LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

LLNL-JRNL-417614

Source Mechanisms of Mine-Related Seismicity, SavukaMine, South Africa

Jordi Julia, Andrew A. Nyblade, Ray Durrheim, Lindsay Linzer, Rengin Gok, Paul Dirks, William Walter

October 6, 2009

Bulletin of Seismological Society

Disclaimer

This document was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor Lawrence Livermore National Security, LLC, nor any of their employees makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or Lawrence Livermore National Security, LLC. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or Lawrence Livermore National Security, LLC, and shall not be used for advertising or product endorsement purposes.

Source Mechanisms of Mine-Related Seismicity, Savuka Mine, South Africa

by

Jordi Julià*, Andrew A. Nyblade, Ray Durrheim, Lindsay Linzer, Rengin Gök, Paul Dirks, and William Walter

*Corresponding Author: Department of Geosciences, Penn State University, 407 Deike Bldg., University Park, PA 16802, U.S.A., TEL: (814) 863-3419, e-mail: **jjulia@geosc.psu.edu**.

Reviewed manuscript for the Bulletin of the Seismological Society of America.

Abstract

We report full moment tensor solutions for 76 mine tremors with moment magnitudes (Mw) between 0.5 and 2.6 recorded by a network of 20 high-frequency geophones in a deep gold mine in South Africa. Source mechanisms convey important information on how in-mine stresses are relaxed, and understanding the nature of such mechanisms is essential for improving our assessment of rock mass response to mining. Our approach has consisted of minimizing the L2 norm of the difference between observed and predicted P, SV, and SH spectral amplitudes, with visually assigned polarities, to constrain all six independent components of the seismic moment tensor. Our results reveal the largest principal stresses in the mine are compressive, oriented near-vertically, and relaxed through a mix of volumetric closure and normal faulting, consistent with a gravity-driven closure of the mined-out areas. Previous moment tensor studies in deep mines had suggested that the distribution of seismic sources in terms of the volumetric-shear mix was bimodal. A bimodal distribution is compatible with our moment tensor solutions only for moment magnitudes above 2.2. Events in the $0.5 < Mw < 2.2$ moment magnitude range display a continuous distribution of their volumetric-shear mix.

Introduction

An early debate in mining seismology concerned the need for invoking non-deviatoric failure mechanisms to explain the radiation patterns generated by mine-induced seismic events (see e.g. Wong and McGarr [1990], and references therein). This debate was settled in the early 1990s by McGarr [1992a; 1992b], who convincingly demonstrated the necessity of an implosive component to fully explain the radiation patterns of 7 events observed in deep, tabular gold mines in two mining districts in South Africa, and by Feigner and Young [1992], who showed a significant volumetric component was required to explain the radiation patterns of 19 micro-seismic events generated during a "mine-by" experiment at the Underground Research Laboratory (URL) in Manitoba, Canada. In his study of South African source mechanisms McGarr [1992b] noted that the distribution of moment tensor solutions in the deep mines was bimodal: the mine tremors were either pure shear failures, or had a ratio of co-seismic closure to average shear slip around 0.71. McGarr [1992b] also noted that the moment tensor solutions reported by Feigner and Young [1992] had a more continuous distribution of source mechanisms in terms of their volumetric-shear mix.

A number of differences between those two studies could explain the apparent paradox posed by the source-mix distribution of the focal mechanisms. First, the size of the database utilized by Feigner and Young [1992] was significantly larger (33 events) than that utilized by McGarr [1992b] (10 events), and this difference makes the first study more relevant statistically. Second, the mining conditions under which the seismic events were induced were radically different; the deep gold mines described by McGarr [1992b] have a maximum principal stress (in absolute sense) that is negative (i.e. compressive) and near-vertical, which could explain the predominance of implosive sources; the URL in Canada is characterized by a maximum

principal stress that is positive and sub-horizontal [Feigner and Young, 1992], which could explain the existence of tensile sources. Third, the moment magnitude range for the deep-mine events is 1.9 < Mw < 3.3, while the magnitude range for the URL events is –3.3 < Mw < -2.3. The differences could thus be the result of a magnitude-dependent distribution for the source-mix. And fourth, the observations utilized in both studies to infer the moment tensor solutions were different and could have biased the solutions in different ways; McGarr [1992b] utilized select waveform amplitudes in the time domain, while Feigner and Young [1992] utilized spectral amplitudes with polarities attached.

More recently, a moment tensor study of 6 shallow events, with moment magnitudes between 1.6 and 1.8, in a coal mine in Utah revealed a continuous variation of the source-mix [Fletcher and McGarr, 2005]. The moment tensor solutions were obtained from select time-domain amplitudes, as in McGarr [1992b], thus demonstrating that the bimodal distribution reported in McGarr [1992b] did not reflect any bias introduced by the inversion method. The lack of overlap in the magnitude range, the small size of the data set, and the different mining conditions (shallow *vs* deep), however, can still explain the observed differences in the source-mix distributions.

In this study, we report moment tensor solutions for 76 well-recorded seismic events, with moment magnitudes in the 0.5-2.6 range, and recorded in a deep gold mine in South Africa (Savuka Mine, Carletonville mining district). The moment tensor solutions have been obtained by inverting spectral amplitudes with polarity attached and verified in the time domain with synthetic seismograms. Our results demonstrate that the source-mix distribution is continuous under deep mining conditions in the 0.5 < Mw < 2.2 moment magnitude range. At larger

magnitudes our results are less conclusive but still compatible with a bimodal distribution of the volumetric-shear mix, as reported by McGarr [1992b].

