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Comparison of Joint Modeling Approaches Including Eulerian Sliding Interfaces

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Foreword

This paper describes a portion of the work to be presented by the author at the Workshop on Ground Shock in Faulted Media, McLean, VA, 12 January 2009

Introduction

Accurate representation of discontinuities such as joints and faults is a key ingredient for high fidelity modeling of shock propagation in geologic media. The following study was done to improve treatment of discontinuities (joints) in the Eulerian hydrocode GEODYN (Lomov and Liu 2005). Lagrangian methods with conforming meshes and explicit inclusion of joints in the geologic model are well suited for such an analysis. Unfortunately, current meshing tools are unable to automatically generate adequate hexahedral meshes for large numbers of irregular polyhedra. Another concern is that joint stiffness in such explicit computations requires significantly reduced time steps, with negative implications for both the efficiency and quality of the numerical solution. An alternative approach is to use non-conforming meshes and embed joint information into regular computational elements. However, once slip displacement on the joints become comparable to the zone size, Lagrangian (even non-conforming) meshes could suffer from tangling and decreased time step problems. The use of non-conforming meshes in an Eulerian solver may alleviate these difficulties and provide a viable numerical approach for modeling the effects of faults on the dynamic response of geologic materials.

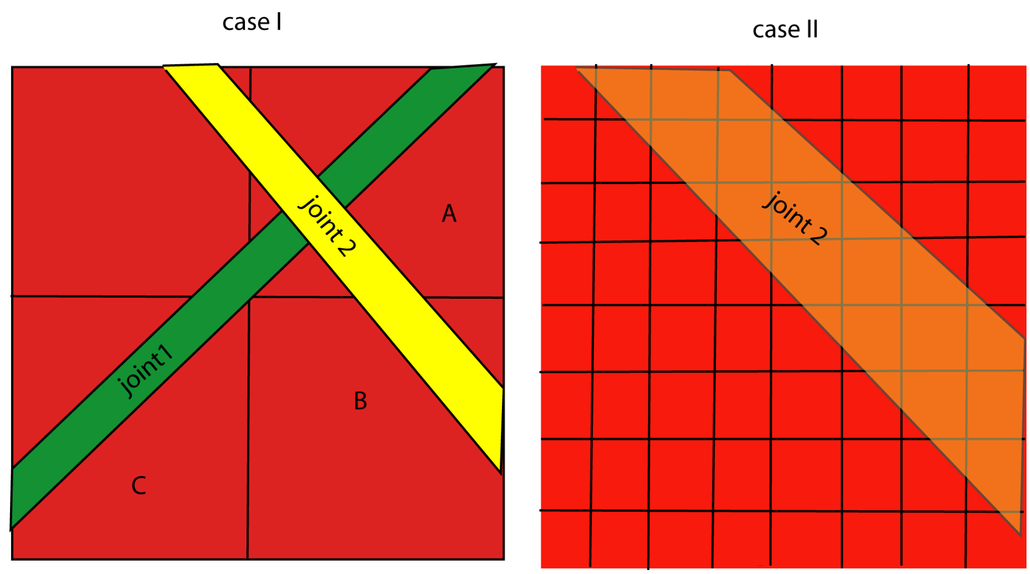

Figure .1 Possible cases for joints treatment:
case I – under-resolved joints, case II – resolved joint

We studied shock propagation in jointed/faulted media using a Lagrangian and two Eulerian approaches. To investigate the accuracy of this joint treatment the GEODYN calculations have been compared with results from the Lagrangian code GEODYN-L which uses an explicit treatment of joints via common plane contact. We explore two approaches to joint treatment in the code, one for joints with finite thickness and the other for tight joints. In all cases the sliding interfaces are tracked explicitly without homogenization or blending the joint and block response into an average response. In general, rock joints will introduce an increase in normal compliance in addition to a reduction in shear strength. In the present work we consider the limiting case of stiff discontinuities that only affect the shear strength of the material.

GEODYN Code

For Eulerian studies, we used GEODYN, a multi-material Eulerian Godunov shock physics code featuring material strength and adaptive mesh refinement (AMR) for numerical simulations. GEODYN can model large deformations of solids, capture shocks, and track material interfaces with piecewise linear interface reconstruction (PLIC). GEODYN has a flexible material library with analytic and tabular equations of state and a wide range of constitutive models. Adaptive mesh refinement can be used to resolve important features in simulations, like shock waves, material boundaries and major joints. Unfortunately, even with AMR usually it is impossible to satisfactorily resolve all relevant features of jointed medium.
Since the joint size typically is much smaller then the characteristic problem size, it is not always possible to allocate enough numerical cells to resolve the joint (see Fig.1 case I). Yet in GEODYN, it is possible to use AMR techniques to resolve the most important joints (see Fig1. case II). The goal of the present study is to evaluate how to model the main effects of joints on an Eulerian grid.

