



LAWRENCE  
LIVERMORE  
NATIONAL  
LABORATORY

# On the Accuracy of Techniques for Determining Neutron Compound-nucleus Formation Cross Sections

F. S. Dietrich

November 5, 2009

Second International Workshop on Compound Nuclear  
Reactions and Related Topics  
Bordeaux, France  
October 5, 2009 through October 8, 2009

## **Disclaimer**

---

This document was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor Lawrence Livermore National Security, LLC, nor any of their employees makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or Lawrence Livermore National Security, LLC. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or Lawrence Livermore National Security, LLC, and shall not be used for advertising or product endorsement purposes.

# On the accuracy of techniques for determining neutron compound-nucleus formation cross sections

Frank S. Dietrich<sup>a</sup>

Lawrence Livermore National Laboratory, L-414, P.O. Box 808, Livermore, CA 94551, USA

**Abstract.** We consider three methods for determining neutron nonelastic cross sections: direct measurement by transmission of neutrons through a spherical shell; subtraction of the angle-integrated elastic cross section from the total cross section; and a modification of the subtraction technique using Wick's limit that in favorable cases can significantly reduce the errors in the subtraction method. We show new results using the modified subtraction technique for nonelastic cross sections at 21.6 MeV neutron energy over a wide mass range, and discuss criteria that should be satisfied in order for the modified subtraction technique to be reliable.

## 1 Introduction

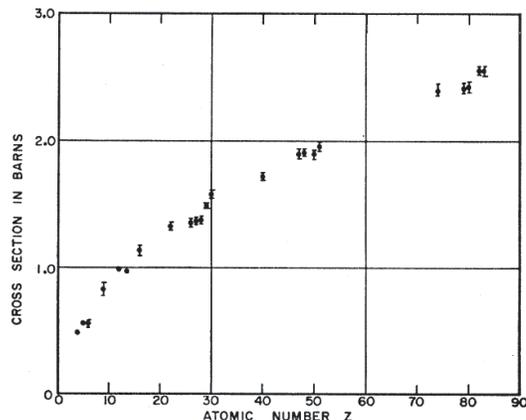
Total neutron cross sections can be measured fairly easily to an accuracy in the neighborhood of 1%. However, the nonelastic (or reaction) cross sections, which are more relevant than the totals for calculating compound-nuclear reactions, are rather poorly known experimentally; measurements typically scatter by 5 to 10%. Herein we discuss three techniques that can be used to determine nonelastic cross sections, and show examples of the results that may be obtained with them. For simplicity we limit the discussion to spherical nuclei, although extensions to deformed nuclei are straightforward.

The classic technique for direct determination of non-elastic cross sections is by measurement of transmission through a spherical shell surrounding the detector. This method has been widely applied, but requires extreme care to details such as detector stability and corrections for absorption and multiple scattering. A well documented set of measurements in the 8–26 MeV range [1–4] was made at Livermore *circa* 1960; results of their measurements at 14.2 MeV are shown in Fig. 1. A smooth curve could be drawn through these data that would be consistent with the claimed  $\approx 3\%$  uncertainty. On the other hand, the complete set of sphere-transmission measurements on Fe, shown in the bottom part of Fig. 2, shows significantly greater uncertainties.

As an alternative method for determining the nonelastic cross section, we consider the direct subtraction of the angle-integrated elastic scattering from the total cross section,

$$\sigma_{\text{reac}} = \sigma_{\text{tot}} - \sigma_{\text{elas}}, \quad (1)$$

where the nonelastic cross section is designated by  $\sigma_{\text{reac}}$ . This method is subject to enhancement of uncertainties arising from the subtraction of two numbers, and is also dependent on the reliability of the systematic error estimates