Data

The data set utilized in this study has been obtained from a network of 20 seismic stations deployed underground by Integrated Seismic Systems International (ISSI) to monitor mine-related seismic activity at the AnglogoldAshanti Savuka gold mine, South Africa (Fig. 1). The network consists of a combination of three-component sensor types: the G4.5 geophone, with a flat response in velocity between 4 Hz and 2000 Hz and a sensitivity of 28 V/m/s; the G14 geophone, with flat response in velocity between 12 Hz and 2000 Hz and a sensitivity of 80 V/m/s; and the G28 geophone, with a flat response in velocity between 28 Hz and 2000 Hz and a sensitivity of 15 V/m/s. The seismic stations are operated at sampling rates ranging from 2,000 to 10,000 samples per second, with timing synchronized through common GPS time-keeping. Except for one surface sensor, the sensors are deployed at depth along the two gold-bearing horizons being mined: the Ventersdorp Contact Reef (VCR) and the Carbon Leader Reef (CLR). The VCR is mined at depths of around 3000 m, with Ventersdorp Lavas in the hanging wall and a thin layer of shale in the footwall. The CLR is mined at depths of around 3500 m, and is separated from the shallower VCR by a ~500 m thick layer consisting predominantly of quartzite.

Data Selection

A total of 11,224 mine tremors, with local magnitudes between –3.4 and 4.4, were catalogued by ISSI during 2007 through the Savuka Mine, in-mine network. This large catalog

includes events induced by mine activity at Savuka Mine itself, as well as events induced in nearby mines. Good azimuthal coverage is critical for accurately mapping the seismic radiation patterns of the recorded mine tremors, so a selection was made for those events with locations falling near or within the volume covered by the in-mine network. Furthermore, the moment tensor inversion procedure employed in this study assumes the propagating medium is a whole space of constant wave-speed. This is a reasonable assumption, as the seismic waves from the selected events are expected to propagate through the quartzite layer separating the two gold-bearing horizons. Nonetheless, to ensure that the selected seismic phases propagate at constant wave-speed, Wadati diagrams were constructed for each event from the ISSI, hand-picked P and S travel-times. Wadati diagrams are representations of S-P *vs* P travel-times and, for media of constant wave-speed, follow a straight line with a slope that is directly related to the Vp/Vs ratio of the medium [Wadati, 1933]. For each event, a search was run for the largest subset of stations that yielded slopes with associated Vp/Vs ratios between 1.55 and 1.70, which are appropriate for quartzites at shallow depths [Christensen, 1996]. Outliers were removed from the dataset.

To illustrate this 'Wadati filter', Figure 2 displays a Wadati diagram for an Mw = 1.8 event recorded at as many as 16 stations. Note how stations SAV34 and SAV64 have been rejected by our 'Wadati filter' due to inconsistent S-P or P travel-times. The corresponding waveforms are shown on a record section in Figure 3, along with the manual travel-time picks reported by ISSI. Except for the S-wave arrival at station SAV64, all the recordings show clear P- and S-wave arrivals. We think the discrepancy highlighted by the Wadati diagrams for SAV64 is the result of an unclear S-wave onset in the recordings; for SAV34, the discrepancy could be caused by either a misidentification of the S-wave arrival in the North component or a deviation from propagation at uniform wave-speed.

Sensor Orientation

Sensor orientations were verified by comparing theoretically rotated waveforms with empirically rotated waveforms (Fig. 4). Theoretical rotations were performed through the rotation angles calculated as

$$\tan \varphi = (y_S - y_0)/(x_S - x_0)$$
$$\cos \phi = (z_S - z_0)/[(x_s - x_0)^2 + (y_S - y_0)^2 + (z_S - z_0)^2]^{1/2}, \quad (1)$$

where (x_s, y_s, z_s) and (x_0, y_0, z_0) are the station and source coordinates, respectively, with 'x' being North, 'y' being East, and 'z' being Up, and ϕ and φ are the take-off angle and azimuth, respectively. The empirical rotation angles, on the other hand, where obtained from the eigenvectors associated to the eigenvalues of the covariance matrix defined by Montalbetti and Kanasewich [1970]

$$\begin{bmatrix} \mathrm{Var}[x] & \mathrm{Cov}[x,y] & \mathrm{Cov}[x,z] \\ \mathrm{Cov}[x,y] & \mathrm{Var}[y] & \mathrm{Cov}[y,z] \\ \mathrm{Cov}[x,z] & \mathrm{Cov}[y,z] & \mathrm{Var}[z] \end{bmatrix}, \quad (2)$$

where $\mathrm{Var}[x] = \Sigma\, (x_i - x^*)^2/(n-1)$, $\mathrm{Cov}[x,y] = \Sigma\, [(x_i - x^*)(y_i - y^*)]/(n-1)$, with * denoting the average within the P-wave window of the in-mine recordings and 'n' denoting the number of data points. This 'polarization filter' only yields the directions of the P-, SV-, and SH-axis, and cannot resolve the sense of motion due to the symmetry of the covariance matrix with respect to reflections about the origin of coordinates.

The sensor orientation could only be verified for 13 stations out of the 20 in the network, as 7 of them had at least one component failing for the entire recording period, which prevented the rotation into the local ray-coordinate system. Additionally, we observed that the theoretically

and empirically rotated waveforms did not compare well for 7 of those 13 stations. Fortunately, corrections could be obtained for 4 of them through rotations around the vertical axis, yielding a total of 10 stations available for moment tensor inversion (Table 1). However, an ambiguity of 180° around the vertical axis is still possible. Indeed, we observed that waveforms recorded at stations SAV79 and SAV81 were, for some events, more consistent with the moment tensor solutions when rotated by 180°. Therefore, these stations were also not used to constrain focal mechanisms. A final event selection was done after sorting the events by the number of recording stations with verified sensor orientations, in descending order, and selecting the top 100. The hypocentral locations and origin times for the selected events are listed in Table E1 (online at http://www.geosc.psu.edu/~jjulia/julia-esupp.html).

