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# The nonlinear Landau damping rate of a driven plasma wave

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In this Letter, we discuss the concept of the nonlinear Landau damping rate, $\nu$, of a driven electron plasma wave, and provide a very simple, practical, analytic formula for $\nu$ which agrees very well with results inferred from Vlasov simulations of stimulated Raman scattering. $\nu$ actually is more complicated an operator than a plain damping rate, and it may only be seen as such because it assumes almost constant values before abruptly dropping to 0. The decrease of $\nu$ to 0 is moreover shown to occur later when the wave amplitude varies in the direction transverse to its propagation.

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Landau damping is a collisionless, linear, phenomenon resulting from the global acceleration of electrons by a plasma wave which occurs, for example, in an initially Maxwellian plasma. The corresponding damping rate, $\nu_L$, was first derived in the famous 1946 paper Ref. [1], using complex contour deformation and analytic continuation. As is well known, if $\nu_L \ll \omega_{pe}$, where $\omega_{pe}$ is the plasma frequency, $\nu_L$ is approximately proportional to the derivative, $f_0'(v_\phi)$, of the unperturbed distribution function at the wave phase velocity. This is because Landau damping is a resonant process, predominantly due to the electrons such that $|v_0 - v_\phi| \lesssim \nu_L/k$, where $k$ is the wave number and $v_0$ is the unperturbed electron velocity.

A nonlinear counterpart of $\nu_L$ was first calculated by O'Neil in Ref. [2], who considered an electron plasma wave (EPW) of constant and uniform amplitude, $E_0$, which grew infinitely quickly in an initially Maxwellian plasma. When $\omega_B \gg \nu_L$, where $\omega_B = \sqrt{ekE_0/m}$, $-e$ being the electron charge and $m$ its mass, most of the electrons which contributed to $\nu_L$ in the linear regime are now trapped and oscillate in the wave trough. Within one oscillation period, a trapped electron neither gains nor loses momentum in the wave frame, so that the mechanism which gave rise to Landau damping vanishes. Actually, O'Neil showed that the EPW non collisional damping rate oscillated with time, $t$, and was almost 0 whenever $\omega_B t \gtrsim 30$.

A countless number of papers, addressing both the linear and nonlinear regimes, have been written since these two seminal works were published. In the linear regime, many articles discussed the physical origin of Landau damping and tried to provide a simpler derivation of $\nu_L$ without resorting to contour deformation (see Ref. [3] and references therein). In this Letter, we provide such a derivation which, we believe, is quite simple. In the nonlinear regime, several papers recently discussed the very work of O'Neil, eventually leading to its experimental check (see Ref. [4] and references therein). Although the situation considered by O'Neil is physical and could be reproduced experimentally, it is not the most general one since a plasma wave amplitude usually depends on both space and time, even when the wave induces nonlinear electron motion. Yet, despite the importance of the subject and the number of papers devoted to it, we are not aware of any simple analytic expression, supported by numerical simulations, for the nonlinear non collisional damping rate of an EPW whose amplitude is neither uniform nor constant. This is what we provide in this Letter. Usually, an EPW either results from a plasma instability or an external drive. However, only in the latter case may the EPW grow in an initially Maxwellian plasma, and may global electron acceleration, at the origin of Landau damping, occur. This is the case we shall consider in this Letter. Moreover, recent numerical [5, 6] and experimental [7] papers on stimulated Raman scattering (SRS) reported reflectivities far above what could be inferred from linear theory, with direct implication to inertial confinement fusion. This so-called "kinetic enahncement" was attributed to the nonlinear reduction of the Landau damping rate, although no theory, nor analytic formula, was available to support this assumption. The present Letter fills this gap.

