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Modeling of the LIFE minichamber theta pinch experiment

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Modeling of the LIFE minichamber theta pinch experiment

Anomalous Absorption 2010

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Loosmore, Jeff Latkowski, Greg Moses

LIFE minichamber theta pinch experiment

In the LIFE fusion chamber, Xe buffer gas protects the chamber wall from X-rays and debris produced by implosion of the fuel capsules (15 Hz rep. rate.) The Xe cools radiatively between shots and its conditions affect target survival and drive beam propagation. Xe atomic physics is not well known in this regime — 4 $\mu\text{g}/\text{cc}$ and 0.1 to 10 eV.

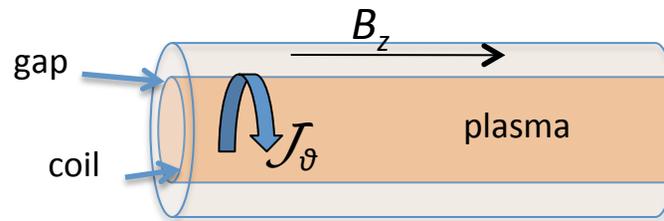
Goal of minichamber effort: heat a few cm^3 of Xe gas, observe how it cools to below 1 eV, and how beam propagation is affected.

Approach: we are building a magnetically driven theta pinch, which may permit sustained Joule heating of the Xe over hundreds of ns and balance radiative losses. Initially, Thomson scattering will be used to measure n_e and T_e .

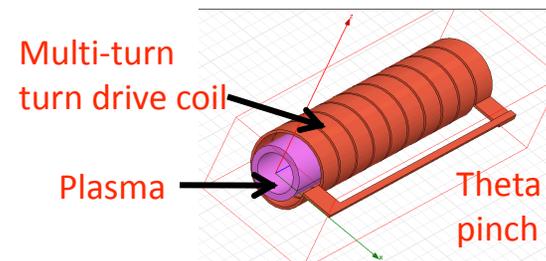
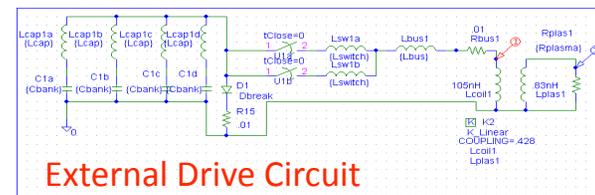
Current modeling: we are using

- the EM code Maxwell to design external circuit and drive
- the MHD code HYDRA to predict the plasma response

A simple 1D model of the theta pinch is coaxial infinite-length solenoids



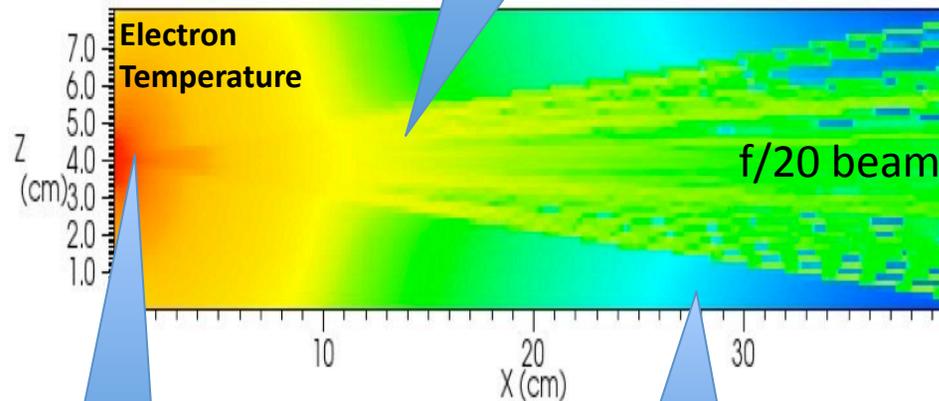
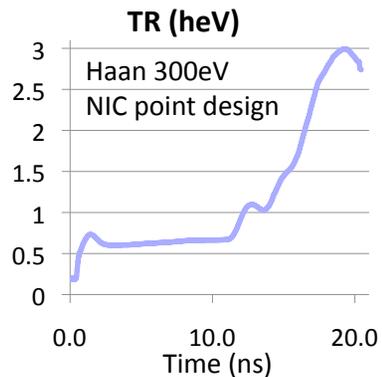
The coil current produces a solenoidal field B_z that diffuses into the Xe plasma, producing an azimuthal plasma current



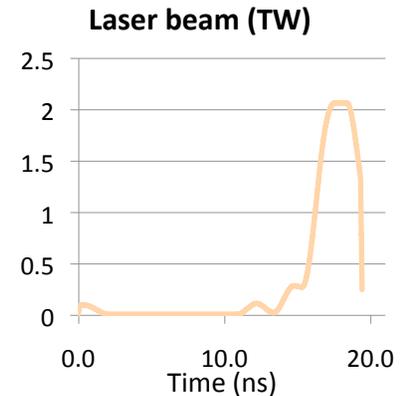
NIF beams propagate through 500 cm of vacuum;
LIFE beams propagate through 500 cm of ~ 0.5 torr Xe gas/cold plasma en route to TCC

- What is the maximum density of Xe allowable along the beam path?
 - minimize power loss
 - minimize beam deflection (pointing) and aberration (spot quality)

During the 20ns ignition pulse, the hohlraum LEHs leak soft X-rays that ionize Xe over 10s of cm



The laser can couple to the Xe plasma through inverse Bremsstrahlung absorption



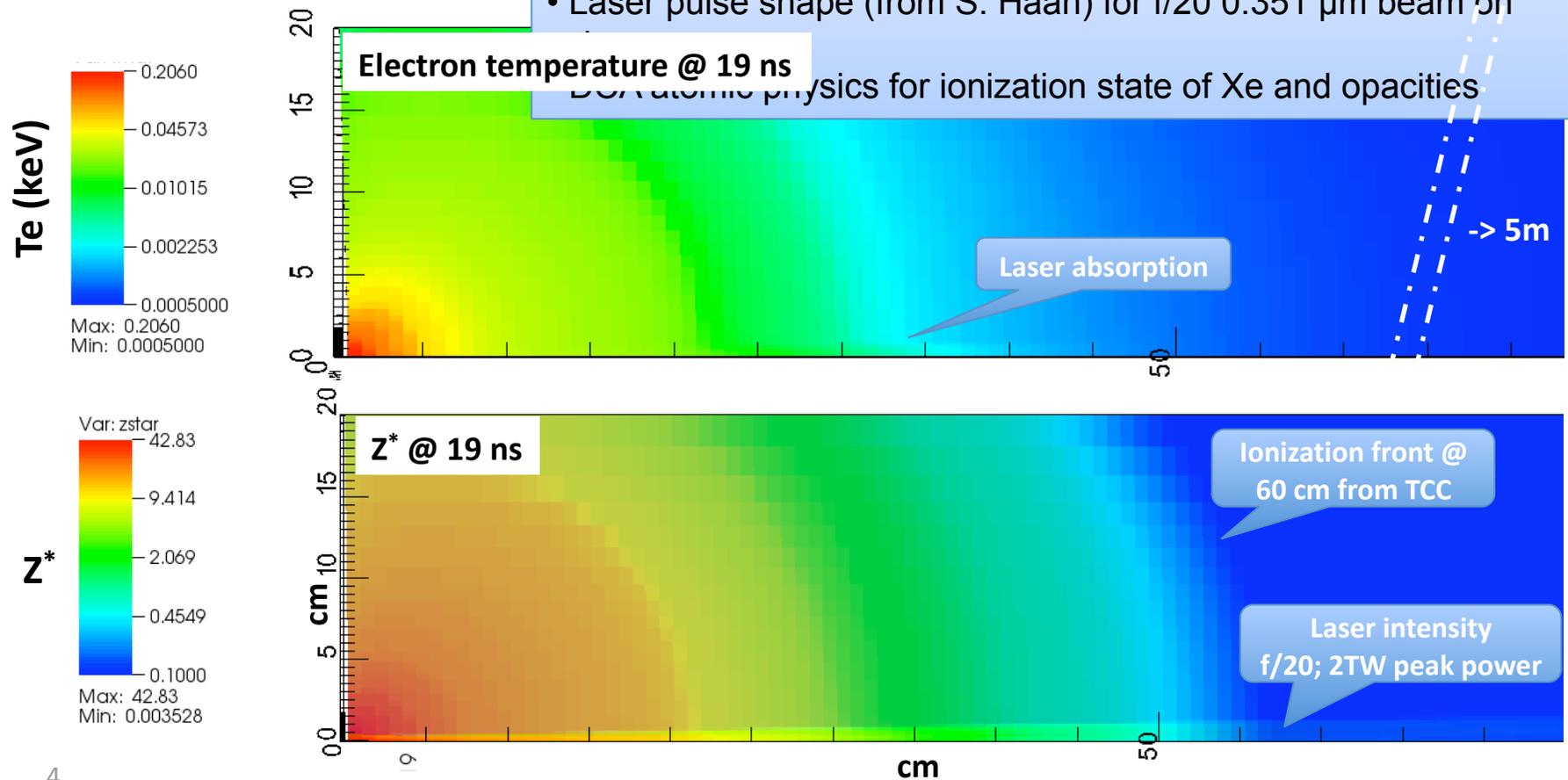
Radiation source (Hohlraum)