Spectral Amplitudes

Spectral amplitudes for the P, SV, and SH pulses were obtained in the time domain from the theoretically rotated waveforms through the integrals of squared velocities and squared displacements according to [Trifu et al., 2000]

$$u = 2S_{D2}^{3/4}/S_{V2}^{1/4}, \qquad (3)$$

where 'u' is the spectral amplitude, $S_{D2} = \int D^2(t)\,dt$, $S_{V2} = \int V^2(t)\,dt$, and 'D' is displacement, 'V' is velocity, and the integrals are evaluated within finite time windows. Following Trifu et al. [2000], P-waveforms were windowed between the P-wave and the S-wave arrivals and S-waveforms were windowed between the S-wave arrivals and a time window two times the S-P travel-time difference in length. Polarities for the spectral amplitudes were visually assigned by comparing the low-pass filtered instrument response in the time domain to the observed low-pass filtered P- and S-wave pulses (Fig. 5). Although the spectral amplitudes are evaluated in the time

domain, they are equivalent to measuring the height of the spectral plateau in the frequency domain [Urbancic et al., 1996]. Consequently, the low-pass filter must go below the corner frequency to make the observed pulse shape similar to the shape of the instrument response in the time domain.

Typically, we were successful in assigning polarities after low-pass filtering below 10 Hz using two passes of a third order Butterworth filter. However, difficulties were sometimes experienced with the 14 and 28 Hz geophones and with stations very close to the source. In such cases, we generally succeeded by low-pass filtering below 20 or 40 Hz, which suggests that the frequency content of the filtered waveform was still within the flat part of the source spectrum. Time shifts between the SV and SH pulse, probably due to anisotropic effects, were observed during this process, but because our moment tensor solutions involve only spectral amplitudes the time shifts have no effect in our moment tensor solutions.

Inversion Method

Estimates for the 6 independent components of the seismic moment tensor were obtained by inverting P, SV, and SH spectral amplitudes with polarities attached. As explained before, the ray-paths for the selected seismic events are mostly contained within the ~500 m thick layer of quartzite separating the two gold-bearing reefs, so the forward problem can be safely formulated for a homogenous whole space. Following Trifu et al. [2000], the forward problem can be expressed as

$$\mathbf{u} = c\ \mathbf{F}:\mathbf{M}, \qquad (4)$$

where **u** is the vector of spectral displacements in the local ray-coordinate system, $c=1/(4\pi\rho v^3 R)$ with 'v' being either the P- or S-wave velocity, 'R' the hypocentral distance, ρ the density, **M** the

matrix defining the seismic moment tensor in the geographic system, and **F** the excitation matrix given by

$$\mathbf{F}^{P} = \begin{bmatrix} \sin^2\phi \cos^2\varphi & 1/2 \sin^2\phi \sin 2\varphi & 1/2 \sin 2\phi \cos \varphi \\ 1/2 \sin^2\phi \sin 2\varphi & \sin^2\phi \sin^2\varphi & 1/2 \sin 2\phi \sin \varphi \\ 1/2 \sin 2\phi \cos \varphi & 1/2 \sin 2\phi \sin \varphi & \cos^2 \phi \end{bmatrix}$$

$$\mathbf{F}^{SV} = \begin{bmatrix} 1/2 \sin 2\phi \cos^2\varphi & 1/4 \sin 2\phi \sin 2\varphi & \cos^2\phi \cos \varphi \\ 1/4 \sin 2\phi \sin 2\varphi & 1/2 \sin 2\phi \sin^2\varphi & \cos^2\phi \sin \varphi \\ - \sin^2\phi \cos \varphi & \sin^2\phi \sin \varphi & -1/2 \sin 2\phi \end{bmatrix} \qquad (5)$$

$$\mathbf{F}^{SH} = \begin{bmatrix} -1/2 \sin \phi \sin 2\varphi & - \sin \phi \sin^2\varphi & - \cos \phi \sin \varphi \\ \sin \phi \cos^2\varphi & 1/2 \sin \phi \sin 2\varphi & \cos \phi \cos \varphi \\ 0 & 0 & 0 \end{bmatrix},$$

where ϕ and φ are, again, the take-off angle and azimuth, respectively.

Due to the symmetry of the moment tensor **M**, equation (4) can be rewritten as

$$u_j = \sum_k c^i_j f^i_k m_k, \quad i = P,SV,SH \qquad j=1,\ldots,n \qquad (6)$$

where 'n' is the number of data points, $m_1 = M_{11}$, $m_2 = M_{12} = M_{21}$, $m_3 = M_{22}$, $m_4 = M_{13} = M_{31}$, $m_5 = M_{23} = M_{32}$, and $m_6 = M_{33}$, and where $f_1 = F_{11}$, $f_2 = F_{12} + F_{21}$, $f_3 = F_{22}$, $f_4 = F_{13} + F_{31}$, $f_5 = F_{23} + F_{32}$, and $f_6 = F_{33}$. Equation (6) defines a linear problem, which is solved for the moment tensor elements m_k with no constraints (i.e. full moment tensor solution) after a Singular Value Decomposition (SVD) of the matrix $c^i_j f^i_k$ [see e.g. Menke, 1984]. Since the only moment tensor

solutions that do not require a truncation of the singular value spectrum are capable of independently constraining all moment tensor components, in practice, the moment tensor solutions reported in this study minimize the difference between observations and predictions in a least squares sense.

The ‘c’ factor in equations (4) and (6) does not include any corrections for anisotropy or attenuation. As noted before, time shifts were observed between the SV and SH amplitudes, which we attribute to anisotropic propagation effects due to horizontal layering. The SV and SH components coincide with the S1 and S2 components in a transversely isotropic medium, which may delay the pulses with respect to each other but will not introduce any pulse doubling [Backus, 1962]. Since the inversions are performed in the frequency domain, where phase information is dropped (except for polarity), we do not believe that anisotropy strongly maps into our solutions. Attenuation, on the other hand, could potentially have a stronger effect. As shown later, our reported moment tensor solutions correctly predict the polarities of the observed amplitudes, so our mechanism types should not be strongly affected by attenuation effects. On the other hand, attenuation corrections would make the observed amplitudes bigger, and our moment tensor sizes could be somewhat underestimated.