Before proceeding, it is necessary to clarify what one means by the "nonlinear Landau *damping* rate" of a wave which, since it is driven, grows. Actually, the driven EPW accelerates electrons exactly the same way as if it were freely propagating, which hampers its growth, and one would like to account for this through an effective damping rate that could be used in an envelope equation. More precisely, when the EPW electric field is $\vec{E}_{EPW} = (E_p e^{i\varphi} + c.c)\hat{x}$, with $|E_p^{-1}\partial_x E_p| \ll k \equiv \partial_x\varphi$, and $|E_p^{-1}\partial_t E_p| \ll \omega \equiv -\partial_t\varphi$, and similarly the driving electric field is $\vec{E}_{drive} = (E_d e^{i(\varphi+\delta\varphi)} + c.c)\hat{x}$ with$|E_d^{-1}\partial_x E_d| \ll k$, $|E_d^{-1}\partial_t E_d| \ll \omega$, and $\delta\varphi \ll \varphi$, one would like to write the follwing envelope equation for the EPW amplitude,

$$\partial_t E_p + v_g\partial_x E_p + \nu E_p = E_d\cos(\delta\varphi)/\partial_\omega\chi^r_{\text{env}} \qquad (1)$$

where $\nu$ is called the (nonlinear) Landau damping rate of the driven plasma wave. Actually, the nonlinear envelope equation of an EPW has already been derived in Ref. [8], and is, when $\mathrm{Re}(\chi) \approx -1$ and $|\mathrm{Im}(\chi)| \ll 1$,

$$\mathrm{Im}(\chi)E_p - k^{-1}\partial_x E_p = E_d \cos(\delta\varphi) \tag{2}$$

where $\chi$ is the electron susceptibility, $\chi \equiv i\rho_0/(\varepsilon_0 k E_0)$, where $\rho_0$ and $E_0$ are, respectively, the amplitudes of the charge density and of the total longitudinal electric field (including the plasma wave and the drive). In this Letter, we derive a theoretical expression for $\mathrm{Im}(\chi)$ showing that Eq. (2) can indeed be cast in the form of Eq. (1), and provide explicit formulas for all the coefficients of this equation. The accuracy of our theoretical estimate for $\mathrm{Im}(\chi)$ can be appreciated in Fig. 1(a), while the nonlinear variations of the coefficients of Eq. (1) are illustrated in Figs. 1(b)-(d). In particular, one can see that $\nu$ remains approximately constant before abruptly dropping to 0. This is very different from the oscillating result found by O'Neil because, in this Letter, we consider slowly varying waves inducing a nearly adiabatic electron motion. As a consequence, electrons with the same initial velocity are all trapped nearly simultaneously, while in the situation considered by O'Neil, electrons with the same initial velocity are not all trapped by the wave, depending on their initial position. Hence, $\nu$ is less efficiently reduced to 0 in the O'Neil situation than in ours, and we find $\nu \approx 0$ whenever $\int \omega_B dt \gtrsim 6$, instead of $\omega_B t \gtrsim 30$ as found by O'Neil.

Let us now derive the envelope equation for the plasma wave. We first assume that the total longitudinal field amplitude, $E_0$, is uniform, while its time variation is such that $|\Gamma| \equiv |E_0^{-1} d_t E_0| \ll \omega_{pe}$. For a freely propagating plasma wave, when $\nu_L \ll \omega_{pe}$, the Landau damping rate may be estimated using the expansion $\mathrm{Im}[\chi(\omega - i\nu_L)] \approx \mathrm{Im}[\chi(\omega - i0)] - \nu_L \partial_\omega \mathrm{Re}(\chi)$. Then $\mathrm{Im}(\chi) = 0$ yields $\nu_L = \mathrm{Im}[\chi(\omega - i0)]/\partial_\omega \mathrm{Re}(\chi)$. Here, we would like to make a similar expansion to get,

$$\mathrm{Im}[\chi(\omega + i\Gamma)] \approx \mathrm{Im}[\chi(\omega + i0)] + \Gamma \partial_\omega \chi^r_{\mathrm{env}} \tag{3}$$