Ionization front

Integrated simulation of a laser drive beam through the target chamber predicts negligible coupling at 0.351 μm

Highest risk is at the end of the pulse (19 ns), when the Xe plasma extent/density is largest

- start with **hot Xe gas 4 $\mu\text{g}/\text{cc}$ @ 0.5 eV** (see Target Chamber talk)
- HYDRA simulation uses 300eV NIC point design radiation source
- Laser pulse shape (from S. Haan) for f/20 0.351 μm beam on



Inverse Bremsstrahlung absorption of 0.351 μm light leads to negligible power loss through 5 m of Xe @ 4 $\mu\text{g}/\text{cc}$

Inverse Bremsstrahlung absorption in low density-low temperature plasmas requires a careful treatment

$$\kappa_{IB} = \left[\left(1 - e^{-\frac{\hbar\omega}{kT}} \right) \frac{kT}{\hbar\omega} \right]$$

Less net absorption when photons are more energetic than electrons (0.351 μm = 3.75 eV)

$$\times \frac{N_e N_i Z^2 r_0^2}{N_c} \frac{4\sqrt{2\pi}}{3} \left(\frac{mc^2}{kT} \right)^{3/2}$$

$$\times \ln \Lambda(Z, T, \omega)$$

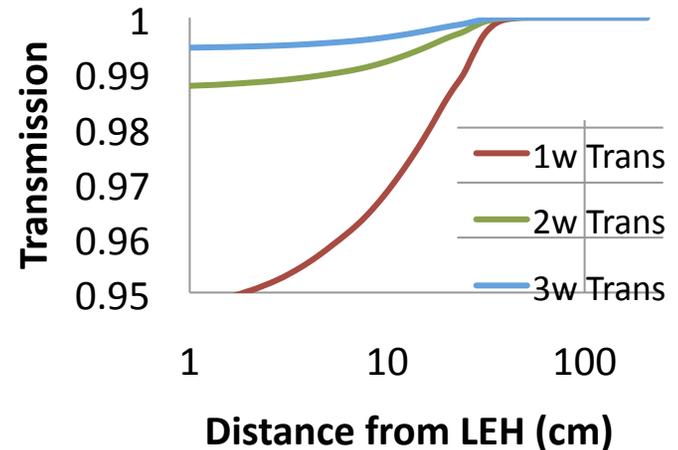
Needs DCA to get correct ionization at low temperature

Correct formula for high frequency/low temperature: Sommerfeld/Dawson

Textbook formula (or HYDRA) tend to overestimate absorption at low temp by 2-5x

Loss by I.B. is calculated by post-processing HYDRA rad-hydro simulations

Xenon @ 19 nsec
4 $\mu\text{g}/\text{cc}$, Te = 0.5 eV, Tr = 300 eV

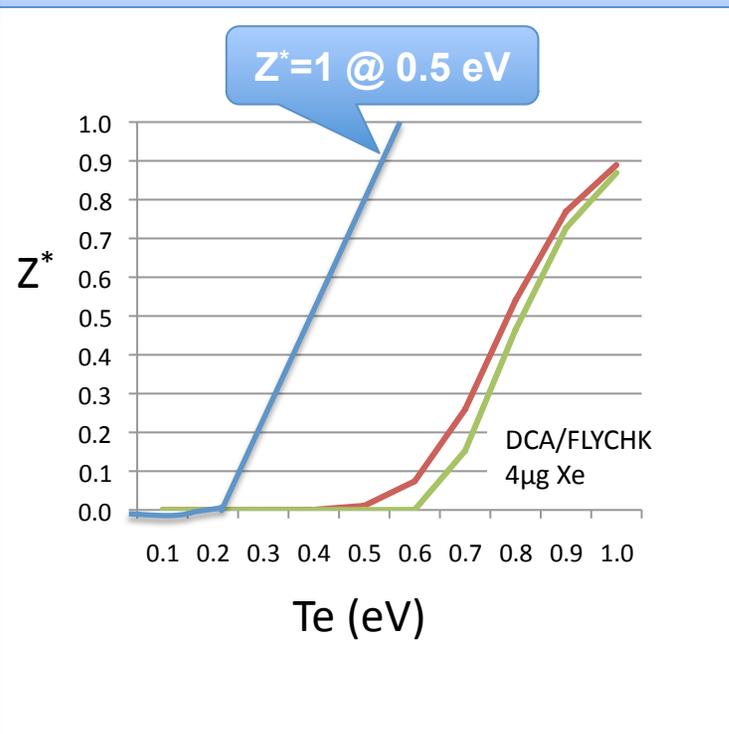


Only the last 60 cm is ionized Xe and absorption at 0.351 μm (3 ω) is 0.5%

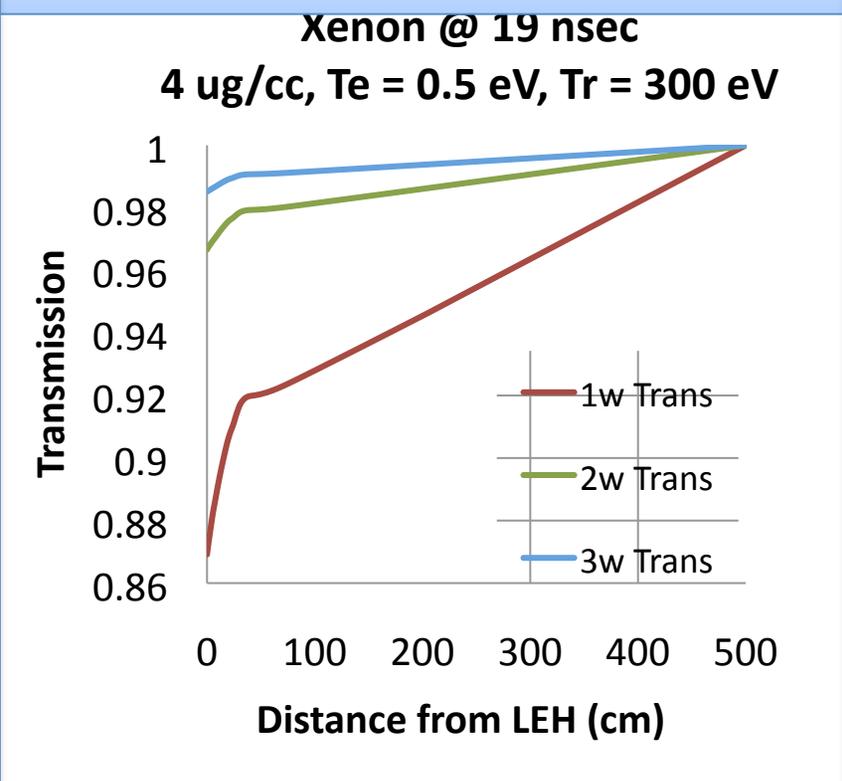
What if we have 5 m of Xe plasma ($Z=1$) @ 0.5 eV instead of gas?
 (i.e. atomic physics and radiative cooling are wrong)

Assume electrons decouple from ions, radiative cooling does not occur as expected and DCA is off by 2x