Formal uncertainties for the components of the seismic moment tensor are not reported in this study. In theory, the linearity of the forward problem defined by equation (6) allows for the matrix of covariance of the spectral amplitudes to be easily propagated into uncertainties of the moment tensor components [e.g. Menke, 1984]. Fletcher and McGarr [2005], for instance, assign a standard deviation to the observed amplitudes equal to 20% of the observed amplitude and propagate the uncertainties into the moment tensor solutions. This approach is successful in conveying the relative accuracy among the moment tensor components, but we find it is

somewhat arbitrary and statistically misleading. A meaningful matrix of covariance for the spectral amplitudes should include uncertainties in the amplitude measurement as well as uncertainties in the velocities of the propagating medium, sensor orientations, and event locations. Our approach has been to conduct numerical experiments in an attempt to falsify those observations that are critical to our study. As shown later, we will be interested to know how well constrained the isotropic part of our moment tensor solutions are. The observed spectral amplitudes will then be re-inverted for a purely deviatoric source in order to assess the necessity of an isotropic component in the moment tensor solutions. This new suite of inversions will be conducted through the same system defined in equation (6) with the additional constraint that the trace of the moment tensor be zero,

$$m_1+m_3+m_6 = 0. \qquad (7)$$

Moment Tensor Solutions

We have applied the inversion of spectral amplitudes with polarity attached described in the previous section to the 100 events selected for this study, after assuming a whole space of Vp=6.0 km/s, Vs=3.70 km/s and ρ=2.69 g/cm^3. Only those moment tensor solutions that correctly predict P, SV, and SH polarities for all the waveforms were accepted (the polarity requirement was relaxed for small amplitudes, which probably represent null measurements), and the quality of the solutions was assessed through the condition number and the correlation between observed and synthetic amplitudes. The accepted moment tensor solutions, totaling 76, and the corresponding condition numbers and correlation coefficients are listed in Table E2 (online at http://www.geosc.psu.edu/~jjulia/julia-esupp.html).

The condition number is defined as the ratio between the smallest and largest singular values in the SVD and is a measure of how robustly constrained the moment tensor components are. A condition number of 1 indicates that all moment tensor components are equally well constrained, while a condition number of 0 indicates that there is at least one component not resolved and that the solution is non-unique. For our accepted moment tensor solutions the condition numbers range between 0.04 and 0.19, just one to two orders of magnitude smaller than the ideal value. As discussed later, the reported moment tensor solutions define a consistent radiation pattern that agrees well with the stress pattern of the mine, suggesting this range of values reflects the solutions are well constrained. The reported condition numbers should then be taken as a measure of the relative robustness among the solutions.

The correlation coefficients were obtained from the cross-correlation between observed and synthetic waveforms and are a measure of the similarity between observations and predictions. Synthetic seismograms were computed from the spectral amplitudes as

$$\mathbf{u}(t) = \mathbf{u}\ \delta(t-R/v), \tag{8}$$

where $\mathbf{u}(t)$ is the displacement vector in the time domain, $\mathbf{u}$ is the spectral amplitude from equation (4), $\delta(t)$ is a unit-area Dirac's delta function, 'v' is either the P- or S-wave velocity, and 'R' is the hypocentral distance, then differentiated to obtain velocities and finally convolved with the instrument response. The correlation coefficients were obtained as

$$\text{cor} = (1/3)\ [C(u^{obs,P},u^{syn,P}) + C(u^{obs,SV},u^{syn,SV}) + C(u^{obs,SH},u^{syn,SH})] \tag{9}$$

where u^{obs} stands for observation, u^{syn} for synthetic, and where

$$C(\tau) = \max\ [\ \textstyle\int u^{obs}(t+\tau)\ u^{syn}(t)\ dt\ ]\ /\ \max\ [\ \textstyle\int u^{obs}(t)\ u^{obs}(t)\ dt\ ], \tag{10}$$

where τ is the time shift, after low-pass filtering both observations and synthetics generally below 10 Hz (20 or 40 Hz for the 14 Hz and 28 Hz instruments). Note that a coefficient of 1.0

implies a perfect agreement between observations and predictions, while larger and smaller values indicate the amplitudes have been either over-predicted or under-predicted, respectively. The correlation coefficients corresponding to our accepted moment tensor solutions range between 0.54 and 1.18, and we found that even coefficients as small as 0.54 give a satisfactory visual match between observations and predictions. As with the condition numbers, the values of the correlation coefficient should be used as a measure of the relative goodness of fit of the solutions. In the following, we discuss a few moment tensor solutions in detail.

Event 2007.02.21.18.21.56.591 (Mw=1.6)

Figure 6 displays the moment tensor inversion results for this event, along with the coverage of the focal sphere. Spectral amplitudes for this event were measured for P, SV, and SH arrivals recorded at six in-mine seismic stations and the polarities were visually assigned for each of them, as described before. The coverage of the focal sphere is characteristic of mining environments and reflects the lack of station coverage ahead of the advancing mine stopes. The measured P- and S-wave amplitudes, however, had both positive and negative polarities, which suggests that more than one energy lobe in the radiation pattern has been sampled. The moment tensor solution obtained for this event is m_{xx}=-1.25x10^{11} Nm , m_{xy}=0.74x10^{11} Nm , m_{yy}=0.09x10^{11}Nm, m_{xz}=1.20x10^{11} Nm, m_{yz}=0.55x10^{11} Nm, and m_{zz}=-2.66x10^{11} Nm, has a large condition number around 0.16, and a correlation coefficient around ~0.70, which is in the middle-to-low portion of the observed range and offers a satisfactory visual match between predictions and observations.