Joint treatment in GEODYN-L

GEODYN-L is a Lagrangian code where the discontinuities can be explicitly included as distinct contact elements. The following function of form is used for the normal modulus on the contact element:
Contacts are considered to be isotropic with a Coulomb friction law and a limited tensile strength. Thus the shear stress at the contact is limited by the yield surface dependent on the normal stress as

$$\sigma_{s\max} = C + \sigma_n \tan(\phi\ ) \tag{1}$$

where C is the shear cohesion and ϕ is the friction angle. For stiff, noncompliant joints the aperture will be small compared to the cell size.

More detailed description of GEODYN-L is gived in the paper in "Equivalent continuum modeling for shock wave propagation in jointed media" in the current proceedings.

Joint represented with gauge material

In the first Eulerian technique, we introduce a special fault material with prescribed properties. The fault is explicitly resolved using Eulerian adaptive mesh refinement, which makes it possible to resolve regions of interest to centimeter-size zones when calculating problems with dimensions of hundreds of meters. This approach has the advantage of accurately representing non-ideal faults that are filled with rock debris. Such detailed fault representation may be necessary near underground structures or tunnel cavities where the presence of the fault affect both wave propagation and structural response. The disadvantage of this approach is that it is computationally intensive requiring a minimum of three cells across the fault to achieve convergence. It is typically impossible to use this resolution on joints everywhere, even with AMR.The joints are initialized using their volume fractions described by the corresponding internal variables. These variables are advected at every time step.
In this approach each joint, *i,* is characterized by its volume fraction, f_i, material *id* and the internal angle of friction, ϕ_i. If the minimum and the maximum principal stresses are known, then

$$\mu = \sin(\phi) = \frac{\sigma_1 - \sigma_3}{(\sigma_1 + \sigma_3)} \tag{2}$$

In the case of triaxial loading the yield surface can be expressed as a linear function of pressure as

$$\begin{aligned} Y &= \alpha P \\ \alpha &= \frac{\partial Y}{\partial P} = \frac{6\mu}{3-\mu} \end{aligned} \tag{3}$$

If many joints intersect the cell (as it is shown in Fig1. case I for cell A), the effective slope of the yield surface, α_{eff} is found as

$$\alpha_{eff} = \frac{\sum f_i}{\sum \frac{f_i}{\alpha_i}} \tag{4}$$

The joint material is treated the same way as the material with corresponding *id* except for one additional step: After the new stress is found, it is checked for consistency with the yield surface for the joints. So, the new von Mises stress, σ_e should satisfy

$$\sigma_e < \alpha_{eff}\, p \tag{5}$$

where p is the average pressure in the cell. The radial return algorithm is used to reduce the deviatoric stress if this condition is violated.

The level set method

The second Eulerian technique treats joints as slip surfaces embedded in regular computational cells. The primary objective of this approach is to account for the effect of joints on wave profiles. In this case we assume that the joint thickness is always smaller then the cell size and use a level set method to track the fault surface. Lagrangian surface represented as the zero level set of a higher-dimensional scalar function ϕ (Osher and Sethian 1988):

$$\Gamma = \left\{ x \in \mathbf{R}^n : \phi(x) = 0 \right\} \tag{6}$$

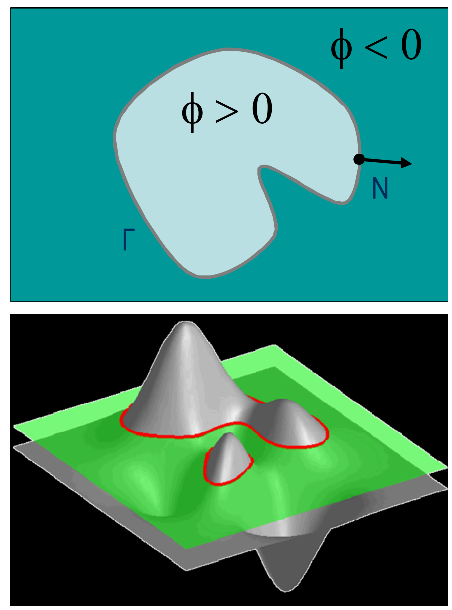

Figure 2. Level set representation of an interface.

In the simplest case ϕ can be initialized as a signed distance from a surface of interest (fault, joint). It is trivial to calculate a surface normal:

$$\mathbf{n} = \nabla\phi / \left\| \nabla\phi \right\|, \tag{7}$$

which can be later used for physics models, as well as to establish a link between motion of the surface and the evolution of the level set function. A Lagrangian ordinary differential equation can be related to an Eulerian partial differential equation on a cartesian grid:

$$\frac{d\Gamma}{d} = \mathbf{v} \qquad x \qquad \Leftrightarrow \qquad \frac{\partial\phi}{\partial t} + \nabla\phi \cdot \mathbf{v} = 0 \tag{8}$$