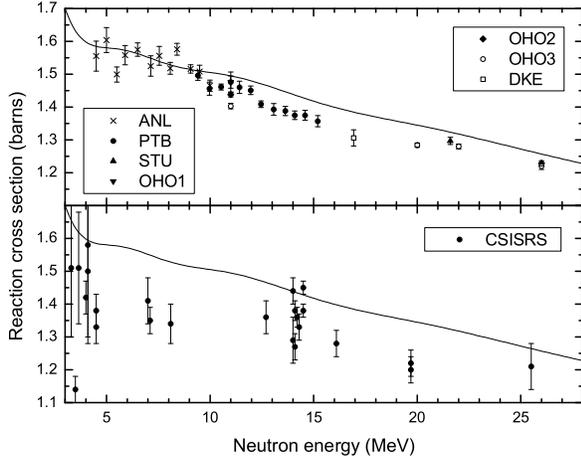
**Fig. 1.** Measurements at 14.2 MeV of the compound formation cross section by the sphere-transmission technique, as reported in Ref. [3].

in two experiments of very different type. In the present work we use a set of total cross sections over a wide mass range [5,6] believed to have uncertainties in the neighborhood of 1%, as well as a 2% measurement on <sup>nat</sup>Ce [7].

We have found a modification of the subtraction technique that in favorable cases can significantly reduce the uncertainties. By using the definition of Wick's limit, we can induce correlations between the two terms in the subtraction expression. We give a brief description of this technique, which has been developed and applied to several cases (<sup>208</sup>Pb, <sup>54,56</sup>Fe, <sup>232</sup>Th, and <sup>238</sup>U) in Refs. [8–10]. We define  $\sigma_0^W$ , the Wick's limit value for the c.m. zero-degree differential elastic cross section, and  $\eta$ , the fractional deviation of the true zero-degree cross section  $\sigma_0$  from its Wick's limit value by

$$\sigma_0^W = \left( \frac{k}{4\pi} \sigma_{\text{tot}} \right)^2 \quad \text{and} \quad \eta = \frac{\sigma_0 - \sigma_0^W}{\sigma_0^W}. \quad (2)$$

<sup>a</sup> e-mail: fsdietrich@sbcglobal.net



**Fig. 2.** Top: Nonelastic cross sections for  $^{54,56}\text{Fe}$  using the modified subtraction technique; angular distributions used to calculate the  $F$  factor are identified in Ref. [9]. Bottom: Nonelastic cross sections for these nuclei from CSISRS database. Solid curves represent the global optical potential of Koning and Delaroche [11].

We also define a quantity

$$F = \frac{\sigma_{elas}}{\sigma_0} = \frac{1}{\sigma_0} \int d\Omega \frac{d\sigma_{elas}}{d\Omega}, \quad (3)$$

which is determined entirely by experiment. Using these definitions for  $\eta$  and  $F$  we can rewrite Eq. 1 as

$$\sigma_{reac} = \sigma_{tot} - (1 + \eta)F \left( \frac{k}{4\pi} \right)^2 \sigma_{tot}^2, \quad (4)$$

which expresses the nonelastic cross section in terms of three independent quantities,  $\sigma_{tot}$ ,  $\eta$ , and  $F$ . The fractional uncertainties due to uncertainties in these quantities, which are to be added in quadrature, are

$$\frac{\Delta\sigma_{reac}^{(1)}}{\sigma_{reac}} = \left| 2 - \frac{\sigma_{tot}}{\sigma_{reac}} \right| \frac{\Delta\sigma_{tot}}{\sigma_{tot}}, \quad (5)$$

$$\frac{\Delta\sigma_{reac}^{(2)}}{\sigma_{reac}} = \left( \frac{\sigma_{tot}}{\sigma_{reac}} - 1 \right) \frac{\eta}{1 + \eta} \frac{\Delta\eta}{\eta}, \quad (6)$$

$$\frac{\Delta\sigma_{reac}^{(3)}}{\sigma_{reac}} = \left( \frac{\sigma_{tot}}{\sigma_{reac}} - 1 \right) \frac{\Delta F}{F}. \quad (7)$$