Further insight can be gained from the eigenvalues (σ_i) and eigenvectors ($\mathbf{e}_i$) associated to the moment tensor solution. The eigenvalues are indicative of the compressional (negative) or

tensional (positive) character of the principal stresses, and the direction is given by the corresponding eigenvectors [e.g. Lay and Wallace, 1995]. For this moment tensor solution, we obtain σ_1=-3.35x10^{11} Nm, σ_2=-1.22x10^{11} Nm, and σ_3=0.74x10^{11} Nm, and $\mathbf{e}_1$ = (0.49,0.04,-0.87), $\mathbf{e}_2$ = (-0.72,0.57,-0.38), and $\mathbf{e}_3$ = (-0.49,-0.82,-0.31). The largest principal stress is thus compressive and oriented near-vertically (deflection from the vertical is ~29°), consistent with the stress conditions expected in a deep mine.

All moment tensors can be decomposed into isotropic and deviatoric components, with the isotropic eigenvalue given by tr(σ_i)/3, where tr(σ_i)=σ_1+σ_2+σ_3 is the trace, and the deviatoric eigenvalues given by $\sigma^*_j = \sigma_j$ – tr(σ_i)/3 [e.g. Jost and Herrman, 1989]. For this particular moment tensor solution tr(σ_i)= –3.83x10^{11} Nm, representative of an implosive volumetric source. The deviatoric eigenvalues are σ^*_1 = -2.07x10^{11} Nm, σ^*_2 = 0.06x10^{11} Nm, and σ^*_3 = 2.01x10^{11} Nm, which are well approximated by a double-couple force equivalent [e.g. Jost and Herrmann, 1989]. The eigenvectors of the deviatoric moment tensor are the same as those for the general moment tensor, so the double-couple solution thus has a sub-vertical pressure axis and a sub-horizontal tension axis characteristic of normal faulting. The decomposition of the moment tensor solution for this event thus indicates that stresses have been relaxed through normal faulting and co-seismic closure [McGarr, 1992a].

Event 2007.02.01.01.49.31.639 (Mw=1.1)

More generally, however, we obtain deviatoric moment tensors that do not follow a clear double-couple pattern, and Figure 7 displays the moment tensor inversion results and the coverage of the focal sphere for one such event. All P-wave amplitudes are negative, although a polarity could not be assigned for SAV35 due to its small amplitude, and polarities of both signs

are found among the S-wave amplitudes. The moment tensor solution obtained for this event is m_{xx}=-2.48x10^{10} Nm , m_{xy}=1.47x10^{10} Nm , m_{yy}=-1.98x10^{10}Nm, m_{xz}=1.50x10^{10} Nm, m_{yz}=-1.78x10^{10} Nm, and m_{zz}=-4.33x10^{10} Nm, has a large condition number of ~0.18, and a correlation coefficient of 0.88 which, once more, offers a satisfactory visual match between observations and predictions.

The eigenvalues for the seismic moment tensor for this event are σ_1=-6.36x10^{10} Nm, σ_2=-1.77x10^{10} Nm, and σ_3=-0.67x10^{10} Nm, and the corresponding eigenvalues are $\mathbf{e}_1$ = (0.47,-0.46,-0.75), $\mathbf{e}_2$ = (-0.72,0.30,-0.63), and $\mathbf{e}_3$ = (-0.51,-0.84,0.20), which again reveal a compressive and sub-vertical maximum principal stress (deflection from vertical is ~41°). The trace of the moment tensor is –8.80x10^{10} Nm, suggestive of an implosive volumetric contribution and consistent with the negative polarities observed for all P-wave amplitudes. The deviatoric eigenvalues are σ^*_1 = -3.43x10^{10} Nm, σ^*_2 = 1.16x10^{10} Nm and σ^*_3 = 2.27x10^{10} Nm, which clearly do not match the double-couple pattern. The deviatoric moment tensor can further be decomposed into major and minor double-couples [e.g. Jost and Herrmann, 1989]; for this particular event, one possible decomposition would be:

$$\begin{bmatrix} -3.43 & & \\ & 1.16 & \\ & & 2.27 \end{bmatrix} = \begin{bmatrix} -1.16 & & \\ & 1.16 & \\ & & 0.00 \end{bmatrix} + \begin{bmatrix} -2.27 & & \\ & 0.00 & \\ & & 2.27 \end{bmatrix} \qquad (11)$$

in 10^{10} Nm. This decomposition would explain the observed radiation pattern through two normal faults, with a common P-axis and two corresponding T-axes forming an angle of 90°, and a co-seismic volumetric closure.

The deviatoric moment tensor can be decomposed in multiple ways and the choice of the decomposition type is arbitrary [see e.g. Jost and Herrmann, 1989]. Another common choice is the decomposition into a compensated linear vector dipole (CLVD) and a double couple, which can be useful in identifying pillar failures (the force equivalent is the compression CLVD combined with an isotropic implosion which annihilates the P-wave compressions in the directions perpendicular to the principal axis of the CLVD) [e.g. Sileny and Milev, 2006; 2008]. The decomposition utilized in equation (11) was the one favored in McGarr [1992a,b], and it has been chosen here for compatibility with those studies.

Discussion and Conclusions

The 76 moment tensor solutions obtained in this study reveal that the largest (in an absolute sense) principal stresses are generally negative (i.e. compressional) and oriented sub-vertically (see Table E3, online at http://www.geosc.psu.edu/~jjulia/julia-esupp.html). This is in agreement with the expected principal stresses in the deep mine being mainly caused by gravitational forces attempting to close the mined-out areas. Indeed, in-situ measurements in the East Rand Proprietary Mine (ERPM), also in the Witswatersrand basin, show that the maximum principal stress is compressional and oriented at an angle between the vertical and the normal to the gold-bearing horizon (which dips around 30° to the SSW in the ERPM area) [Pallister et al, 1971]. If a decomposition of the deviatoric moment tensor into two double couples as in equation 11 is chosen, then the solutions listed in Table E3 indicate that deep mine stresses are relaxed through a combination of co-seismic volumetric closure and normal faulting.