When $E_p \gg E_d$, which is typically the case for SRS (see Ref. [9] for a detailed discussion), then $\Gamma \equiv E_0^{-1} d_t E_0 \approx E_p^{-1} d_t E_p$. Hence, plugging Eq. (3) into Eq. (2), would yield the envelope equation (1) with $\nu = \mathrm{Im}[\chi(\omega + i0)]/\partial_\omega \chi^r_{\mathrm{env}}$. In order to calculate $\mathrm{Im}(\chi)$ we use the expression, $\chi = -i(k\lambda_D)^{-2}\langle e^{-i\varphi}\rangle/\Phi$, derived in Ref. [8], where $\lambda_D$ is the Debye length, $\Phi = eE_0/kT_e$, $T_e$ being the electron temperature, and

$$\langle e^{-i\varphi}\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{-\infty}^{+\infty} f(\varphi, v, t) e^{-i\varphi} d\varphi dv \tag{4}$$

where $f$ is the electron distribution function, and $\varphi$ may be seen as a dynamical variable such that, for each electron, $d\varphi/dt = kv - \omega$, where $v$ is the electron velocity and $\omega$ the EPW frequency. Let us first give an estimate of $\langle e^{-i\varphi}\rangle$ obtained through the means of a first order perturbation analysis. This amounts to using the following expansion $\varphi(x,t) = \varphi_0 + (v_0 - v_\phi)\tau + \delta\varphi$, where $\tau = k\lambda_D \omega_{pe} t$, velocities are normalized to the thermal one, $v_{th} \equiv \lambda_D \omega_{pe}$, and, at 0-order in the time variations of $v_\phi$,

$$\delta\varphi = -\int_0^\tau \int_0^u \Phi(\xi) e^{i(\varphi_0 + w\xi)} d\xi du \tag{5}$$

where we have denoted $w \equiv v_0 - v_\phi$. As shown in Ref. [8], deeply trapped electrons do not contribute to $\mathrm{Im}(\chi)$, and one may therefore calculate $\mathrm{Im}(\chi)$ by only accounting for electrons with initial velocity $|v_0 - v_\phi| < V_{\mathrm{lim}}$, where $V_{\mathrm{lim}} = [4\omega_B/(\pi k v_{th})]\left[1 - 3/\int_0^t \omega_B(u)du\right]$, and the latter expression is supposed to be 0 if negative. Then, using the expansion, $\langle e^{-i\varphi}\rangle \approx \langle -i\delta\varphi e^{-i(\varphi_0 + w\tau)}\rangle$, we find

$$\langle e^{-i\varphi}\rangle = i\int_{|w|\geq V_{\mathrm{lim}}}\int_0^\tau\int_0^u \Phi(\xi) e^{iw(\xi - \tau)} f_0(w + v_\phi) du d\xi dw \tag{6}$$

where $f_0$ is the electron distribution function in the limit $\Phi \to 0$. When $\Phi$ monotonically increases as a function of time, $f_0$ is the electron distribution function at $t = 0$. It is therefore the unperturbed distribution function, usually a Maxwellian. However, when $\Phi$ decreases to 0 after reaching high enough a value to induce nonlinear electron motion, perturbation analysis only makes sense if one uses for $f_0$ the electron distribution function in the limit $t \to +\infty$, and integrates the electron motion from $t = +\infty$ by taking advantage of the time-reversal invariance of the dynamics. Then, as explained in Ref. [8], $f_0$ is nearly symmetric with respect to $v_\phi$ in the interval $|v_0 - v_\phi| \leq \max(V_{\mathrm{lim}})$ (an example of such a symmetric distribution function for a decaying wave amplitude may be seen in Fig. 4 of Ref. [10]). This implies that once trapped electrons never contribute to $\mathrm{Im}(\chi)$ again, even after being detrapped. Eq. (6) may therefore be simplified by using for $f_0$ the unperturbed distribution function and by replacing $V_{\mathrm{lim}}$ by $\max(V_{\mathrm{lim}})$. Such a simplfication will be implicitly used throughout the remainder of this Letter. In order to derive an expression similar to Eq. (3) for $\mathrm{Im}(\chi)$, we now use the decomposition $\langle e^{-i\varphi}\rangle \equiv I_1 + I_2$ with

$$I_1 = f_0'(v_\phi)\int_0^\tau\int_0^u \Phi(\xi)\int_{|w|\geq V_{\mathrm{lim}}} iwe^{iw(\xi-\tau)} dw d\xi du \tag{7}$$

$$\begin{aligned} I_2 = {} & i\int_{|w|\geq V_{\mathrm{lim}}}\int_0^\tau\int_0^u \Phi(\xi) e^{iw(\xi-\tau)} \times \\ & \times [f_0(w + v_\phi) - wf_0'(v_\phi)] dw d\xi du \end{aligned} \tag{8}$$