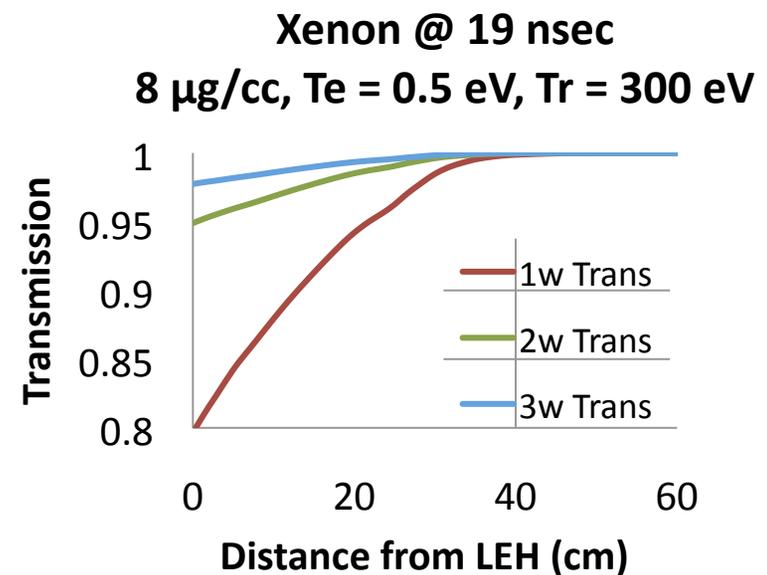
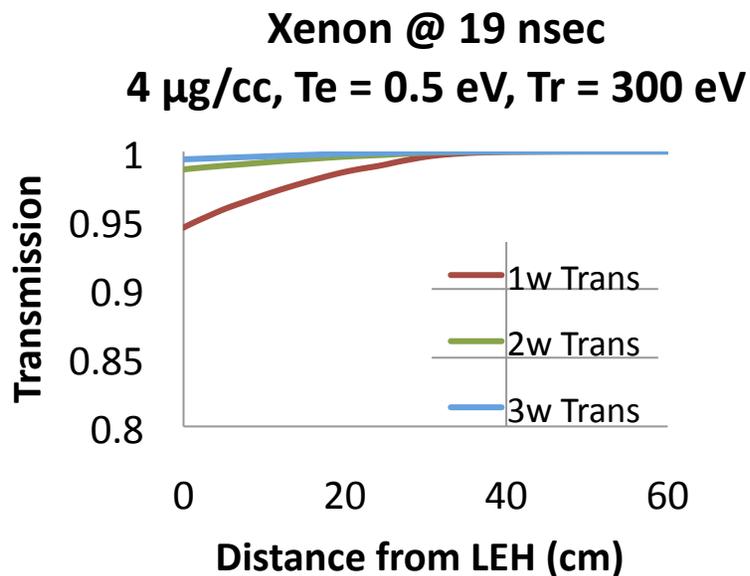
2x



Only 1.5% loss for 0.351 μm light and 4 $\mu\text{g}/\text{cc}$ Xe if the entire target chamber is ionized



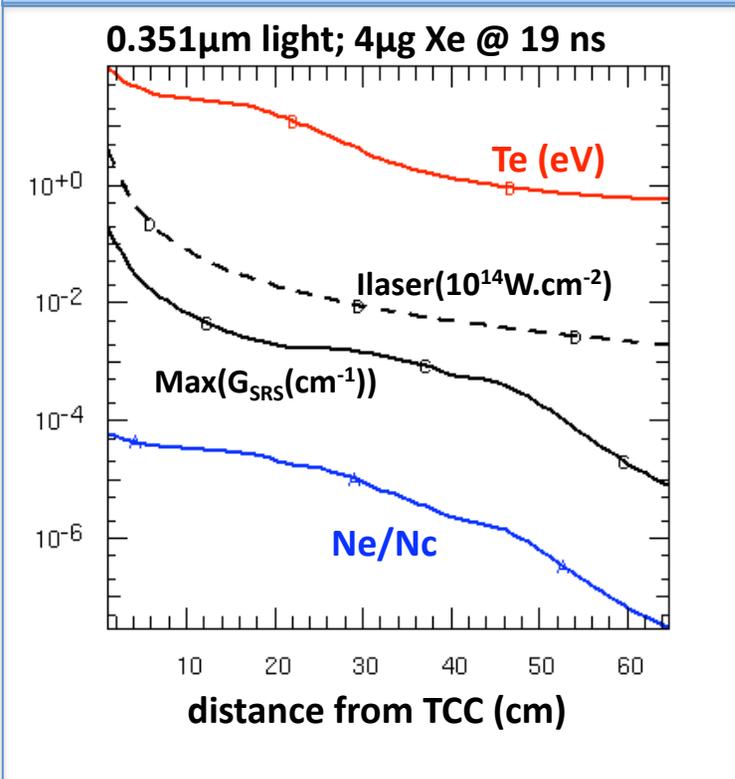
Doubling the Xe density to 8 $\mu\text{g}/\text{cc}$ leads to 2% power loss for 0.351 μm light through the target chamber



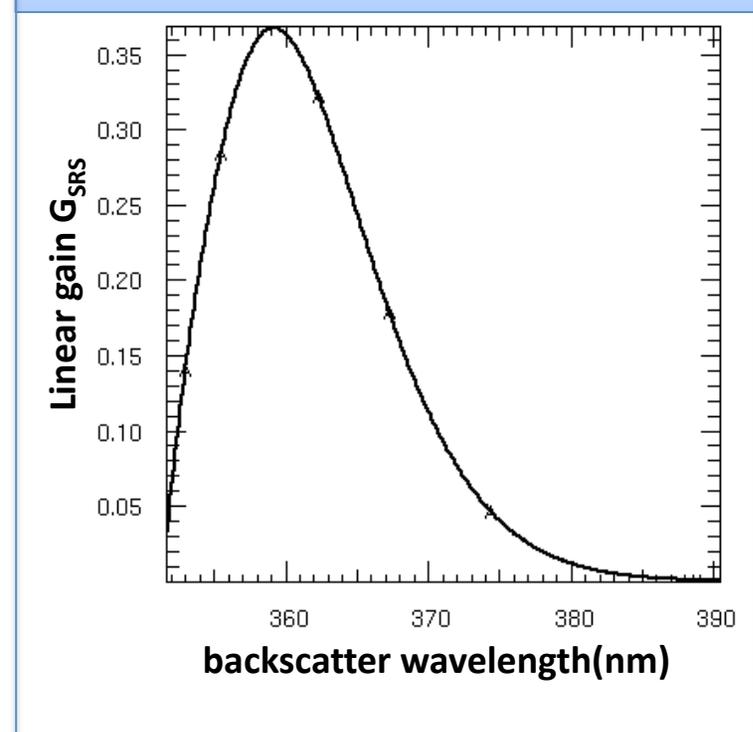
But the loss is 20% for 1 μm light...(Petawatt laser for Fast Ignition)

LPI: SRS/SBS gain calculated with HYDRA+LIP is < 1
No backscatter should occur

Low electron density and laser intensity keep Laser-plasma instabilities below threshold

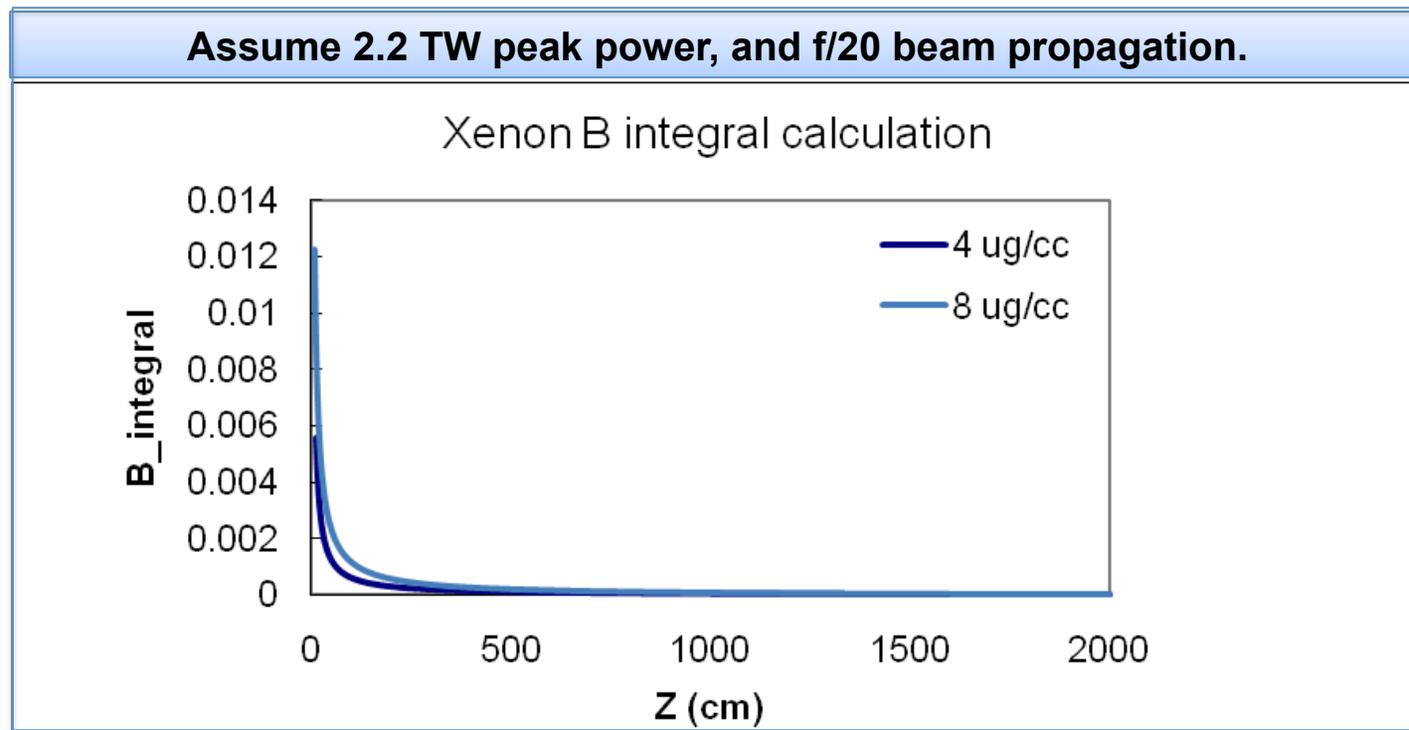


SRS peak gain is only shifted by 9 nm due to the very low Xe plasma density



B-integral looks to be negligible for parameters of interest.