We have obtained isotropic (m^I), deviatoric (m^D), total (m^T) scalar moments, and moment magnitudes from our moment tensor solutions. Assuming that the eigenvalues of the general

moment tensor are given by σ_1, σ_2, and σ_3, then the scalar moments are defined as [Bowers and Hudson, 1999]

$$m^I = |1/3\ tr(\sigma_i)|,\ m^D = max(|\sigma^*_j|),\ \text{and}\ m^T = m^I + m^D, \tag{12}$$

respectively, where $tr(\sigma_i) = \sigma_1+\sigma_2+\sigma_3$ and $\sigma^*_j = \sigma_j - tr(\sigma_i)/3$. The moment magnitude is obtained from the total scalar moment as [Hanks and Kanamori, 1979]

$$Mw = 2/3\ (\log_{10}(m^T) - 9.1), \tag{13}$$

where m^T is expressed in Nm. The values for the scalar moments and moment magnitudes can be found in Table E4 (online at http://www.geosc.psu.edu/~jjulia/julia-esupp.html).

A careful inspection of our moment tensor solutions (Table E4) reveals that a total of 66 solutions have a significant isotropic moment representing more than 10% of the total moment (computed as m^I/m^Tx100). Before drawing any conclusions about the volumetric shear-mix, however, it is important to assess how well constrained the volumetric components are and, as explained before, this is investigated by re-running the moment tensor inversions after imposing a purely deviatoric moment tensor solution (equation 7). This is illustrated in Figure 8 through event 2007.03.04.06.16.04.098, where low-frequency synthetic seismograms predicted by the full and constrained moment tensor solutions are compared to the corresponding observed traces. Both moment tensor solutions correctly predict the polarities for the P, SV, and SH amplitudes and the correlation coefficients between observed and predicted amplitudes are similar, 0.86 for the full solution and 0.77 for the constrained solution. The condition number of the full moment tensor solution is around 0.15.

The results of this exercise reveal that 47 solutions require a non-zero trace to fully explain the polarities for the observed amplitudes, while the polarities for the remaining 19 events can be explained just as well through a purely deviatoric source. As expected, the match

between observed and predicted amplitudes is generally better for the full moment tensor solutions (correlation coefficients range between 0.54 and 1.18) than for the constrained solutions (correlation coefficients range between 0.42 and 1.32), as the full solutions have an additional degree of freedom compared to the constrained solution. Occasionally, the change in the correlation coefficient is big enough that it could be used to favor one solution over the other but, in general, we observe the change is not significant. The condition number for the solutions with an uncertain volumetric component ranges between 0.05 and 0.17, similar to the range observed for the full moment tensor solutions for the whole dataset. As argued before, we think that this range of values reflects well-constrained moment tensor solutions, as the similarity suggests uncertain volumetric contributions are the result of uncertainties in the amplitude measurements rather than an insufficient coverage of the focal sphere. The deviatoric P-wave radiation patterns for these moment tensor solutions displayed in Figure 9, superimposed to the Savuka mine plan.

McGarr [1992b] reported a bimodal distribution of the volumetric-shear mix for 10 events with moment magnitudes in the 1.9-3.3 range recorded in two deep mines in South Africa. The distribution peaked at values of 0.0 and 0.71 for the volumetric to shear slip ratio ($-\Delta V/\Sigma AD$), with no events having ratios between 0.0 and 0.5. Following McGarr [1992b], the volumetric closure ($-\Delta V$) and the average shear slip (ΣAD) are calculated as

$$\Delta V = tr(\sigma_i)/(\lambda+2\mu)$$
$$\Sigma AD = (|\sigma^*_1|+|\sigma^*_2|)/2\mu, \qquad (14)$$

where $tr(\sigma_i)$ is the trace of the moment tensor, σ^*_1 and σ^*_2 are the eigenvalues of the major and minor double couples of the deviatoric moment tensor, λ and μ are Lamé's elastic constants, and 'A' and 'D' represent the fault area and average slip, respectively. We have assumed that $\lambda+2\mu =$

1.63×10^{11} Pa and $\mu = 3.76 \times 10^{10}$ Pa for the Witswatersrand quartzite [McGarr, 1992b]. The $-\Delta V$, ΣAD, and $-\Delta V/\Sigma AD$ values for the 76 moment tensors can be found in Table E5 (online at http://www.geosc.psu.edu/~jjulia/julia-esupp.html). Note that negative values of the source-mix ratio indicate an explosive volumetric component, null values indicate a purely deviatoric source, and positive values indicate an implosive volumetric component. A quick inspection of the ratios reveals a more or less continuous variation of volumetric closure to average shear slip ratios ($-\Delta V/\Sigma AD$), with values ranging between –0.4 and 0.9, and peaking around 0.2 and 0.6 (Fig. 10). Note the moment tensors with an uncertain volumetric contribution include all the explosive sources.

The magnitude dependence of the volumetric closure to shear slip ratios is investigated in Figure 11. The figure includes the ratios for the 76 events reported in this study, plus the 10 measurements obtained by McGarr [1992b]. The diagram shows that the distribution of volumetric to shear slip ratios is continuous for moment magnitudes below 2.2. If we remove the poorly constrained events from our database, we observe that the ratios for events with moment magnitudes below 2.2 still display a more or less continuous range of values for the $-\Delta V/\Sigma AD$ ratio. For moment magnitudes above 2.2 the distribution seems bimodal, as reported by McGarr [1992b], but this magnitude range is clearly under-sampled by our measurements.