Provided that $(d\Phi/d\tau)_{\tau=0}$ may be neglected, integrating Eq. (8) by parts yields, at first order in the time variations of $\Phi$,

$$\mathrm{Re}(I_2) \approx 2\frac{d\Phi}{d\tau}\int_{|w|\geq V_{\mathrm{lim}}} \frac{f_0(w + v_\phi) - wf_0'(v_\phi)}{w^3} dw \tag{9}$$

$$\equiv -(k\lambda_D)^2 (d\Phi/dt)(\partial\chi_1^r/\partial\omega) \tag{10}$$

where the integral in Eq. (9) has to be taken in the sense of Cauchy's principal part when $V_{\rm lim}=0$.

Setting $V_{\rm lim}=0$ in Eqs. (7) and (9) just yields the linear value of $\text{Im}(\chi)$. Then $\chi_1^r$ just is the adiabatic approximation of the linear value of $\text{Re}(\chi)$. As for $I_1$, since $\int_{-\infty}^{+\infty} iwe^{iw(\xi-\tau)}dw = 2\pi\partial_\xi\delta(\xi-\tau)$, where $\delta$ is the Dirac distribution, one easily finds $I_1=\pi f_0'(v_\phi)\Phi(\tau)$. Hence, in the linear limit, $\text{Im}(\chi)=-\pi(k\lambda_D)^{-2}f_0'(v_\phi)+\Gamma\partial_\omega\chi_1^r$, which has the same form as Eq. (3). Therefore, Eq. (2) may indeed be cast in the same form as Eq. (1), with $\chi^r_{\rm env}=\chi_1^r$ and $\nu=-\pi(k\lambda_D)^{-2}f_0'(v_\phi)/\partial_\omega\chi_1^r$. The preceding linear value of $\nu$ is just the Landau damping rate, $\nu_L$, in the limit $\nu_L\ll\omega_{pe}$. Hence, our linear calculation is one derivation of the Landau damping rate which does not resort on complex contour deformation.

In the nonlinear regime, and when $V_{\rm lim}^{-1}$ is much smaller than the typical timescale of variation of $\Phi$, $\tau_\phi$, integrating Eq. (7) by parts yields,

$$\text{Re}(I_1)=f_0'(v_\phi)[4V_{\rm lim}^{-1}d\Phi/d\tau+O(V_{\rm lim}^{-3}d^3\Phi/d\tau^3)] \quad (11)$$

Hence, when $V_{\rm lim}\gg\tau_\phi^{-1}$ which, for a slowly varying wave is typically the case when $\int\omega_B dt\gg1$, $\text{Re}(I_1)$ is nearly proportional to $d\Phi/d\tau$. As a consequence, $\text{Im}(\chi)$ is nearly proportional to $\Gamma$ and, in Eq. (1), $\nu\approx0$. The decrease of $\nu$ towards 0 has the same physical origin as in the situation considered by O'Neil : nearly resonant electrons, which predominantly contributed to Landau damping, are the first to be trapped as the EPW grows, and no longer contribute to $\nu$ while oscillating within the wave trough. When $\int\omega_B dt\gg1$, $\text{Im}(\chi)$ may therefore be approximated by $\text{Im}(\chi)\approx\Gamma\partial_\omega\chi^r_{\rm eff}$ where $\chi^r_{\rm eff}$ is the real part of some effective susceptibility obtained by removing the contribution of the deeply trapped electrons. How to calculate $\partial_\omega\chi^r_{\rm eff}$ very accurately, without resorting to perturbation theory, is explained in Ref. [8]. Note that the $I_1$ term which, in the linear limit, yields the damping rate $\nu$, renormalizes the term $\partial_\omega\chi^r_{\rm env}$ in Eq. (1) when $\int\omega_B dt\gg1$. In the strong damping limit, when $\nu_L\gg\Gamma$, $\partial_\omega\chi^r_{\rm env}$ may then increase by more than one order of magnitude, as illustrated in Fig. 1 (c). As for for the perturbative estimate $\text{Im}(\chi_{\rm per})$ of $\text{Im}(\chi)$, yielding Eqs. (7) and (9), it is valid provided that $\int\omega_B dt\lesssim1$. Hence, to get an expression of $\text{Im}(\chi)$ whatever the wave amplitude, we just need to connect values of $\text{Im}(\chi)$ obtained when $I_{\omega_B}\equiv\int\omega_B dt\lesssim1$, and when $I_{\omega_B}\gg1$, the following way,