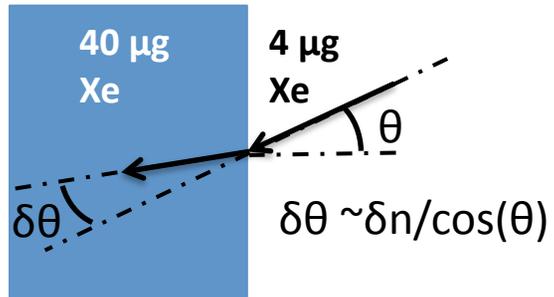
$$B = \frac{2\pi}{\lambda} \int_0^L n_2 I(z) dz \quad \text{is the total on-axis nonlinear phase shift accumulated}$$



When B-integral > 1 radian, there is a problem: we are well below that.

a cold jet of Xe @ 40 $\mu\text{g}/\text{cc}$ in the beam path leads to 30 μm mispointing at TCC in the worst case

laser beam crosses a localized cold jet 5m from TCC



Gas: $n(\text{Xe}, 4\mu\text{g})_{3\text{w}} = 1 + 0.5 \cdot 10^{-6}$

$\delta n = 4.5 \cdot 10^{-6}$

$\Delta x = 5 \text{ m} \times \delta\theta \sim 30 \mu\text{m}$

Plasma: $n(\text{Xe}^+, 4\mu\text{g})_{3\text{w}} = 1 - 10^{-6}$

$\Delta x = 50 \text{ cm} \times \delta\theta \sim 6 \mu\text{m}$

Laser beam rides along a density gradient for 5m



$\delta\theta \sim \delta n / f\#$

$\Delta x = 5 \text{ m} \times \delta\theta \sim 1.5 \mu\text{m}$

This can be easily corrected with adaptive optics (pointing & tracking)

Experiments should verify our assumptions about Xe atomic physics @ 1 eV and laser propagation in neutral Xe

A theta-pinch could produce 1-3 eV plasma: vary density and temperature

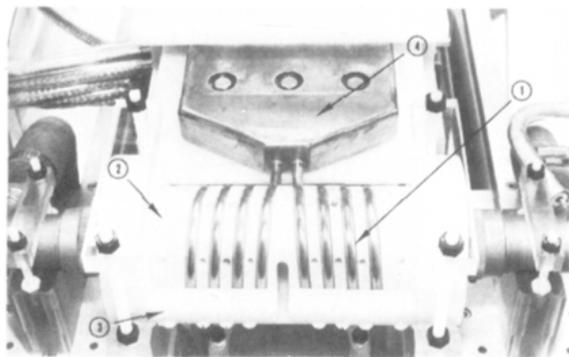
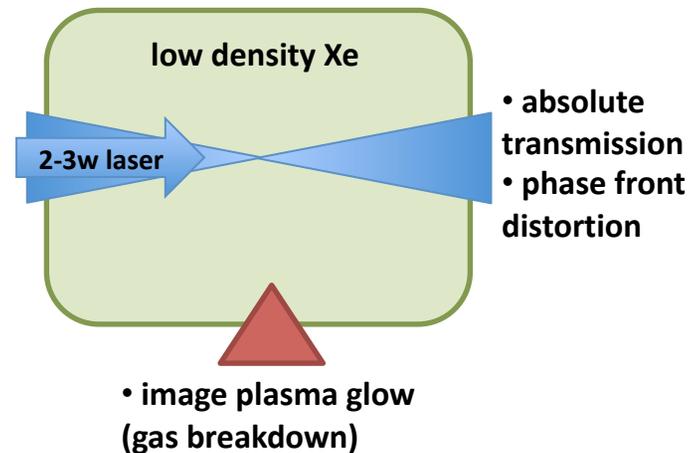


Figure 2. Helical theta coil assembly: (1) helical winding; (2) nylon winding form; (3) nylon clamp (lower half); (4) coil/upper transmission plate transition.

G. Kamis and A. Scheeline, Anal. Chem. 1986

measure Z^* ; study cooling/recombination of Xe^+ into Xe gas

Laser propagation through 1 meter of Xe gas: vary density, intensity, mixture

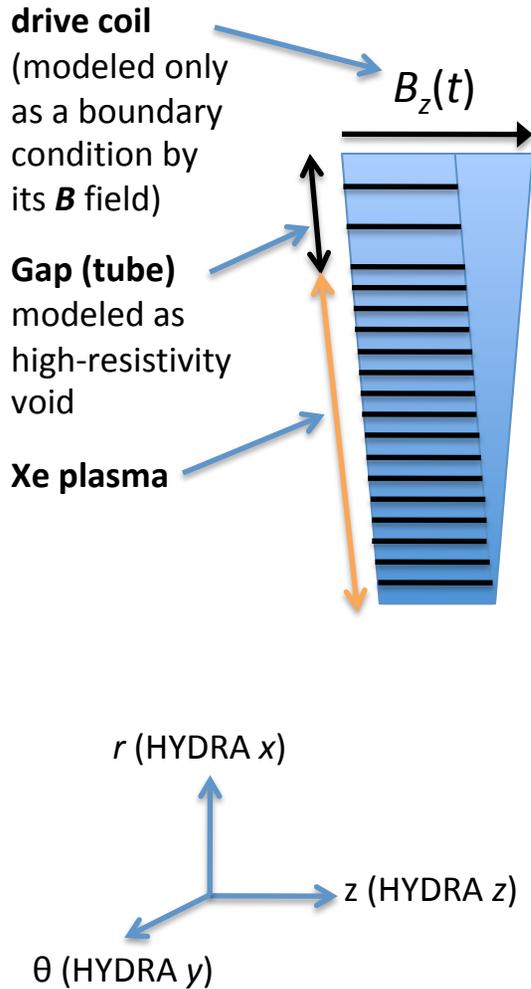


Quantify laser induced breakdown; effect of impurities; absorption

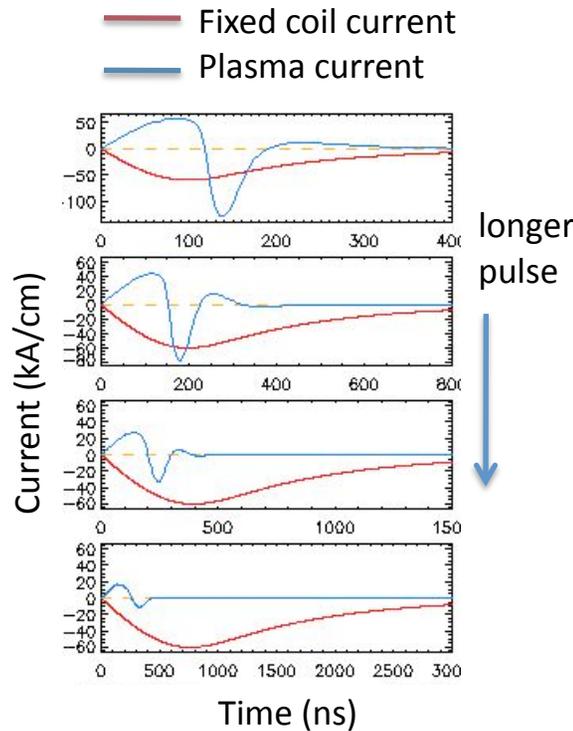
Beam propagation through a low-density-Xe-filled target chamber appears to not be an issue for 0.351 μ m light

- Only the last 60 cm of beam path is ionized and calculated laser absorption by inverse Bremsstrahlung is 0.5% @ 4 μ g/cc Xe and 2% @ 8 μ g/cc Xe
- Electron density is so low ($\sim 2 \cdot 10^{-5}$ critical for Z=1) that linear (refraction) and nonlinear (B-integral and laser-plasma instabilities) laser-plasma effects are negligible
- Problems could arise for 1 μ m light if Xe density has to be increased to 8 μ g/cc (20% absorption)
- We would be fools not to check our modeling/key assumptions in the laboratory:
 - atomic physics of cooling Xe plasma at a few eV
 - laser propagation through meters of Xe gas at low pressures (~ 1 torr) and increasing intensity
- We have not considered the last centimeter of propagation so far (i.e. what is the impact of the Xe background on LPI near the hohlraum LEH)

