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# Predicting the total charm cross section

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We discuss the energy dependence of the total charm cross section and some of its theoretical uncertainties including the quark mass, scale choice and the parton densities.

## I. INTRODUCTION

Extracting the total charm cross section from data is a non-trivial task. To go from a finite number of measured  $D$  mesons in a particular decay channel to the total  $c\bar{c}$  cross section one must: divide by the branching ratio for that channel; correct for the luminosity,  $\sigma_D = N_D/\mathcal{L}t$ ; extrapolate to full phase space from the finite detector acceptance; divide by two to get the pair cross section from the single  $D$ s; and multiply by a correction factor to account for unmeasured charm hadrons. Early fixed-target data were at rather low  $p_T$ , making the charm quark mass the most relevant scale. At proton and ion colliders, although the RHIC experiments can access the full  $p_T$  range and thus the total cross section, the data reach rather high  $p_T$ ,  $p_T \gg m$ , making  $p_T$  ( $m_T$ ) the most relevant scale. Here we focus on the total cross section calculation where the quark mass is the only relevant scale.

## II. NEXT-TO-LEADING ORDER PQCD

The hadronic cross section in  $pp$  collisions can be written as

$$\sigma_{pp}(S, m^2) = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^p(x_1, \mu_F^2) f_j^p(x_2, \mu_F^2) \hat{\sigma}_{ij}(s, m^2, \mu_F^2, \mu_R^2) \quad (1)$$

where  $x_1$  and  $x_2$  are the fractional momenta carried by the colliding partons and  $f_i^p$  are the proton parton densities. The partonic cross sections [1] include  $q\bar{q}$  and  $gg$  initial states at both  $\mathcal{O}(\alpha_s^2)$  and  $\mathcal{O}(\alpha_s^3)$  as well as  $qg$  and  $\bar{q}g$  interactions at  $\mathcal{O}(\alpha_s^3)$ . At high energies the  $q\bar{q}$  and the  $\mathcal{O}(\alpha_s^2)$   $gg$  contributions are small while the  $\mathcal{O}(\alpha_s^3)$   $gg$  and  $qg$  contributions plateau at finite values. Thus, at collider energies, the total cross sections are primarily dependent on the small  $x$  parton densities and phase space.

The perturbative parameters are the charm quark mass and the value of the strong coupling,  $\alpha_s$ , while the parton densities are a nonperturbative input. We take  $m = 1.5$  GeV as the central value and vary the mass between 1.3 and 1.7 GeV to estimate the mass uncertainties. The perturbative calculation also depends on the unphysical factorization ( $\mu_F$ ) and renormalization ( $\mu_R$ ) scales. The sensitivity of the cross section to their variation can be used to estimate the perturbative uncertainty due to the absence of higher orders. Since Eq. (1) is independent of the kinematics, we take  $\mu_{R,F} = \mu_0 = m$  as the central value and varied the two scales independently within a ‘fiducial’ region defined by  $\mu_{R,F} = \xi_{R,F}\mu_0$  with  $0.5 \leq \xi_{R,F} \leq 2$  and  $0.5 \leq \xi_R/\xi_F \leq 2$ . In practice, we use the following seven sets:  $\{(\xi_R, \xi_F)\} = \{(1,1), (2,2), (0.5,0.5), (1,0.5), (2,1), (0.5,1), (1,2)\}$ . The

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uncertainties from the mass and scale variations are added in quadrature. The envelope containing the resulting curves,

$$\sigma_{\max} = \sigma_{\text{cent}} + \sqrt{(\sigma_{\mu,\max} - \sigma_{\text{cent}})^2 + (\sigma_{m,\max} - \sigma_{\text{cent}})^2} \quad (2)$$

$$\sigma_{\min} = \sigma_{\text{cent}} - \sqrt{(\sigma_{\mu,\min} - \sigma_{\text{cent}})^2 + (\sigma_{m,\min} - \sigma_{\text{cent}})^2} \quad (3)$$

defines the uncertainty.