Later the level set approach has been extended to vector level sets. Transport of several auxiliary quantities can allow to track more information on Eulerian grid. Let ψ be the level set function of an auxiliary surface, than we advect two functions:

$$
\begin{aligned}
&\frac{\partial \phi}{\partial t} + \nabla \phi \cdot \mathbf{v} = 0, \\
&\frac{\partial \psi}{\partial t} + \nabla \psi \cdot \mathbf{v} = 0
\end{aligned} \tag{9}
$$

This approach allowed to simulate 3D curves with level sets (Burchard, Cheng et al. 2001), which can be used to model reinforcements:

$$\{\phi = 0\} \cap \{\psi = 0\} \tag{10}$$

The vector level sets are also necessary for representing non-persistent surfaces or joints restricted to a specific domain. In this case we define a surface of interest the following way:

$$\{\phi = 0\} \cap \{\psi \le 0\} \tag{11}$$

The vector level sets have been applied for region tracking (Bertalmio, Sapiro et al. 1999) and for modeling open surfaces (Solem and Heyden 2004).
On the other hand, a single level set function can represent a whole family of similar non-intersecting joints:

$$\Gamma_i = \left\{ x \in \mathbf{R}^n : \phi(x) = i\alpha \right\} \tag{12}$$

The important advantages of the level set treatment over the "gauge material" approach are that the fault does not "diffuse" numerically during advection and it is simple to calculate and use surface normal and surface area fraction of a joint in a cell. First, we calculate a parameter which defines that a unit cell contains the surface, in two dimensional case it is:

$$d = \left(\left(|n_x| + |n_y| \right) - \phi \right) \Big/ 2 \tag{13}$$

If $d > 0$, the surface intersects the cell and the normalized surface area within the cell is calculated as

$$f = 1 \Big/ \max\left(1, \min\left(|n_x|, |n_y| \right) \big/ d \right) \tag{14}$$

Three dimensional formulas are more complicated, but are still pretty straightforward.
Directional joint plasticity model (1) is applied to the stress state after the regular update in each cell that contains an interface. For tests throughout this paper we used cohesionless joint model with the Coulomb friction law:

$$
\begin{aligned}
&\mathbf{n} \cdot \mathbf{T}_j \cdot \mathbf{n} \le 0, \\
&\left| \mathbf{T}_j \cdot \mathbf{n} - \left(\mathbf{n} \cdot \mathbf{T}_j \cdot \mathbf{n} \right) \mathbf{n} \right| \le \tan\alpha \left| \mathbf{n}\, \mathbf{T}_j \cdot \mathbf{n} \right|
\end{aligned} \tag{15}
$$

The final stress $\mathbf{T}$ in a cell is a weighted average of intact stress $\mathbf{T}_i$ and modified stress $\mathbf{T}_j$

$$\mathbf{T} = (1 - f)\,\mathbf{T}_i + f\mathbf{T}_j \tag{16}$$

This approach allows to use joint models which relieve stresses based on flow rules. More advanced material models which require modifications of a free energy function (for hyperelastic formulations) or strain-stress dependencies (for hypoelastic formulations) are more difficult to implement in the level set framework.

Test 1. Impact of a square block with a diagonal joint onto a rigid wall

This problem represents a case of joint orientation where the joint is not aligned with the Cartesian mesh. At the same time this problem is simple enough for the verification. The impact velocity was 100 m/s along the Y axis. The rigid wall boundary was set at the top of the mesh (see Fig. 3).The symmetry boundaries are set from the sides. Elastic material with non-compliant cohesionless joint has been used in this study.
A more accurate representation of the joint is provided by the Lagrangian code where the joint was aligned with the mesh boundary (see Fig.3b). The joint aperture, *a*, was 0.001 m to approximate a very stiff (practically incompressible) joint.

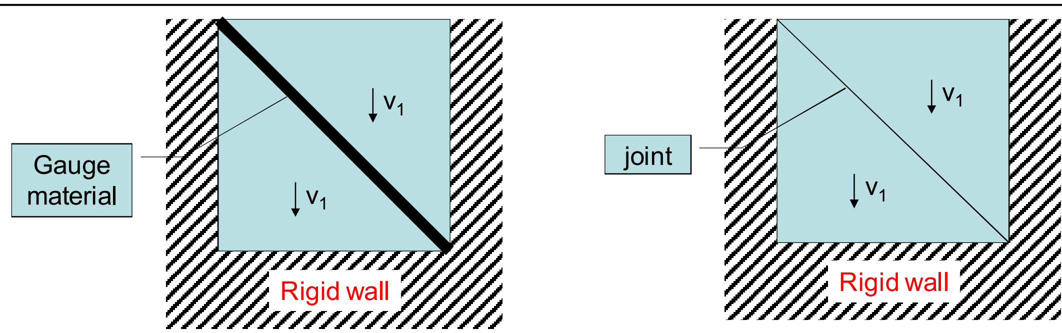

Figure 3.Test cases
a) Joint representation with gauge material
b) Level set joint and Lagrangian representation