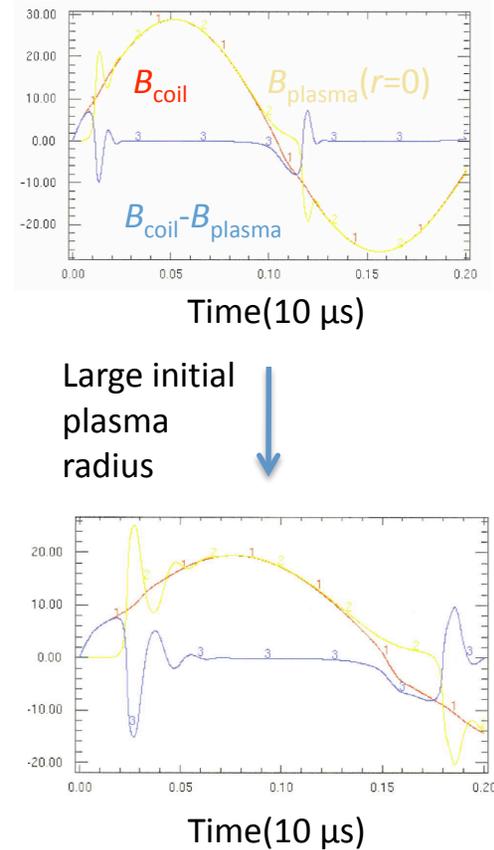
1D HYDRA modeling shows a high- β plasma current that oscillates due to pinching



The oscillation frequency depends upon the plasma radius and only weakly upon the drive rise time



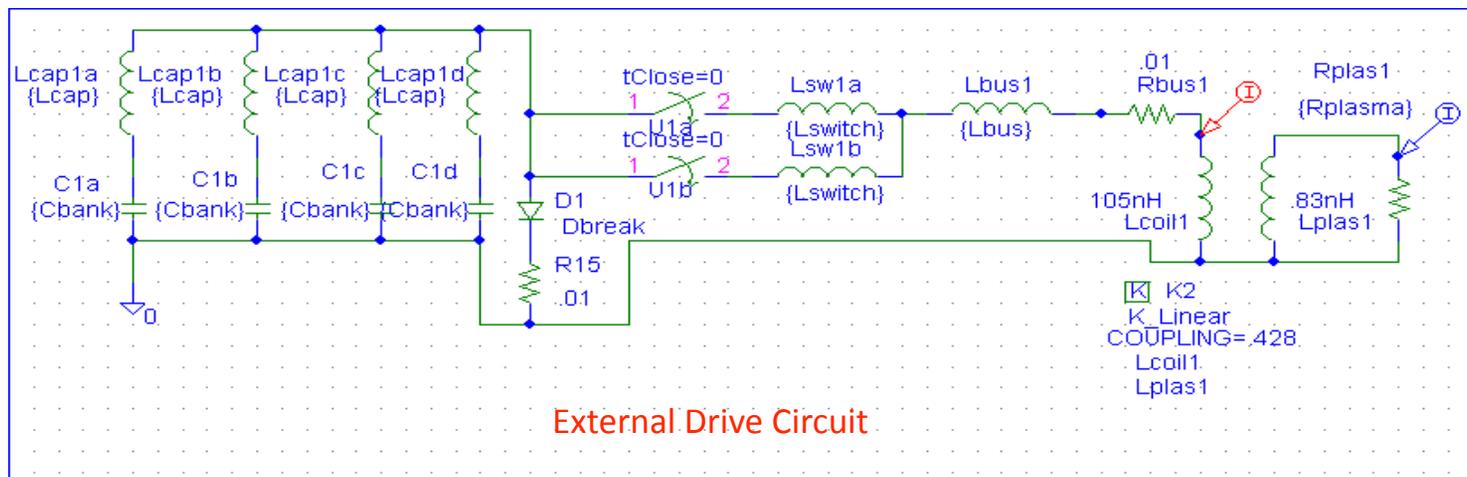
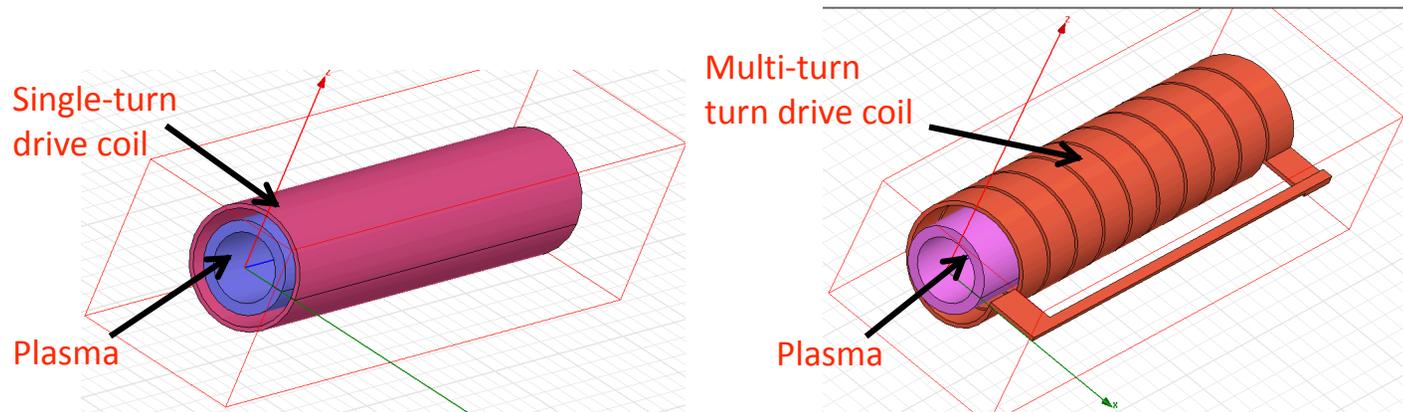
Inductive feedback modifies the assumed drive — a linked circuit model is required



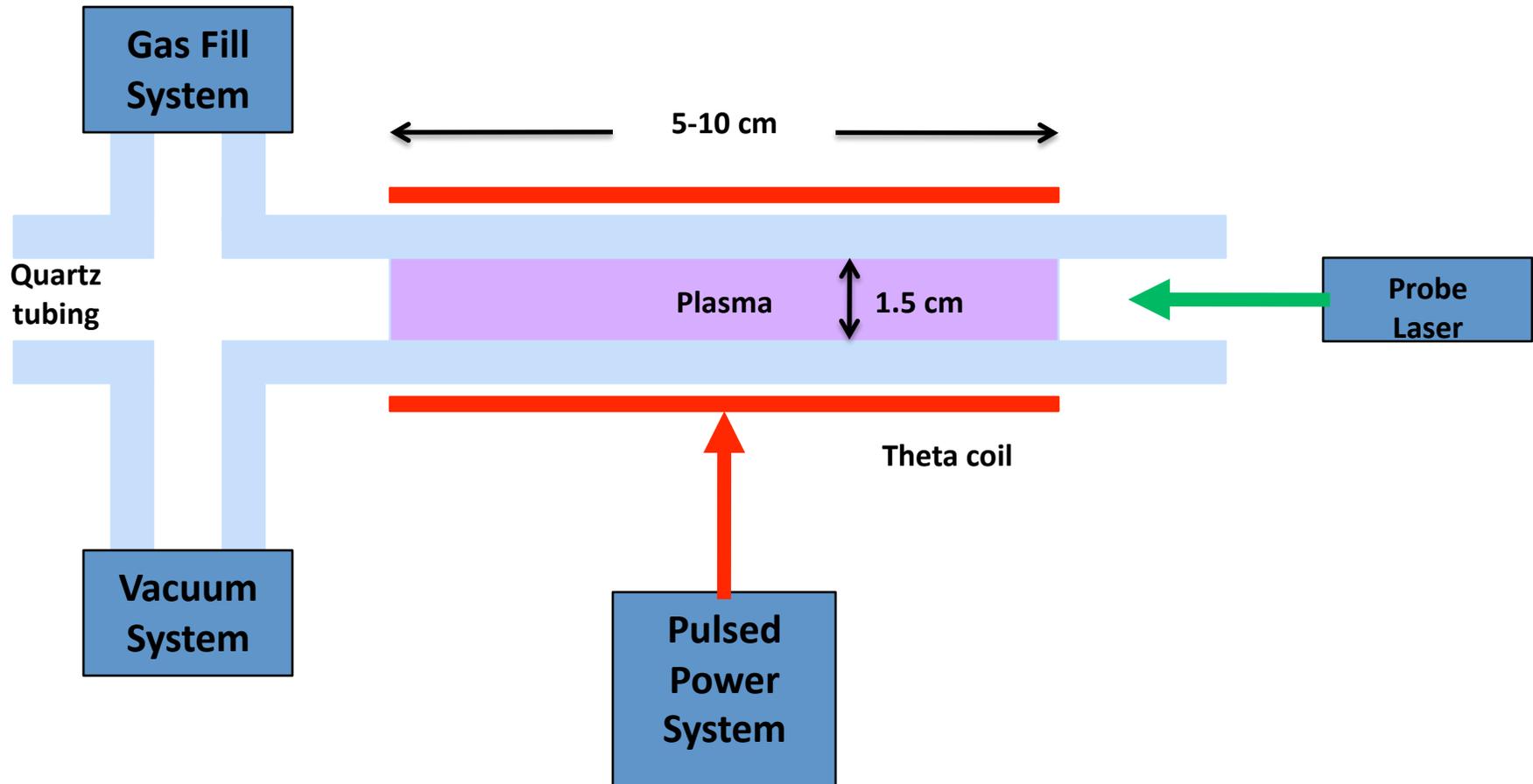
LIFE theta pinch minichamber experiment

Currently: we are instead building a magnetically driven theta pinch, which may permit sustained Joule heating of the Xe over hundreds of ns.