Since  $m$  is the only perturbative scale, the total cross section calculations are quite sensitive to the low  $x$  and low  $\mu$  behavior of the parton densities. Probing the full fiducial range of the uncertainty band is problematic for charm production since  $\xi_F = 0.5$  is below the minimum scale of the CTEQ6M parton densities,  $\mu_0^{\text{CTEQ6M}} = 1.3$  GeV [2]. Thus, for this scale, backward evolution is required. The behavior of the gluon density at low scales and low  $x$  is atypical, especially for  $x < 10^{-2}$ . Instead of increasing with decreasing  $x$ , for  $x < 0.01$ , the density decreases and, for  $\xi_F = 0.5$ ,  $xg(x)$  can even become zero, as shown on the left-hand side of Fig. 1. This accounts for the high  $\sqrt{S}$  behavior of the lower bound on the uncertainty band. The low  $x$ , low  $\mu_F$  behavior of the gluon density depends strongly on how the group performing the global analysis extrapolates to unmeasured regions. All that is required is minimization of the global  $\chi^2$  and momentum conservation.

The energy dependence of the total cross section, calculated with the CTEQ6M parton densities [2], is shown on the right-hand side of Fig. 1. The central value is indicated by the solid curve while the upper and lower edges of the band are given by the dashed curves. The dotted curve on the left-hand side is calculated with  $\mu_F = \mu_R = 2m$  and  $m = 1.2$  GeV. The uncertainty band broadens as the energy increases. The lower edge of the band grows more slowly with  $\sqrt{S}$  above RHIC energies while the upper edge is compatible with and even above the reported total cross sections at RHIC [3, 4]. The uncertainty band is reduced at higher energies if the GRV98 parton densities [5] are used. However, in this case, only the upper edge of the uncertainty band is in agreement with the data. The NLO cross sections in  $pp$  collisions at  $\sqrt{S} = 200$  GeV and 5.5 TeV are given in Table I for the CTEQ6M and GRV98 parton densities.

$\sqrt{S}$ (GeV)	$\sigma^{\text{NLO, CTEQ6M}}$ ( $\mu\text{b}$ )	$\sigma^{\text{NLO, GRV98}}$ ( $\mu\text{b}$ )
200	$301^{+1000}_{-210}$	$2585^{+13125}_{-2260}$
5500	$178^{+300}_{-122}$	$3562^{+7321}_{-3321}$

TABLE I: Summary of the uncertainty on the charm total cross sections calculated from the NLO partonic total cross sections at RHIC and the LHC.

One obviously important contribution to the uncertainty is the difference in the number of flavors in the two calculations, especially for charm since the fiducial range,  $0.5 \leq \xi_R \leq 2$ , is in a region where  $\alpha_s$  is changing rapidly with  $\mu_R$ . Although increasing the number of light flavors involves more than just changing a parameter in the calculation of  $\alpha_s$ , we can get an estimate of the importance of the value of  $\alpha_s$  to the uncertainty in the total cross section by looking at the dependence of  $\alpha_s$  on the renormalization scale. When calculated with the 5 flavor QCD scale for CTEQ6M,  $\Lambda_5 = 0.226$  MeV, and using a scheme where  $\alpha_s$  is continuous across mass thresholds, we have the values shown in Table II. It is clear, based on these values alone, that the charm uncertainty is larger than that for bottom since  $\alpha_s(\xi_R = 0.5)/\alpha_s(\xi_R = 2) = 2.63$  for charm and 1.56 for bottom. The real difference in coupling strength between the two heavy quarks is even larger since the leading order cross section is proportional to  $\alpha_s^2$  while the next-order contribution is proportional to  $\alpha_s^3$ . We note that the GRV98 set has a smaller value of  $\Lambda_5$ , reducing the value of  $\alpha_s$  in the cross section for this set.