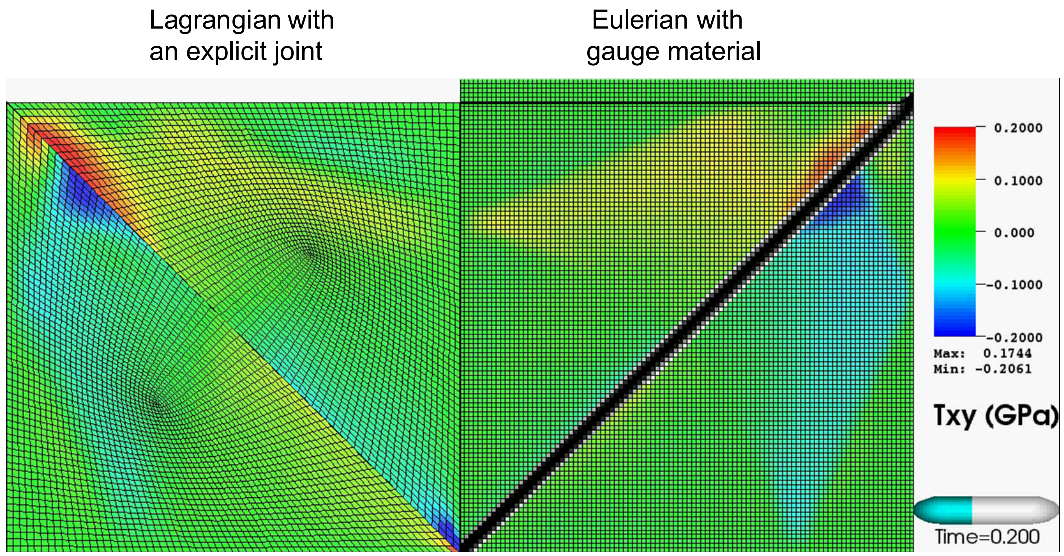

Figure 4. Eulerian joint represented with gauge material vs Lagrange: Txy stress component comparison

It was found that the response of the joint in the Eulerian representation with gauge material (see Fig.3a) was mesh sensitive if the joint is under-resolved. Figure 4 shows the calculated stress field in the problem and corresponding snaphot from a Lagrangian simulation. The frictional coefficient used in these calculations was $\alpha = 11°$. The thinner the gauge layer the more cells are needed for the converged solution. There should be more than 3 cells across the joint thickness in order to get the converged solution. Under-resolved joints tend exhibit more friction than the resolved ones. For very coarse zoning there is no slip between block at all.

Figure 5 shows quantitative comparison between the two codes. It is clear Eulerian model with 3 cells of gauge material gives good agreement with Lagrangian joint model. There is a substantial difference in a waveform at 1.5 cell wide joint and no slip at all for 0.75 cells per joint thickness. Since the codes use different models for the joints, the results may differ even if the same internal friction angle is used in both codes. The best agreement can be obtained for quasi hydrostatic loading where the difference between the