$$\text{Im}(\chi)\approx\text{Im}(\chi_{\rm per})\left[1-Y\left(I_{\omega_B}\right)\right]+\Gamma\partial_\omega\chi^r_{\rm eff}Y\left(I_{\omega_B}\right) \quad (12)$$

where $Y$ is a function rising from 0 to 1 as $I_{\omega_B}$ increases. From the preceding equation, we then derive

$$\chi^r_{\rm env} = (1-Y)\times\chi_1^r+Y\times\chi^r_{\rm eff} \quad (13)$$
$$\nu = Y\times I_1/\partial_\omega\chi^r_{\rm env} \quad (14)$$

To complete our calculation, we now need to provide a practical formula for $I_1$, simpler than Eq. (7), and to specify a choice for the function $Y$ in Eq. (12). Let us first consider the case when $\Gamma$ is a strictly positive constant, which is a relevant limit since our theory is only for slowly varying wave amplitudes. As shown in Ref. [8], when the EPW grows exponentially with time, $\Gamma\partial_\omega\chi^r_{\rm env}$ is very close to $\text{Im}(\chi)$ whenever $I_{\omega_B}>6$ and quickly diverges away from it when $I_{\omega_B}<6$. Hence, $Y$ must be such that $\text{Im}(\chi)$ defined by Eq. (12) quickly changes from $\text{Im}(\chi_{\rm per})$ to $\Gamma\partial_\omega\chi^r_{\rm env}$ when $I_{\omega_B}$ increases from a little less than 6 to a little more than 6. This is the case if we choose $Y(x)=\tanh^5[(e^{x/6}-1)^3]$. Moreover, as shown in Ref. [11], such a choice for $Y$ yields an excellent agreement between the theoretical values of $\text{Im}(\chi)$ and those inferred from test particles simulations. This is therefore the choice we make in the general case. As for $I_1$, when $\Gamma$ is a strictly positive constant, one easily finds,

$$\frac{\text{Re}(I_1)}{f_0'(v_\phi)}=\Phi(\tau)\left[\pi-2\tan^{-1}\left(\frac{V_{\rm lim}}{\gamma}\right)+\frac{2\gamma V_{\rm lim}}{\gamma^2+V_{\rm lim}^2}\right] \quad (15)$$

where $\gamma\equiv\Gamma/k\lambda_D\omega_{pe}$. In order to generalize the preceding formula we use the expansion $\text{Re}(I_1)=f_0'(v_\phi)[\pi\Phi(\tau)+\delta I_1]$ and find, from Eq. (15), $\delta I_1\approx-(4/3)(V_{\rm lim}/\gamma)^3$ when $V_{\rm lim}\ll\gamma$, while when $V_{\rm lim}\ll\tau_\phi^{-1}$ Eq. (7) yields $\delta I_1\approx-4(V_{\rm lim}^3/3)\int_0^\tau\int_0^u\int_0^\xi\Phi(\xi')d\xi'd\xi du$. Hence, while for an exponential growing wave $\gamma\equiv\Phi^{-1}d\Phi/d\tau=\Phi/\int\Phi d\tau$, we find that, when $V_{\rm lim}\ll\tau_\phi^{-1}$, Eq. (15) still holds in the general case provided that $\gamma$ is expressed in terms of the time integral of $\Phi$. When $V_{\rm lim}\gg\gamma$, Eq. (15) yields $\text{Im}(I_1)\approx4\gamma\Phi f_0'(v_\phi)/V_{\rm lim}$, which is the same as Eq. (11) provided that $\gamma=\Phi^{-1}d\Phi/d\tau$. Having clarified the actual meaning of $\gamma$ in Eq. (15), we may generalize this equation by plugging into it