- Can produce the high input power needed to balance the radiation loss
- Known to be (relatively) MHD stable
- Ends are open to allow laser propagation experiments, target injection, and active venting cooling
- Modularly scalable in length



The experimental setup to create the plasma is relatively simple



Challenges to current design of LIFE minichamber experiment

Challenges:

- Diagnostics may be limited initially — mainly Thomson scattering, so modeling is needed to interpret experimental data
- design is difficult because unknown Xe atomic properties determine opacity and conductivity
- Pinching of the plasma may make it difficult to create homogeneous conditions
- The finite length of the Xe pinch causes end hydro effects that propagate into the Xe
- The plasma is potentially unstable to the ‘kink’ mode

Other work:

LANL has done theta pinch work in the context of fusion:

A STUDY OF THE VUV EMISSION FROM HIGHLY IONIZED KRYPTON IN A THETA PINCH PLASMA

L. A. JONES (LANL), E. KALLNE (Harvard),

Quant. Spectrosc. Radiat. Transfer Vol. 30. No 4, pp. 117-326, 1983

Current modeling approach

Current modeling: we are using

- the EM code Maxwell to design the theta pinch external circuit and determine the drive
- the MHD code HYDRA to predict the plasma response

Issues

- The unknown Xe atomic physics make the modeling challenging.
- The drive rise time, the time scales for **B**-field diffusion into the Xe, and ion-electron equilibration time are similar, suggesting NLTE is needed
- DCA NLTE in HYDRA not targeted at this regime so we are currently using DCA LTE
- Plasma response may feed back inductively to external circuit, modifying the assumed drive
- The plasma may not be well modeled by resistive MHD, eg. the Hall term may be significant (James)

Some assumptions

- The plasma and drive coil are modeled reasonably well as 1D as infinite length solenoids
 - fringing (return) fields for a finite length solenoid diminish for length:diameter ratios above ~4:1
- The Xe is adequately modeled as a single fluid with 3T (this assumption is in question)
- Resistive MHD is adequate (in question)
- Heat conduction to the walls is negligible

Status of the LIFE minichamber theta pinch modeling

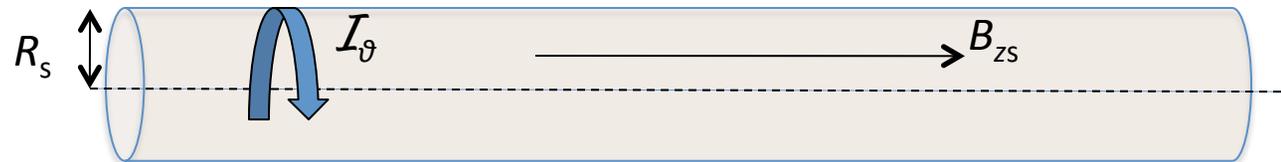
Results so far:

- 1D HYDRA models suggest that
 - the plasma current may oscillate and decay much faster than predicted by a simplified lumped circuit model, reducing the Joule heating
 - Radiative losses and variations in resistivity do not appear to be the cause
 - Pinching may be the cause — possibly simple flux compression
 - the $\mathbf{J} \times \mathbf{B}$ term may be significant, ie. the small-Hall resistive MHD assumption may not be satisfied, throwing the modeling results into question
- Preliminary 2D HYDRA models suggest that end effects may propagate in slowly enough to permit diagnosis of an unaffected region of Xe.

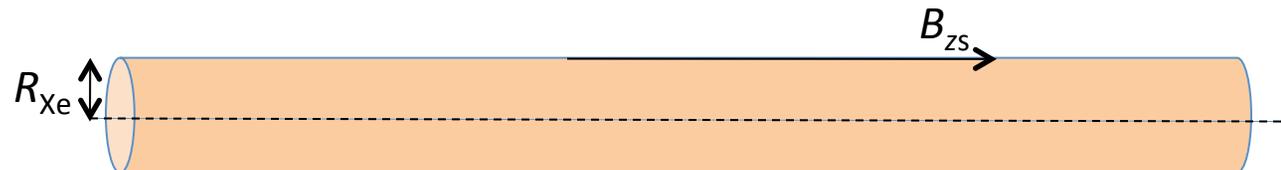
Ongoing work: we are

- trying to understand the cause of the oscillations and early decay of the plasma current (this talk)
- creating a LASNEX model to
 - investigate the effect of the $\mathbf{J} \times \mathbf{B}$ term (this talk)
 - possibly use DCA NLTE
 - Investigate feedback of the plasma response to the external circuit drive
- continuing 2D investigations of end effects (James)
- Implementing Greg Moses's simpler Xe atomic physics model to compare to DCA (James)
- Investigating other plasma effects in a generalized Ohm's Law (James)

A simple 1D model of the theta pinch is coaxial infinite-length solenoids

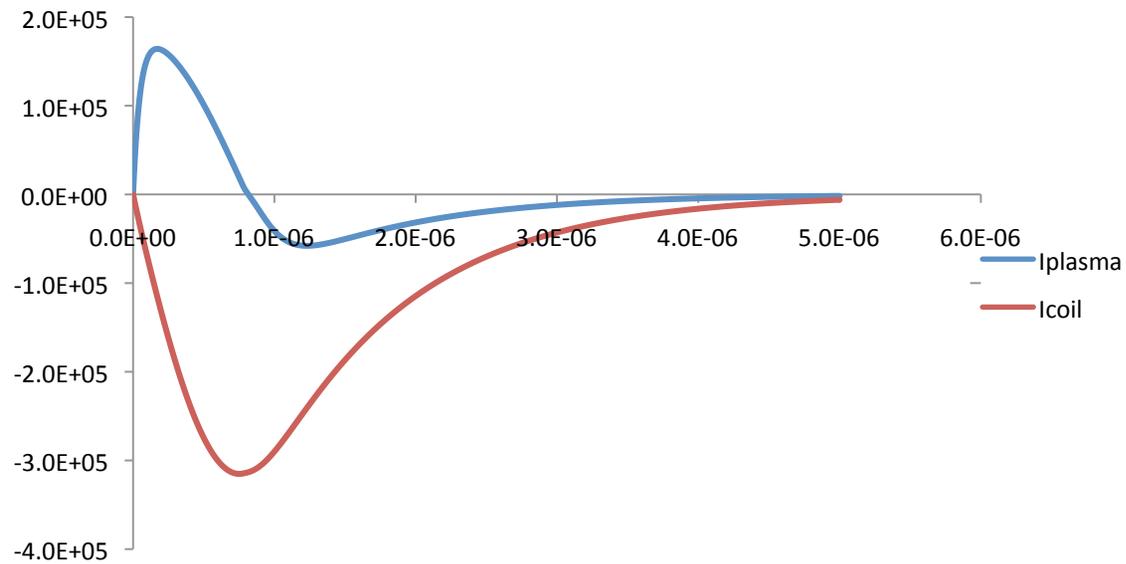
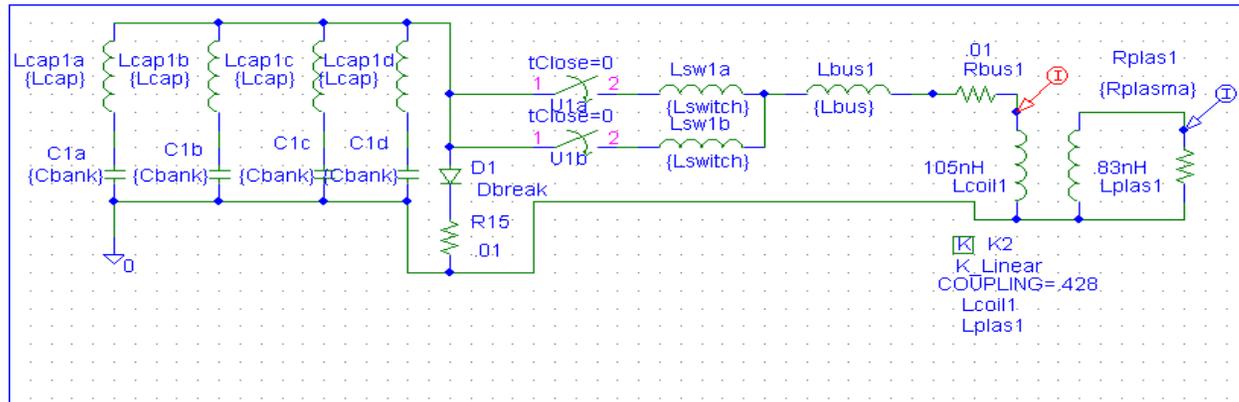