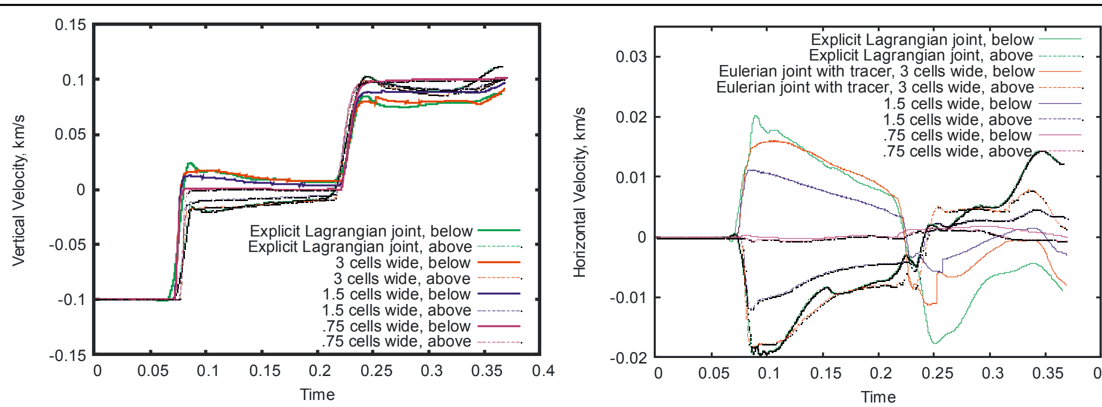

Figure 5. Wedge Test: Results with gauge material joint

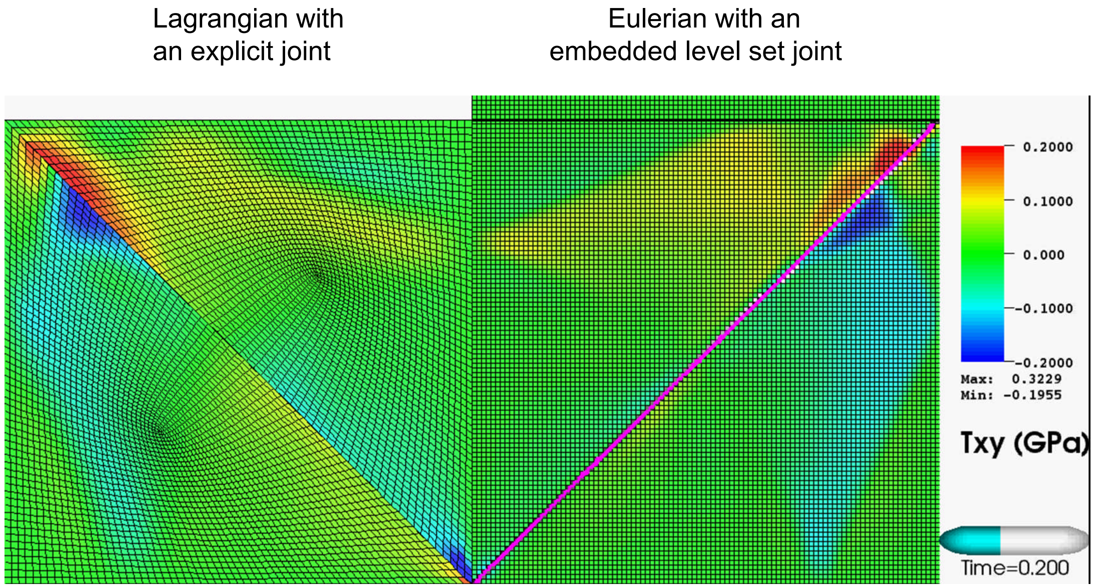

Figure 6. Eulerian level set vs. Lagrange:
Txy stress component comparison

pressure and the normal stress at the joint is minimal. Another concern is that it may not be possible or practical to have enough resolution in real world problems

Figure 6 shows a similar comparison between Lagrangian and Eulerian level set formulation. We can see that results are much closer to each other that in in the former comparison. In fact, most of the differences can be attributed to not to joint response but to differences of numerical scheme for free field wave propagation. Velocity records (Fig. 7) show almost perfect match between Lagrangian and level set profiles. The biggest difference comes at late time, when it is necessary to describe separation of the block on rebound. In this case single-valued velocity and stress fields may not be appropriate and have to be extended similar to XFEM techniques.

Figure 8 shows stress path and displacement on the joint in the simulations. We can see that stress strictly follows prescribed Coulomb friction law and joint displacement tracks Lagrangian simulation with a large plastic component.

This test provide verification that an Eulerian approach with an appropriate technique

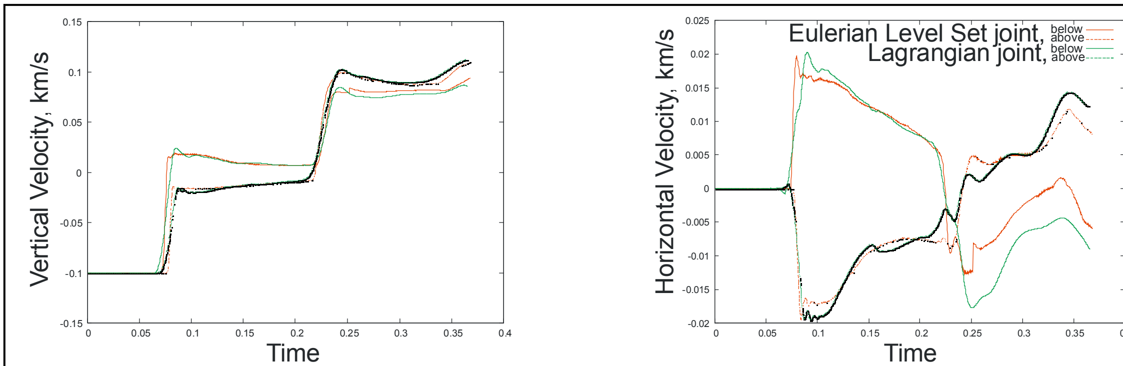

Figure 7. Eulerian level set results agree with Lagrange

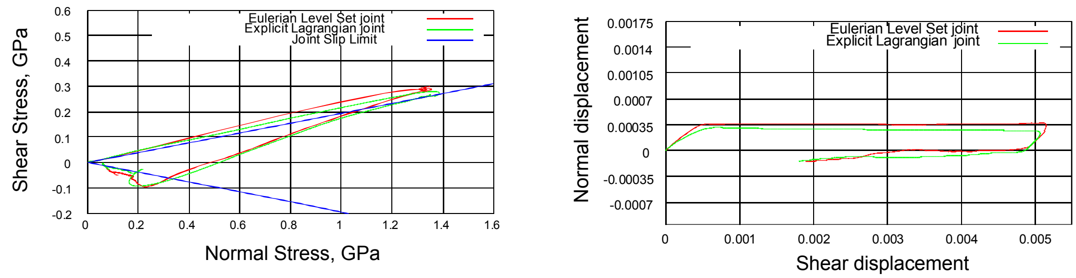

Figure 8. Stress path and displacement on the joint for the wedge problem.