$$\gamma\equiv\frac{\Phi(\tau)-\Phi(\tau-\pi/V_{\rm lim})}{\int_{\tau-\pi/V_{\rm lim}}^{\tau}\Phi(u)du} \quad (16)$$

which has the required properties $\gamma\approx\Phi/\int\Phi d\tau$ when $V_{\rm lim}\ll\tau_\phi^{-1}$, and $\gamma\approx d\Phi/d\tau$ when $V_{\rm lim}\gg\tau_\phi^{-1}$. As shall be seen, Eqs. (15) and (16) provide quite a precise estimate for $\nu$. The accuracy can even be improved by using the high order perturbative results of Ref. [8] instead of Eq. (15). We will not show here the corresponding, huge, formulas, but Fig. 1 illustrates the improvement.

The previous results are easily generalized to account for one dimensional (1-D) space variations of the EPW amplitude. Indeed, by using a Fourier expansion of the charge density then, as shown in Ref. [8], one finds

$$\text{Im}(\chi)=\nu+\Gamma\partial_\omega\chi^r_{\rm env}-\kappa[\partial_k\chi^r_{\rm env}+\text{Re}(\chi)/k] \quad (17)$$

where $\kappa\equiv E_0^{-1}\partial_xE_0\approx E_p^1\partial_xE_p$, and $\nu$ and $\chi^r_{\rm env}$ are still defined by Eqs. (13-16) except that $I_{\omega_B}$, $\gamma$ and $\max(V_{\rm lim})$ need now be evaluated in the wave frame. Plugging Eq. (17) into Eq. (1) we find, provided that $[1+\text{Re}(\chi)]\approx0$, the following expression for the EPW group velocity,

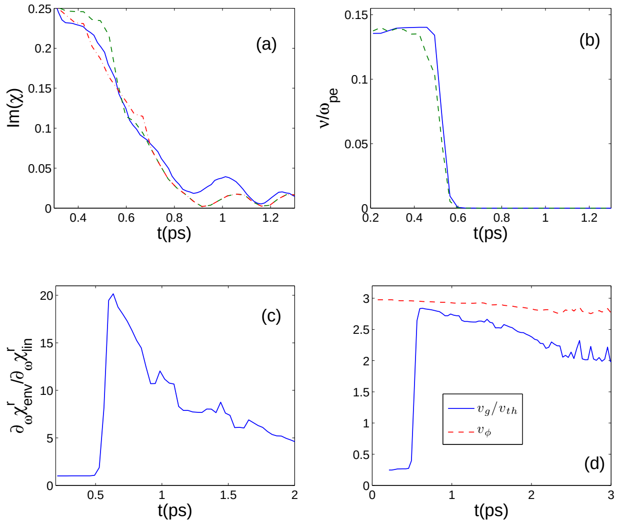


FIG. 1: (Color online) Panel (a), $\mathrm{Im}(\chi)$ calculated numerically (blue solid line) and theoretically using for $\mathrm{Im}(\chi_{\mathrm{per}})$ a $1^{\mathrm{st}}$ order (green dashed line) or an $11^{\mathrm{th}}$ order (red dashed-dotted line) perturbation analysis, panel (b), the nonlinear Landau damping rate normalized to the plasma frequency from a $1^{\mathrm{st}}$ order (blue solid line) or an $11^{\mathrm{th}}$ order (green dashed line) perturbation analysis, panel (c), $\partial_\omega \chi^r_{\mathrm{env}}$ normalized to its linear value and, panel (d), the EPW group velocity (blue solid line) and phase velocity (red dashed line) normalized to the thermal one.