Summarizing, the moment tensor solutions obtained in this study for 76 mine tremors in South Africa have demonstrated that the distribution of co-seismic volumetric closure to shear-slip ratios is continuous in the $0.5 < Mw < 2.2$ moment magnitude range. A continuous distribution of the volumetric-shear mix had been previously reported for mine tremors at overlapping and lower magnitude ranges in shallow mines in Canada and Utah [Feigner and Young, 1992; Fletcher and McGarr, 2005], which suggests the continuous distribution of the

volumetric-shear mix is independent of mining conditions. At moment magnitudes above 2.2, our measurements do not have enough overlap with those of McGarr [1992b] to either confirm or call into question their postulated bimodal distribution. The number of measurements in this magnitude range is significantly smaller than at lower magnitudes and more measurements are required to confidently assess the bimodality of the source-mix distribution.

Data and Resources

Seismic waveforms used in this study were obtained as part of the Mine Seismicity Project of AfricaArray (http://africaarray.psu.edu). Event locations and local magnitudes were obtained from Integrated Seismic Systems International (ISSI). The moment tensor inversion codes were developed by J. Julià. Some plots were made using the Generic Mapping Tools, version 4.3.3 (http://gmt.soest.hawaii.edu; Wessel and Smith, 1998).

Acknowledgements

AngloGold Ashanti is thanked for the sharing of the proprietary data for Savuka mine used in this study. We are also indebted to Integrated Seismic Systems International for their role in providing the in-mine data analyzed in this study. The manuscript has greatly benefited from detailed and insightful reviews from two anonymous referees, as well as from comments by Steve Spottiswoode and Alex Milev at the Council for Scientific and Industrial Research in South Africa. Roger Stewart is thanked for helping produce a printable version of the Savuka mine plans. Support for this work has been provided by the U.S. Department of Energy, contract number DE-FC52-06NA27320.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

References

Backus, G.E. (1962). Long-wave elastic anisotropy produced by horizontal layering, *J. Geophys. Res.,* **67**, 4427-4440.

Baker, C., and R.P. Young (1997). Evidence for extensile crack initiation in point source time-dependent moment tensor solutions, *Bull. Seism. Soc. Am.,* **87***,* 1442-1453.

Bowers, D., and J.A. Hudson (1999). Defining the scalar moment of a seismic source with a general moment tensor, *Bull. Seism. Soc. Am.,* **89***,* 1390-1394.

Christensen, N.I. (1996). Poisson's ratio and crustal seismology, *J. Geophys. Res.,* **101***,* 3139-3156.

Feigner, B., and R.P. Young (1992). Moment tensor inversion of induced microseismic events: Evidence of non-shear failures in the –4 < M < -2 moment magnitude range, *Geophys. Res. Lett.,* **19,** 1503-1506.

Fletcher, J.B., and A. McGarr (2005). Moment tensor inversion of ground motion from mining-induced earthquakes, Trail Mountain, Utah, *Bull. Seism. Soc. Am.,* **95,** 48-57.

Hanks, T.C., and H. Kanamori (1979). A moment magnitude scale, *J. Geophys. Res.,* **84,** 2348-2350.

Jost, M.L., and R.B. Herrmann (1989). A student's guide to and review of moment tensors, *Seism. Res. Lett.*, **60**, 37-57.

Lay, T., and T.C. Wallace (1995). *Modern Global Seismology*, Academic Press, San Diego, California, 521 pp.

McGarr, A. (1992a). An implosive component in the seismic moment tensor of a mining-induced tremor, *Bull. Seism. Soc. Am.,* **19,** 1579-1582.

McGarr, A. (1992b). Moment tensors of ten Witwatersrand mine tremors, *Pure appl. Geophys.,* **139,** 781-800.

Menke, W. (1984). *Geophysical Data Analysis: Discrete Inverse Theory*, Academic Press, Orlando, Florida, 260 pp.

Montalbetti, J.F., and E.R. Kanasewich (1970). Enhancement of teleseismic body phases with a polarization filter, *Geophys. J. R.A.S.,* **21***,* 119-129.

Pallister, G.F., N.C. Gay, and N.G.W. Cook (1971). Measurements of the virgin state of stress in rock at depth, *Proc. Int. Soc. Rock Mech., Congr. 2nd*.

Sileny, J., and A. Milev (2006). Seismic moment tensor resolution on a local scale: Simulated rockburst and mine-induced seismic events in the Kopanang gold mine, South Africa, *Pure appl. Geophys,* **163,** 1495-1513.

Sileny, J., and A. Milev (2008). Source mechanism of mining induced seismic events-Resolution of double couple and non double couple models, *Tectonophysics,* **456,** 3-15.

Trifu, C-I., D. Angus, and V. Shumila (2000). A fast evaluation of the seismic moment tensor for induced seismicity, *Bull. Seism. Soc. Am.,* **90,** 1521-1527.

Urbancic, T.I., C-I. Trifu, R.A. Mercer, A.J. Feustel, and J.A.G. Alexander (1996). Automatic time-domain calculation of source parameters for the analysis of induced seismicity, *Bull. Seism. Soc. Am.,* **86,** 1627-1633.

Wadati, K. (1933). On the travel time of earthquake waves, Part II, *Geophysical Magazine,* **7**, 101-111.

Wessel, P., and W.H.F. Smith (1998). New, improved version of the Generic Mapping Tools released, *Eos Trans. AGU***, 79,** 579.

Wong, I., and A. McGarr (1990). Implosional failure in mining-induced seismicity: A critical review, in *Rockbursts and Seismicity in Mines*, C. Fairhurst (Editor), Balkema, Rotterdam, The Netherlands, 45-51.

Authors Affiliations and Addresses

Jordi Julià and Andrew A. Nyblade

Department of Geosciences

The Pennsylvania State University

University Park, PA 16802, U.S.A.

Rengin Gök and William Walter

Lawrence Livermore National Laboratory

Livermore, CA 94551, U.S.A.

Ray Durrheim

Council for Scientific and Industrial Research and School of Geosciences, University of Witswatersrand

Johannesburg, South Africa.

Lindsay M. Linzer

MeerCAT Geophysics and School of Geosciences, University of the Witswatersrand

Johannesburg, South Africa.