In the energy range of interest here, the nonelastic cross section is approximately equal to the nuclear area, and the total cross section oscillates with energy about twice this value. Thus the quantity between straight brackets in Eq. 5 is typically very small, and can even be zero at specific energies; the sensitivity to errors in  $\sigma_{tot}$  is consequently very weak. A similar argument indicates that the expressions in parentheses in Eqs. 6 and 7 are approximately unity. The dependence on  $\eta$ , which is calculated from an optical model, introduces a model dependence which has been studied in [8] and shown to be very weak over a wide range of target masses and energies. In this method the largest

uncertainty typically comes from the factor  $F$ , which is computed from experimental elastic scattering angular distributions by Legendre-polynomial fitting to extrapolate to zero degrees and to determine the angle-integrated elastic cross section. Sufficiently accurate extrapolation to zero degrees requires angular distributions with many angular points and with a rather small minimum angle ( $\approx 10$ – $15$  degrees).

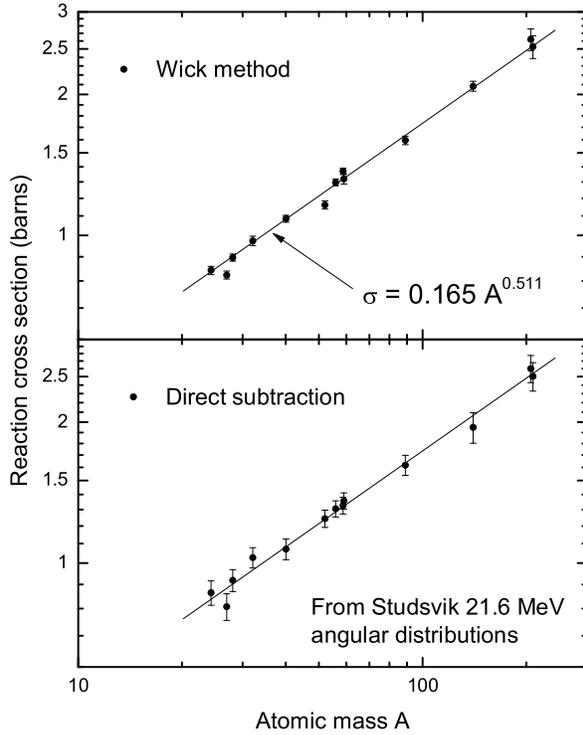
Results for  $^{54,56}\text{Fe}$  are shown in the upper portion of Fig. 2, together with sphere-transmission measurements obtained from the CSISRS database in the lower portion. Both are compared with the predictions of the Koning-Delaroche global optical-model potential [11]. The results from the modified subtraction technique exhibit smaller errors than those from CSISRS, and the cross sections are significantly larger. There is good agreement between the new results near 14 MeV and those from the Livermore sphere transmission measurements [1]. There is also rather good consistency between the results using  $F$  factors calculated with angular distributions from different laboratories.

It is evident that the sources of uncertainty in the three techniques are very different. As noted, the uncertainty in the modified subtraction method is dominated by  $F$ . This quantity is independent of the absolute normalization of the underlying angular distribution measurement; however, the reliability of the extrapolation to zero degrees is crucial. The direct subtraction has the opposite properties: the normalization of the angular distribution is the dominant error, while the extrapolation to small angles carries very little weight in the integration of the differential cross section over solid angle.