The applied sheath current $I_\theta(t)$ (per unit length) produces a solenoidal field B_z parallel to the axis, where $B_z(r > R_s) = 0$, $B_z(r < R_s) = B_{zs}(t) = (4\pi/c) I_\theta(t)$

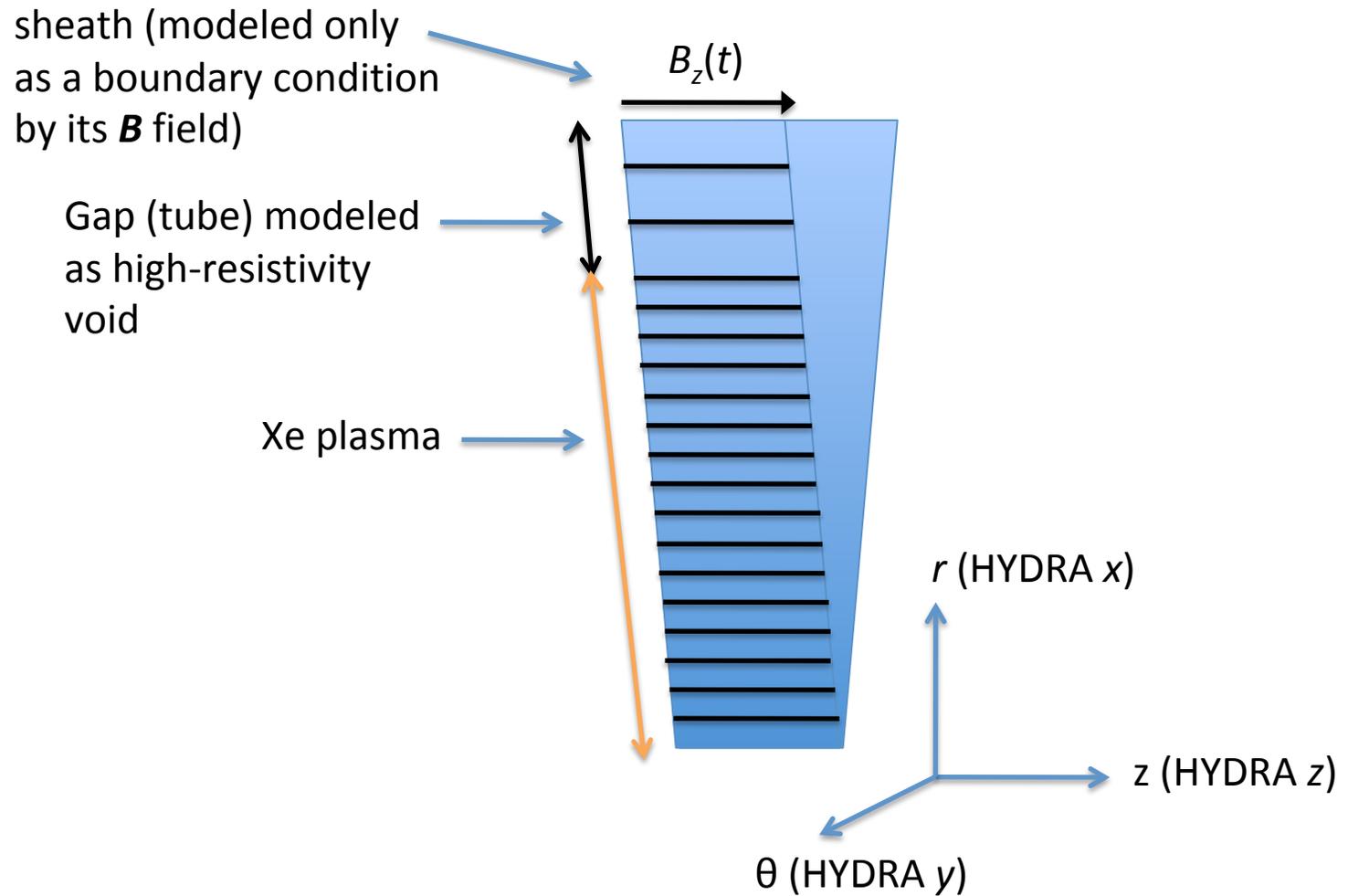


That solenoidal field is the boundary condition $B_z(r = R) = B_{zs}(t)$ at the outer radius of the gap, and is the B field in the gap (tube wall), which is an insulator. The B field diffuses into the Xe plasma. The Xe plasma can be viewed as many thin, inductively coupled coaxial solenoids, each potentially carrying a different (including in direction) azimuthal current, and with each producing no net B field outside itself. The sheath and plasma are also inductively coupled.

MAXWELL predicts the drive current and pulse using a lumped circuit model



In 1D HYDRA the theta pinch is modeled as a thin wedge



The MHD model assumes no displacement current

$$\mathbf{E} = \frac{q}{r^2} \hat{\mathbf{r}} \quad (1) \text{ electric field due to a point charge}$$

$$\mathbf{F} = \frac{q\mathbf{v}_D}{c} \times \mathbf{B} \quad (2) \mathbf{B} \text{ force on a point charge}$$

$$\phi_t + \vec{\nabla} \cdot \mathbf{J} = 0, \quad \mathbf{J} \equiv \phi \mathbf{v} \quad (3) \text{ continuity equation for charge density } \phi$$

We assume averaged macroscopic fields
 $\mathbf{B} = \mathbf{H}, \mathbf{E} = \mathbf{D}$
 $(\mu=1, \epsilon=1).$

$$\vec{\nabla} \cdot \mathbf{E} = 4\pi\phi \quad (4) \text{ Coulomb's law}$$

$$\vec{\nabla} \cdot \mathbf{B} = 0 \quad (5) \text{ no magnetic monopoles}$$

$$\vec{\nabla} \times \mathbf{B} - \frac{1}{c} \mathbf{E}_t = \frac{4\pi}{c} \mathbf{J} \quad (6) \text{ Ampere's law including displacement current [required by (3)]}$$

$$\vec{\nabla} \times \mathbf{E} + \frac{1}{c} \mathbf{B}_t = 0 \quad (7) \text{ Faraday's law}$$

Maxwell's equations

$$\eta \mathbf{J} = \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \quad (8) \text{ Ohm's law [Lorentz transformation of } \eta \mathbf{J} = \mathbf{E}' \text{]}$$

We can neglect the displacement current if the wavelength of field variations λ is much greater than the characteristic length ℓ of the system:

$$\ell \ll \lambda = c\tau < 3 \times 10^{10} \text{ cm/s} \cdot 10^{-9} \text{ s} = 300 \text{ cm}$$

In the theta pinch the system size is < 20 cm and $\tau = 10^{-9}$ s is conservatively small.

The evolution of the \mathbf{B} field and the current \mathbf{J} can be viewed as diffusion of the \mathbf{B} field into the Xe

3D system

$$\vec{\nabla} \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}$$

$$\vec{\nabla} \times \mathbf{E} + \frac{1}{c} \mathbf{B}_t = 0$$

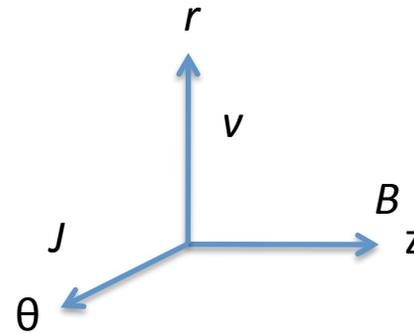
$$\eta \mathbf{J} = \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \Rightarrow$$

$$\mathbf{B}_t + c \vec{\nabla} \times \mathbf{E} = 0$$

$$\mathbf{B}_t + c \vec{\nabla} \times \left(-\frac{\mathbf{v}}{c} \times \mathbf{B} + \eta \mathbf{J} \right) = 0$$

$$\mathbf{B}_t - \vec{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \vec{\nabla} \times (c\eta \mathbf{J}) = 0$$

$$\mathbf{B}_t - \vec{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \vec{\nabla} \times (D \vec{\nabla} \times \mathbf{B}) = 0, D \equiv \frac{c^2 \eta}{4\pi}$$



$$\mathbf{B} = B(r) \hat{\mathbf{z}}, \quad \mathbf{v} = v(r) \hat{\mathbf{r}} \Rightarrow$$