Paul Dirks and Ray Durrheim

School of Geosciences

University of the Witswatersrand

Wits 2050, South Africa.

Table 1 - List of stations

ID	South (m)	West (m)	Down (m)	Rot.
SAV08	27950	-39827	1869	?
SAV21	27816	-40454	226	?
SAV27	28676	-39582	2003	?
SAV29	29617	-40545	3649	-90
SAV34	28330	-40660	2142	180
SAV35	29024	-40247	2285	55
SAV36	27958	-40226	2960	ok
SAV40	27738	-39822	2964	ok
SAV61	27854	-40568	2010	ok
SAV77	28220	-39753	3089	90
SAV79	28109	-40220	3733	ok*
SAV80	29114	-41600	3657	ok
SAV81	29000	-41638	3493	ok*

*Not utilized due to orientation ambiguity

Figure Captions

Figure 1 – Plan of Savuka Mine, South Africa, with the location of the in-mine geophones (circles) and the 100 seismic events (grey squares) considered in this study. The black circles denote the stations utilized for moment tensor inversion and the grey circles indicate the stations that could not be utilized. The thin solid lines delineate the outline of the mine while the shaded areas represent the areas that were mined during 2007. The hypocentral locations were obtained from ISSI.

Figure 2 – Wadati diagram for event 2007.01.03.18.31.21.223. The reference time in the horizontal axis is the trigger time from one of the recording stations, randomly selected by ISSI to label the event. Note that two stations, SAV34 and SAV64, have been left out of the Wadati line. The Vp/Vs ratio, origin time correction, and regression coefficient from the least squares fit to the data points (not including the rejected stations) is shown in the plot. Confidence bounds are given at the 95% confidence level.

Figure 3 – Velocity recordings from the Savuka in-mine network for event 2007.01.03.18.31.21.223, sorted by P-wave travel-time. The traces have been normalized by their maximum absolute amplitude value for displaying purposes. The P- and S-wave travel-time picks reported by the in-mine network operator are overlain on the traces. Note that half of the recording stations (SAV64, SAV80, SAV63, SAV09, SAV78, SAV44, SAV41, SAV43) have at least one failing channel.

Figure 4 – Recordings of event 2007.01.03.18.31.23.223 at station SAV77 in the locar ray-coordinate system. The vertical scale is the same in all plots. The left column shows the theoretically rotated waveforms before applying the correction reported in Table 1. Note that

most of the P-wave energy is wrongly mapped into the SH component. The middle column shows the empirically rotated waveforms, which correctly display most of the P-wave energy in the P component. The right column shows the theoretical rotations on the corrected components, which compare well with the empirical ones.

Figure 5 –Assignment of polarity for the P, SH and SV amplitudes recorded at station SAV61 and corresponding to event 2007.02.01.18.08.14.177. The top trace is the instrument response in the time domain (positive polarity) normalized to unit amplitude, and the traces below are the P, SH and SV components, respectively. All traces have been low-pass filtered below 10 Hz. The similarity of the impulse response to the P and S pulses demonstrates the frequency content of the pulses is within the flat part of the source spectrum, and suggests the polarities are negative, positive, and negative for the P, SH and SV amplitudes, respectively.

Figure 6– Moment tensor inversion results and coverage of the focal sphere for event 2007.02.21.18.21.56.591. The upper left diagram compares predicted and observed spectral amplitudes for P, SV, and SH components. The lower left diagram shows the P-wave radiation pattern for the isotropic and deviatoric moment tensors, along with the coverage of the focal sphere (overlying the deviatoric "beach-ball"). The waveforms on the right are shown to compare the corresponding observed (black) and predicted (grey) P, SV and SH amplitudes in the time domain. The event origin time is at t = 0 s and the synthetic waveforms have been shifted by the amount noted above each trace, to facilitate the comparison with the observations. The time shifts between observed and synthetic waveforms are due to inaccuracies in the event location, assumed wave-speed, and/or anisotropic effects. Note that only spectral amplitudes were inverted, so the observed time shifts did not influence our moment tensor solutions. The waveforms are shown in velocity.

Figure 7 – Same caption as in Figure 6, but for event 2007.02.01.01.49.31.639. The P-wave and SH-wave spectral amplitudes for station SAV35 and SAV34, respecteively, were not included in the inversion due to difficulties in assigning a polarity, but the corresponding synthetic seismograms predicted by our moment tensor solution are shown as a grey, dotted line.

Figure 8 – Observations (black) and synthetic seismograms (grey) for event 2007.03.04.06.16.04.098 corresponding to a full moment tensor solution (left) and a purely deviatoric moment tensor solution (right). The event origin time is at t = 0 s and the synthetic waveforms have been shifted by the amount noted above each trace. Note that both solutions satisfactorily explain the P-wave amplitudes in all recordings. The isotropic component of the full moment tensor solution is explosive and represents ~29% (m^I/m^Tx100) of the total moment tensor.

Figure 9 – Histogram of volumetric to shear-slip ratios ($-\Delta V/\Sigma AD$) for the values listed in Table E5. Negative and positive values correspond to explosive and implosive volumetric contributions, respectively. Note the continuous range of values of the source-mix, from –0.4 to 1.0.

Figure 10 – Magnitude dependence of the volumetric-shear mix for deep mine events in South Africa. The solid squares denote full moment tensor solutions with a poorly constrained isotropic component. The events within the dotted lines have small isotropic components in the full moment tensor solution, and are considered to be well-constrained deviatoric sources [McGarr, 1992b].

Figure 11 – Spatial distribution of the deviatoric focal mechanisms for the Savuka mine events considered in this study (only those events with a well-constrained deviatoric part have been included). The black "beach-balls" represent those events with a significant volumetric

component (more than 10% of the total scalar moment); the grey “beach-balls” represent those events with a near-deviatoric solution (less than 10% of the total scalar moment). The mine plan is the same as in Figure 1, with active areas highlighted.