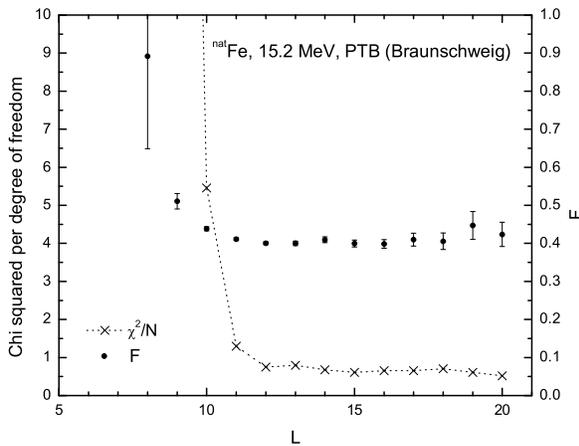
## 2 New results and discussion

We have continued the study of the modified subtraction method by calculating the nonelastic cross sections from the angular distributions measured at the Studsvik laboratory [12] at 21.6 MeV and the total cross sections of Refs. [5–7]. The Wick deviation  $\eta$  was calculated from the Koning-Delaroche global potential [11]. The angular distributions were measured for 13 elemental samples from Mg to Bi. The results are shown in the upper portion of Fig. 3, and the results using direct (*i.e.* unmodified) subtraction are shown in the lower portion. As seen in the original study of Pb [8], the results of the two methods are consistent (although just barely for Cr), but the uncertainties in the modified-subtraction results are significantly smaller.

There are now enough results from the modified subtraction technique to allow some criteria to be developed to obtain reliable measurements. The most important is that the extrapolation of the angular distribution to zero degrees should be stable. An example of a favorable case is shown in Fig. 4, for a 15.2 MeV angular distribution measured [13] on Fe. With increasing  $L$ , the maximum order of the Legendre polynomial fit, the  $\chi^2$  per degree of freedom has a sharp knee beyond which it is stable. At the same critical value of  $L$  (12 in this example), the value of

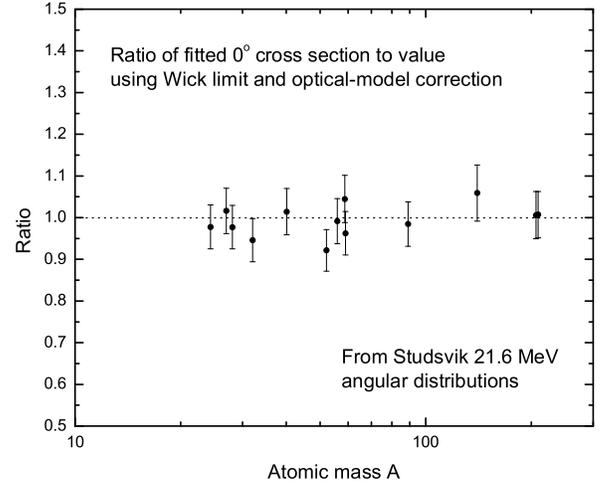


**Fig. 3.** Nonelastic cross sections derived from angular distributions of Ref. [12] at 21.6 MeV neutron energy. Top: Results from modified subtraction. Bottom: Results from direct subtraction. The line is a power-law fit to the results using modified subtraction.



**Fig. 4.**  $\chi^2$  per degree of freedom (crosses) and the factor  $F$  (solid circles) as a function of the maximum order of the Legendre fit to the angular distributions for 15.2 MeV neutrons on natural Fe [13].

$F$  stabilizes, and larger values of  $L$  only yield larger uncertainties for  $F$ . The critical  $L$  is roughly proportional to  $kR$ , the grazing angular momentum for the incident neutron. An empirical expression based on this relation can be used to predict the energy and mass dependence of the critical value of  $L$ .



**Fig. 5.** Ratio of zero-degree cross section from Legendre fits to value using Wick's limit (obtained from experimental total cross section), corrected by optical-model calculation.

A useful check on the validity of the extrapolated zero-degree cross section can be obtained by comparing it with an estimate of the zero-degree cross section determined from the total cross section and the Wick deviation  $\eta$ . This quantity is

$$\sigma_0^{est} = (1 + \eta) \left( \frac{k}{4\pi} \right)^2 \sigma_{tot}^2, \quad (8)$$

in which the factor  $1 + \eta$  is just the factor that converts the Wick limit to the true zero-degree cross section, calculated from an optical model. The ratio of the extrapolated zero-degree cross section to this estimate is shown in Fig. 5 for the 13 angular distributions in the Studsvik 21.6 MeV measurements. The ratios are close to unity, and the errors are dominated by the approximately 5% normalization uncertainty in the angular distribution measurements. We note that the low point in the  $A \approx 50$  region corresponds to Cr, which also showed a slight discrepancy in the reaction cross sections determined by the direct and modified subtraction methods. These results, together with the fact that the Cr angular distribution does not show a suitably well-developed a knee and subsequent flat behavior in the  $\chi^2$  and  $F$  values as a function of  $L$ , suggests that the modified-subtraction value of the nonelastic cross section for Cr may be questionable.