(level set in this study) to track sharp discontinuities and provide surface normals for material models is on par with Lagrangian approach for wave propagation problems. Problems with significant separation of material blocks may require more complex formulation in Eulerian case.

Test 2. Comparison to a SRI experiment in a jointed limestone

To validate the modeling approaches for real geologic materials, we performed simulations of two sets of spherical wave experiments (Gefken and Florence 1993) performed at SRI with Salem limestone were simulated (one with a joint and the other one without). Radial velocity (along the joint surface) at various ranges was measured in these experiments as shown in Fig.9.I. The spherical charge consisted of 3/8 g of PETN explosive initiated at its center. A single joint placed 2.6 cm away from the charge was studied. The set has been simulated in 2D axisymmetric formulation (Fig.9.II). Two computational domains interacting though a contact boundary were used in Lagrangian calculations with a common plane approach. No softening and a zero cohesion were

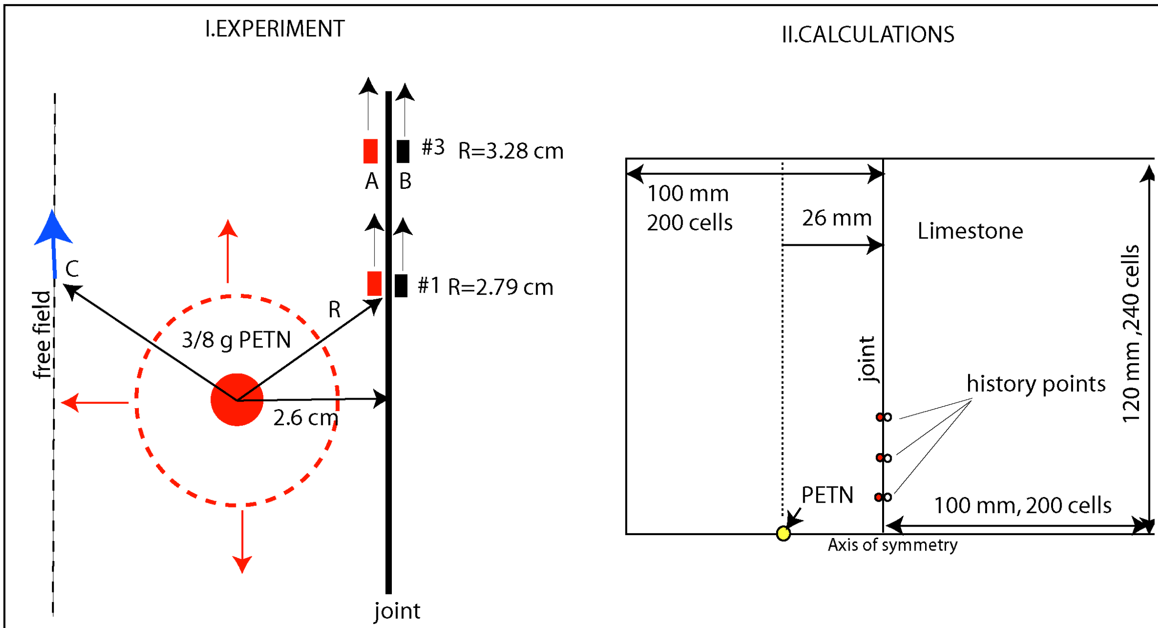

Figure 9. Experiment set-up (I) and calculation set-up (II). Radial velocities were measured before the joint (location A), after the joint (location B) and at the same range in the free field (location C)

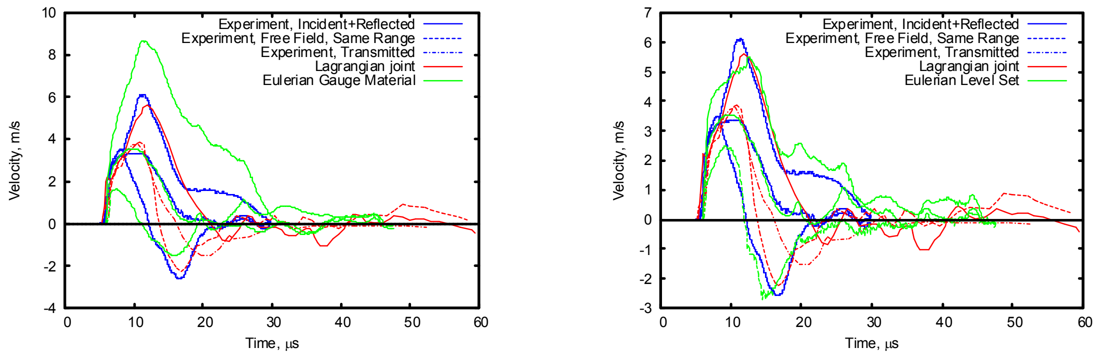

Figure 10. Measured and calculated radial velocity histories at a range R=2.79 cm..

assumed in the calculations. The joint aperture size of 0.1 mm was used. Parametric study showed a weak effect of the aperture size on the calculation results. Eulerian studies have been performed with the gauge material and the level set approaches. Figures 10, 11, 12 show comparisons of the experimental records, Lagrangian simulation and two Eulerian approaches, with gauge material on the left and with the level set method on the right. Experiments show that at range 2.79 cm all three velocities (the incident, the transmitted and the free-field velocity) are close until the sliding on the joint is activated.