$v_g = -\partial_k \chi^r_{\mathrm{env}}/\partial_\omega \chi^r_{\mathrm{env}} = \omega/k - 2/[k\partial_\omega \chi^r_{\mathrm{env}}]$. It is noteworthy that, since in the nonlinear regime $\chi^r_{\mathrm{env}} \neq \mathrm{Re}(\chi)$, $v_g \neq d\omega/dk$. Actually, since $\partial_\omega \chi^r_{\mathrm{env}}$ may reach values much larger than in the linear limit, the nonlinear value of $v_g$ may get quite close to the EPW phase velocity, as shown in Fig. 1 (d).

We now compare our theoretical calculations against direct 1-D Vlasov simulations of SRS using the Eulerian code ELVIS [6]. In our numerical simulations, which are detailed in Refs. [6, 9], the EPW results from the interaction of a pump laser, entering from vacuum on the left ($x = 0$), and a small-amplitude counterpropagating "seed" light wave injected on the right. Using a Hilbert transform of the fields, one can numerically calculate the ratio $[E_d \cos(\delta\varphi) + k^{-1}\partial_x E_p]/E_p$, which yields a first, numerical, estimate of $\mathrm{Im}(\chi)$. From Vlasov simulations one can also extract the values of all the quantities, such as $I_{\omega_B}, \gamma, \ldots$, which enter our theoretical formula for $\mathrm{Im}(\chi)$. Using these values we calculate a second, theoretical estimate, for $\mathrm{Im}(\chi)$. Both these estimates are compared in Fig. 1(a). The simulation results of Fig. 1 correspond to a plasma with electron temperature, $T_e = 5$keV, and electron density $n = 0.1n_c$, where $n_c$ is the critical density. The total length of the simulation box is $L = 270\lambda_l$, where $\lambda_l = 0.351\mu m$ is the laser wavelength, and the data of Fig. 1 were measured at $x = 154\lambda_l$. The laser intensity is $I_l = 4 \times 10^{15}\mathrm{W/cm}^2$ while the seed intensity is $I_s = 10^{-5} I_l$ and the seed wavelength is $\lambda_s = 0.609\mu m$. As can be seen in Fig. 1 (a), there is a very good agreement between the theoretical and numerical values of $\mathrm{Im}(\chi)$, especially as regards the decrease of $\mathrm{Im}(\chi)$ from its linear value. Clearly, as defined by Eq. (14), $\nu$ is much more complicated an operator than a plain damping rate. However, as shown in Fig. 1 (b), $\nu$ is nearly constant before abruptly dropping to 0, so that it may indeed be seen as a damping rate.

In a 3-D geometry, $\mathrm{Im}(\chi)$ is just the statistical average, over all the transverse velocities $\vec{v}_\perp$, of the expression Eq. (17), where all the quantities which enter the theoretical formulas for $\nu$ and $\chi^r_{\mathrm{env}}$ are evaluated in the frame moving at velocity $(\omega/k)\vec{x} + \vec{v}_\perp$ with respect to the laboratory frame. If the transverse extent of the EPW is much less than its longitudinal one then it is clear that, for the same maximum wave amplitude, $I_{\omega_B}$ assumes smaller values than for a plane wave. As a consequence, linear theory is valid up to larger wave amplitudes in 1-D than in 3-D.

In conclusion, we derived a very precise theoretical estimate of $\mathrm{Im}(\chi)$ for a slowly varying electron plasma wave, that we compared against results obtained from Vlasov simulations of SRS. From the expression of $\mathrm{Im}(\chi)$ we deduced the group velocity and the nonlinear Landau damping rate, $\nu$, of the EPW, and provided a simple, practical analytic formula for $\nu$. Our results, first derived for uniform wave amplitudes are easily generalized to allow for 3-D space variations of the waves.

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