Using  $n_{\text{lf}} + 1$  in the FONLL and NLO calculations of the inclusive distributions reduces the uncertainty. When the FONLL total cross sections are instead calculated with  $n_{\text{lf}}$ , the upper and

$\xi_R$	$n_f = 3, m = 1.5 \text{ GeV}$	$n_f = 4, m = 4.75 \text{ GeV}$
0.5	0.6688	0.2822
1	0.3527	0.2166
2	0.2547	0.1804

TABLE II: The values of  $\alpha_s$  for charm and bottom production at the given values of  $\xi_R = \mu_R/m$  using  $\Lambda_5 = 0.226 \text{ MeV}$ , as in the CTEQ6M PDFs.

lower limits of the charm uncertainty are in agreement with Table I [6]. Thus whether charm is treated as a heavy ( $n_f$ ) or an active ( $n_f + 1$ ) flavor in the calculation turns out to be one of the most important influences on the limits of the charm uncertainty.

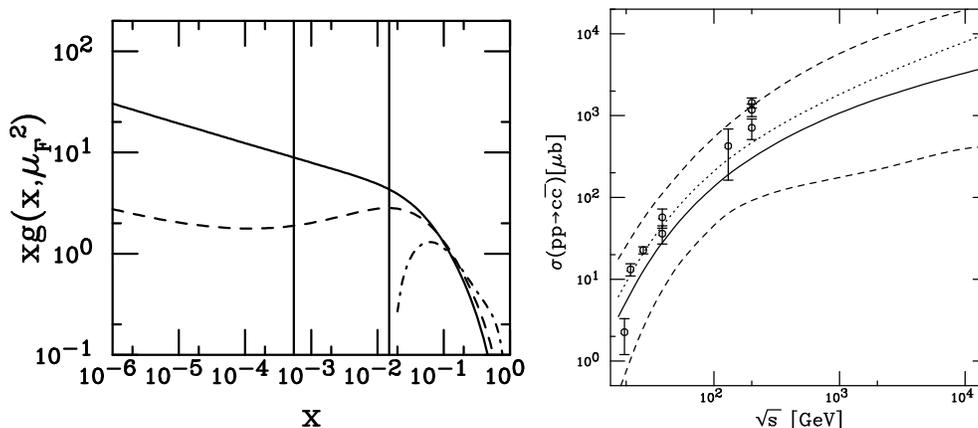


FIG. 1: (Left-hand side) The CTEQ6M parton densities as a function of  $x$  for  $\xi_F = 0.5$  (dot-dashed),  $\xi_F = 1$  (dashed) and  $\xi_F = 2$  (solid) for  $m = 1.5 \text{ GeV}$ . The vertical lines show the value  $x = 2m/\sqrt{s}$  in  $\sqrt{s} = 200 \text{ GeV}$  and  $5.5 \text{ TeV}$   $pp$  collisions at RHIC and the LHC. (Right-hand side) The NLO total charm cross section uncertainty band in  $pp$  interactions calculated with the CTEQ6M PDFs. The central values are given by the solid curves while the dashed curves show the upper and lower limits of the band. The dotted curve is a calculation with  $m = 1.2 \text{ GeV}$ ,  $\mu_F = \mu_R = 2m$ .

### III. CONCLUSIONS

The results are extremely sensitive to the number of flavors, the scale choice and the parton densities, see Ref. [7] for more details. One of the biggest sources of uncertainty at collider energies is the behavior of the gluon density at low  $x$  and low scale, as yet not well determined. Until it is further under control, better limits will be difficult to set. A complete NNLO evaluation of the total cross section may reduce the scale dependence but will still be subject to the same types of uncertainties.

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## References

- [1] P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B **303** (1988) 607.
- [2] J. Pumplin *et al.*, JHEP **0207** (2002) 012 [arXiv:hep-ph/0201195]; D. Stump *et al.*, JHEP **0310** (2003) 046 [arXiv:hep-ph/0303013].
- [3] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **94** (2005) 062301 [arXiv:nucl-ex/0407006].
- [4] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **97** (2006) 252002 [arXiv:hep-ex/0609010]; S. S. Adler *et al.* [PHENIX Collaboration], (2006) arXiv:nucl-ex/0609032.
- [5] M. Glück, E. Reya and A. Vogt, Eur. Phys. J. C **5** (1998) 461.
- [6] M. Cacciari, private communication.
- [7] R. Vogt, Eur. Phys. J. Special Topics **155** (2008) 213 [arXiv:0709.2531 [hep-ph]].