The onset of sliding is mainly controlled by the joint friction. This test is more difficult to analyze than Test 1 since a fairly complicated material model is used for intact material (Vorobiev, Liu et al. 2007) and even free field profiles do not fit experimental data perfectly. Despite of this, all simulations show the correct qualitative behavior on a joint. As expected, Eulerian results with a gauge material are furthest away from the experimental data. Changing the friction angle cannot improve this fit, since it either significantly overpredicts reflected wave or produce a wrong delay for sliding on the joint. The results shown used prescribed friction angle, and could not be improved much beyond shown on Fig. 10-12 by varying this parameter. On the other side, level set and Lagrangian results are much closer to experimental data and it is arguable which results

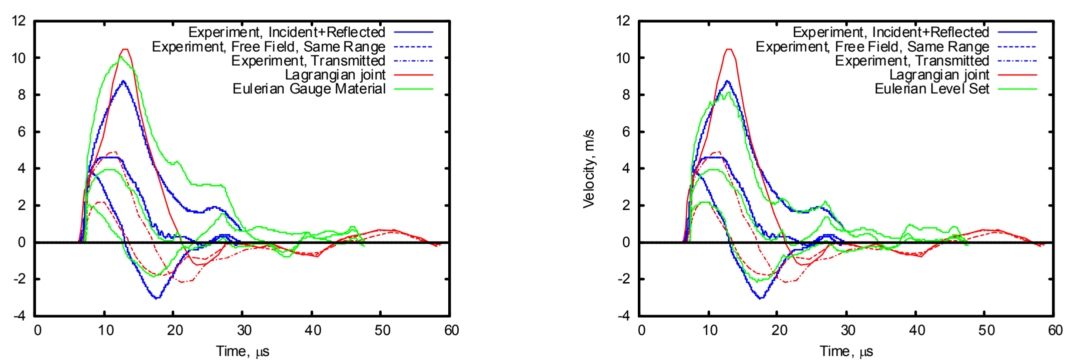

Figure 11. Measured and calculated radial velocity histories at a range R=3.28cm

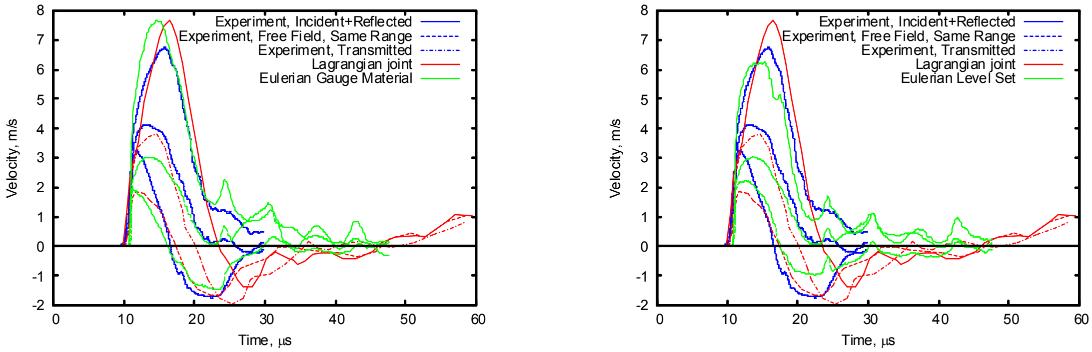

Figure 12. Measured and calculated radial velocity histories at a range R= 4.77cm

are closer to the experiment. In fact, parametric studies show that switching between different joint models (from linear Coulomb law to nonlinear) can produce much larger differences and get us closer to data than differences between Lagrangian and Eulerian approach. Usually very limited datasets are available to characterize jointed rock, so these uncertainties will overwhelm differences and inaccuracies which would come from an Eulerian representation of joints in the faulted medium.

Test 3. Surface explosion in a jointed rock mass

To demonstrate flexibility of the level set approach we ran a simplified simulation of

