. However, given that such shallow falloff is not consistent with most earthquake observations [e.g., *Hough*, 2001] and independent methods [e.g., *Mayeda et al.* 2005; *Walter et al*., 2006] are consistent with non-self similarity for the Hector Mine sequence, that is our preferred interpretation. Finally, we plot the inferred corner frequency against seismic moment in Figure 5, and compare with the teleseismic estimate of the mainshock from *Venkataraman et al.* [2002].

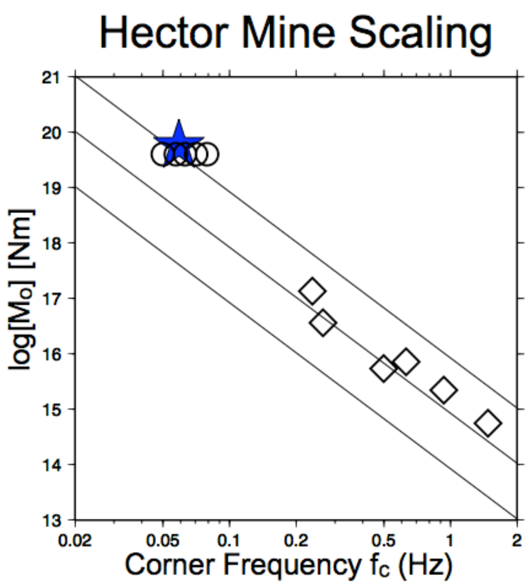

Figure 5. Inferred corner frequency from the MDAC spectral ratio fits are shown for the aftershocks (diamonds) and individual mainshock estimates (circles). We also plot the independent teleseismic estimate of the mainshock (blue star) from *Venkataraman et al.* [2002]. Lines represent f^{-3} scaling which is required for self-similarity.

The mean of the mainshock estimates of corner frequency is 0.063 Hz, in good agreement with the teleseismic estimate of 0.059 Hz from *Venkataraman et al.* [2002]. Though Figure 4 of

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Izutani's [2005] study shows a smooth, non-self-similar trend in moment versus corner frequency ($\propto f^{-3.3}$), our results show more of a step-wise change. For example, within the error of the estimates, the aftershocks appear to cluster around a constant f^{-3} line, whereas the mainshock estimates of corner frequency are clearly not on the same line. However, individual fits between each of the aftershocks and the mainshock results in scaling that ranges between $\sim f^{-3.5}$ and $\sim f^{-4}$.

Discussion and Conclusion:

The averaging nature of coda waves has been shown to provide significantly lower amplitude variance than any traditional direct phase method. In this study, we demonstrate that coda envelope amplitude ratios are an effective tool at isolating the average source effect, with average scatter that is roughly 3 times less than direct phase ratios. Moreover, the minimal move-out for coda allows event pairs to be selected that have significant lateral separation and/or focal mechanism differences, in sharp contrast to direct phase EGF and spectral ratios studies which require co-located events, and ideally, identical focal mechanisms. The stable coda-based source ratios better constrain the source model parameters such as seismic moment, spectral shape, and corner frequency. For the Hector Mine mainshock and selected aftershocks, there are basic conclusions that can be drawn: 1) Inferred corner frequency for the mainshock (f_c=0.063 Hz) is in good agreement with teleseismic estimate (f_c=0.059 Hz) [*Venkataraman et al.* 2002]; 2) The observed source ratios at low frequency are in good agreement with independent moment estimates; 3) At high frequency, none of the spectral ratios is asymptotic to $\left[M_{0_1}/M_{0_i}\right]^{1/3}$, similar to *Izutani's* [2005] finding for the mid Niigata sequence; 4) The scaling parameter, ψ, ranges between 0.15 and 0.25, in agreement with previous studies on the same dataset using different methodologies; 5) The scaling of seismic moment and corner frequency does not follow an f^{-3} dependence, however it is unclear if there is a step-wise change at around M_w 5.0. More work is needed and we plan on applying the methodology to a number of different sequences.

In conclusion, we believe that our spectral ratios are the most stable estimates to date and provide very compelling evidence that a break in self-similar scaling exists between the Hector Mine mainshock and its moderate-sized aftershocks. Though there is evidence to the contrary for smaller events ($<M_w \sim 2$), especially studies that focus on micro earthquakes and repeating earthquakes, there might be a fundamental difference in source dynamics once the event is large.

By virtue of the fact that repeating events are rupturing the same patch of fault, suggests that these events are different than larger, more damaging earthquakes. Finally, the coda's stability and minimal move-out allows for the inclusion of more events that can have significant separation, as well as not having the same focal mechanism. This new methodology will allow for redundancy and lower variance estimates, a point thoroughly discussed recently by *Prieto et al*. [2006].

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