### 3 Summary and conclusions

In the present work we have continued a study of alternative methods to the sphere-transmission technique for determining nonelastic cross sections. These methods are the direct subtraction of the elastic from the total cross section, and a modification in which uncertainties are reduced by introducing Wick's limit to correlate the quantities being subtracted. We have applied these methods to a new set of data, based on a set of angular distribution measurements at 21.6 MeV neutron energy over a wide mass range. We

have also discussed tests and criteria that should be applied when employing the modified subtraction technique. Three of these are:

- The Legendre extrapolation of an angular distribution to zero degrees should be stable for polynomial orders above a critical value, as indicated in the discussion of Fig. 4.
- Since the two methods have very different sources of error, both should be employed and consistency of the results be demanded.
- The extrapolated zero-degree cross section should be consistent with the value obtained from the total cross section, using Wick's limit and an optical-model correction, as shown in Eq. 8.

In order for the last two of these tests to be useful, total cross sections should be used that have uncertainties much smaller than the normalization uncertainties of the angular distributions. It is also important that the model dependence of the modified subtraction technique be small; this means that the optical-model calculation of the Wick deviation  $\eta$  should be well determined; this issue is discussed in Ref. [8], which indicates the regions of neutron energy and target mass for which the technique is suitable.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

## References

1. M.H. MacGregor, W.P. Ball, and R. Booth, *Phys. Rev.* **108**, (1957) 726
2. W.P. Ball, M.H. MacGregor, and R. Booth, *Phys. Rev.* **110**, (1958) 1392
3. M.H. MacGregor, W.P. Ball, and R. Booth, *Phys. Rev.* **111**, (1958) 1155
4. M.H. MacGregor, R. Booth, and W.P. Ball, *Phys. Rev.* **130**, (1963) 1471
5. R.W. Finlay, W.P. Abfalterer, G. Fink, E. Montei, T. Adami, P.W. Lisowski, G.L. Morgan, and R.C. Haight, *Phys. Rev. C* **47**, (1993) 237
6. W.P. Abfalterer, F.B. Bateman, F.S. Dietrich, R.W. Finlay, R.C. Haight, and G.L. Morgan, *Phys. Rev. C* **63**, (2001) 044608
7. H.S. Camarda, T.W. Phillips, and R.W. White, *Phys. Rev. C* **29**, (1984) 2106
8. F.S. Dietrich, J.D. Anderson, R.W. Bauer, and S.M. Grimes, *Phys. Rev. C* **68**, (2003) 064608
9. F.S. Dietrich, J.D. Anderson, R.W. Bauer, and S.M. Grimes, *Proc. Int. Conf. on Nuclear Data for Science and Technology, Santa Fe, 26 Sept. - 1 Oct., 2004*, AIP Conf. Proceedings Vol. 769, R. Haight *et al.*, eds. (Amer. Inst. of Physics, Melville, New York 2005) 300
10. F.S. Dietrich, J.D. Anderson, R.W. Bauer, S.M. Grimes, and D.P. McNabb, *Nucl. Sci. Eng.* **159**, (2008) 213
11. A.J. Koning and J.-P. Delaroche, *Nucl. Phys.* **A713**, (2003) 231, 2003.
12. N. Olsson, B. Trostell, E. Ramström, B. Holmqvist, and F.S. Dietrich, *Nucl. Phys.* **A472**, (1987) 237
13. D. Schmidt, W. Mannhart, H. Klein, and R. Nolte, *Phys. Techn. Bundesanst. Neutronenphysik Reports No.* 20 (1994)