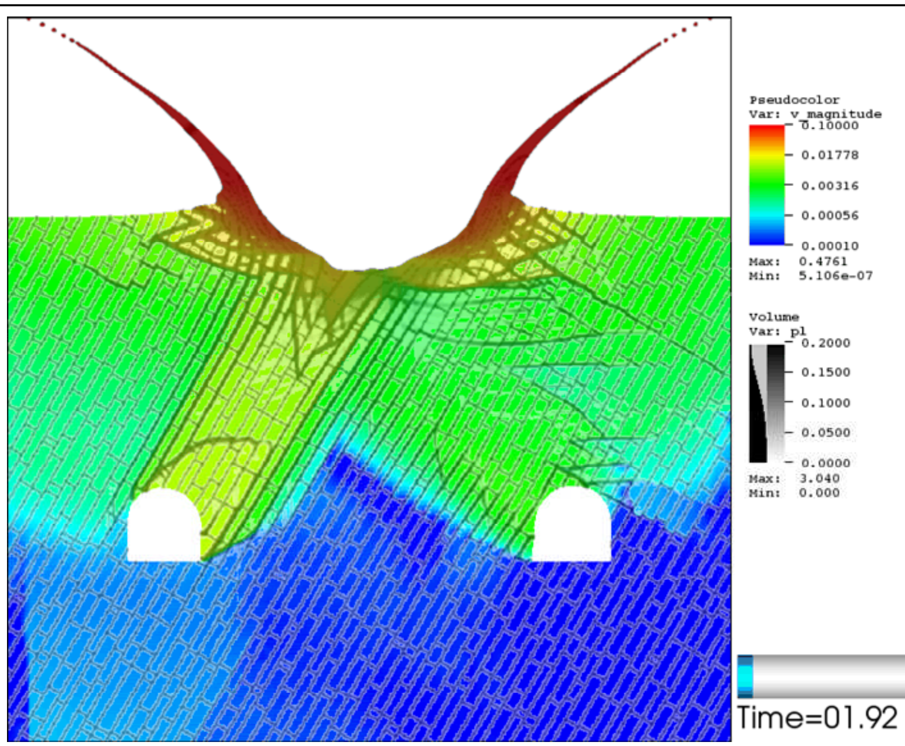

Figure 13. Surface explosion in a jointed rock mass

defeat of an underground structure in geologic medium with nonpersistent joints. Two joint families (each consist of many joints) have been represented only with two scalar level set fields. Since level set functions are inferently smooth, no significant advection errors occur even in the regions with large displacements and deformations. With this approach it is possible to resolve layers with less then 8 cells between joints. For real world problems which can be solved on current computer hardware this translates to ability to resolve joint spacing down to few inches. Figure 13 depicts a velocity field in Test 3. We can identify typical features inherent to wave propagation problems in jointed media: waves mainly propagate in directions of joint normals; damage happens preferentially along joints, but some blocks can break; once "slide lines" are established, underground structures are collapsed by sliding block along those slide paths.

Conclusions and future work

We cross compared results of the Eulerian and Lagrangian approaches for simple test problems and evaluate results against experimental data for explosively driven shocks across discontinuities in a geologic medium. This analysis demonstrate advantages and drawbacks and identifies domains of applicability of each modeling approach. The simple "gauge material" algorithm implemented in GEODYN for joint treatment is in good agreement with more accurate Lagrangian treatment available in Geodyn-L code, provided more than 3 cells span the joint. The Eulerian level set approach is shown to be in excellent agreement with explicit Lagrangian methods for wave propagation problems. Moreover, simulation with embedded joints can reproduce results of explicit Lagrangian contact algorithm without extra constraints on the time step. When it comes to very stiff noncompliant joints, the Eulerian treatment of joints implemented in GEODYN offers clear advantage in comparison to the explicit lagrangian treatment, because in the later case the time step is strongly limited by the joint stiffness. However, the approach is not suitable for calculating discrete blocks and rubble motion where large joint displacements (larger than the cell size) may violate the inherent assumptions of the level set fault modeling approach.

Extended finite element method (XFEM) formulation is developing framework with multivalued stress and velocity values in a zone. While it seems excessive for wave propagation problems, it can be natural way forward to extend Eulerian approach for rubble motion and tunnel collapse problems.

We demonstrated that the level set method is a viable approach to model Lagrangian features in an Eulerian code. Nevertheless this is not the only method to do that. Particle-based methods also have a great potential. Particle tracking is a challenging algorithmic problem, especially coupled with adaptive mesh refinement methods. On the other hand, it is free from some limitations which the level set method can encounter down the line (i.e. compliant joint) Future efforts will also focus on development of optimal parameterization algorithms relating level set fault parameters to measured joint properties.

More work is required to develop a transversely isotropic continuum model with embedded compliant joint. Advanced material model for a cell crossing the joint can be build incrementally, starting with the simplest approach (good for non-compliant joint,used in this study):

1) Calculate the joint surface area and a normal in a cell based on the levelset function

2) Evaluate standard constitutive model for continuum
3) Evalualte transversely isotropic response for joint
With a relatively small effort it can be converted to an Iterative approach, where we iterate between 2) and 3) to come up with a consistent stress state. The ultimate goal for Eulerian technique is a unified model for compliant and non-compliant joints which uses joint surface area explicitly and calibrated with iterative approach
Future work should also focus on combining the "gauge material" and the "level set" approaches, thus making it possible to treat joints with thicknesses larger and smaller then the zone size in a single simulation. A hybrid approach (gauge material in highly resolved regions / level set in coarse region) coupled with adaptive mesh refinement can use best of both methods

Acknowledgements

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