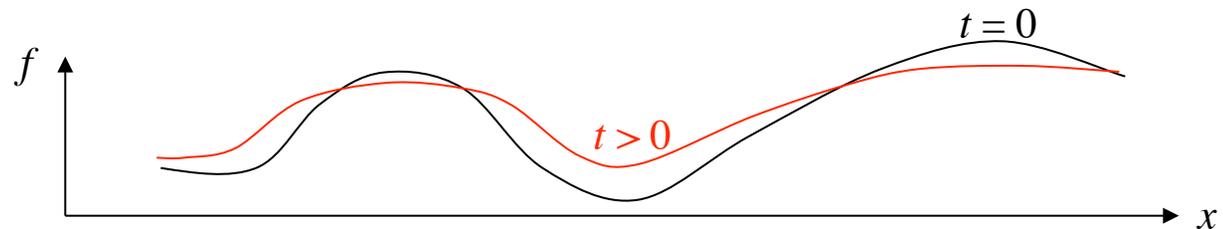
$$J = J_\theta = -\frac{c}{4\pi} \partial_r B,$$

$$\mathbf{v} \times \mathbf{B} \rightarrow v B \hat{\theta},$$

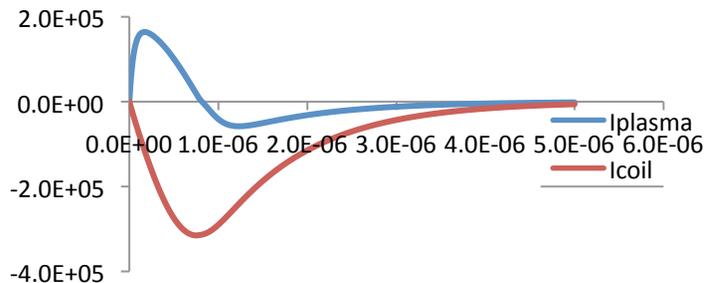
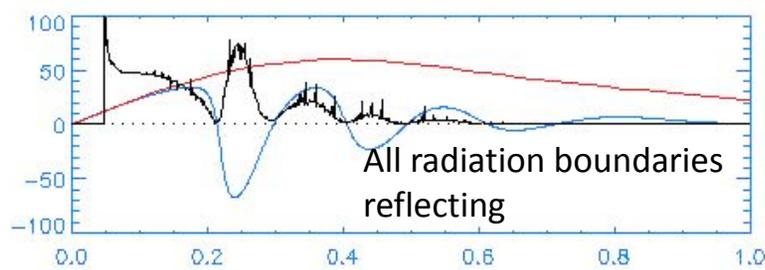
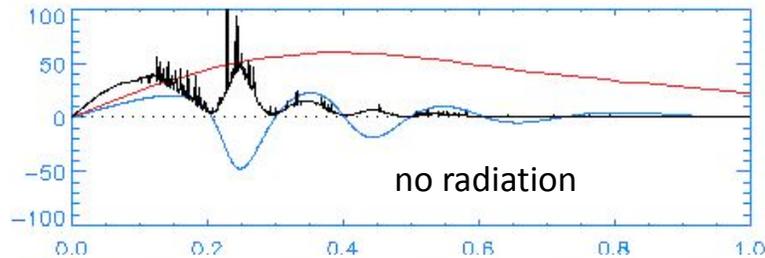
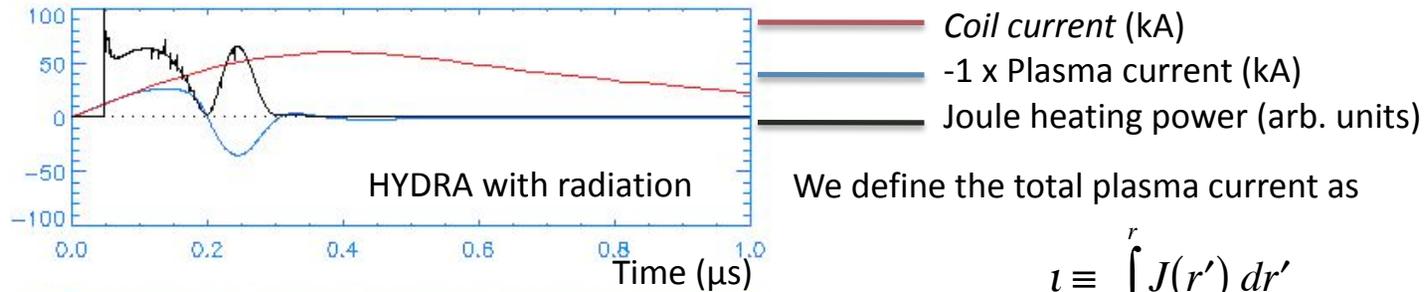
$$\partial_t B = \frac{1}{r} \partial_r [r(c\eta J - vB)]$$

$$E = \hat{\theta}(-\eta J + vB) - \hat{\mathbf{r}} \frac{JB}{en_e}$$

Hall term



For a 400 ns drive rise time, HYDRA predicts that the plasma current drops much more quickly than the coil current, and that radiative losses are not the cause



We define the total plasma current as

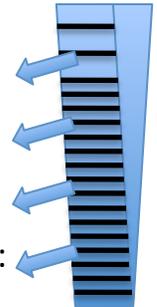
$$I \equiv \int_{r'=0}^r J(r') dr'$$

This is simply per area in the angular direction, and is not weighted by radius. The total Joule heating power per unit length in z is weighted by radius and resistivity:

$$P_J \equiv \frac{1}{\pi R_{\text{plasma}}^2} \int_{r'=0}^r \eta(r') J(r')^2 (2\pi r') dr'$$

Hydra suggests

- The Joule heating power does vary with the total plasma current, so the early drop in total plasma current does correspond to decreased Joule heating.
- The Joule heating occurs in waves.
- Radiative losses speed up the decay of the plasma current, but do not cause the oscillation.



In general, Faraday's Law describes current density, not total current

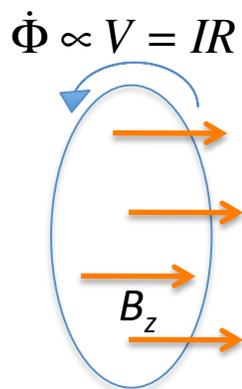
$$\frac{1}{c} \dot{\mathbf{B}}_t = -\vec{\nabla} \times \mathbf{E} \quad \text{Faraday's Law}$$

$$\frac{1}{c} \dot{\Phi}(r) = -\oint \mathbf{E}(r) \cdot d\mathbf{l} = (2\pi r) E_\theta(r) = \varepsilon(r) \quad \text{Integral over circular cross section}$$

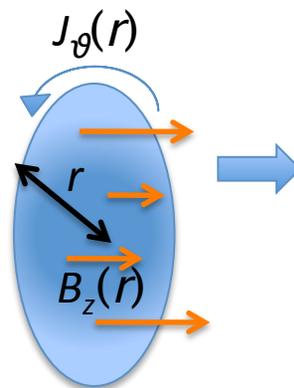
$$\eta \mathbf{J} = \mathbf{E} \rightarrow \eta(r) J_\theta(r) = E_\theta(r) \quad \text{Ohm's Law (fluid frame)}$$

$$\frac{1}{c} \dot{\Phi}(r) = -(2\pi r) \eta(r) J_\theta(r) \quad \text{Faraday's Law for theta pinch geometry}$$

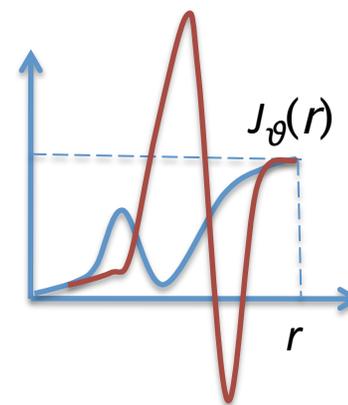
For the special case of the wire loop, we obtain the familiar expression for the total current I



For the theta pinch, Faraday's Law contains only the current *density* $J_\theta(r)$, not the total current



Different Theta pinch $J_\theta(r)$ profiles can satisfy Faraday's Law, with different Joule heating

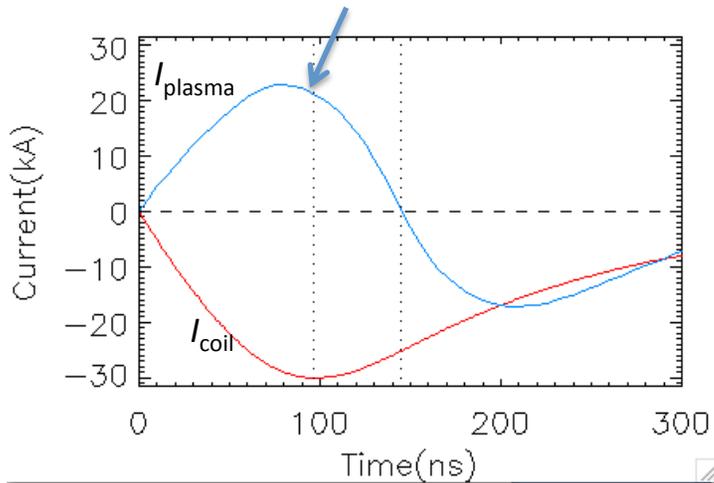


In HYDRA, consistent with Faraday's Law, I_{plasma} lags I_{coil} , and J_{plasma} reverses direction, causing new waves of Joule heating

Conditions:

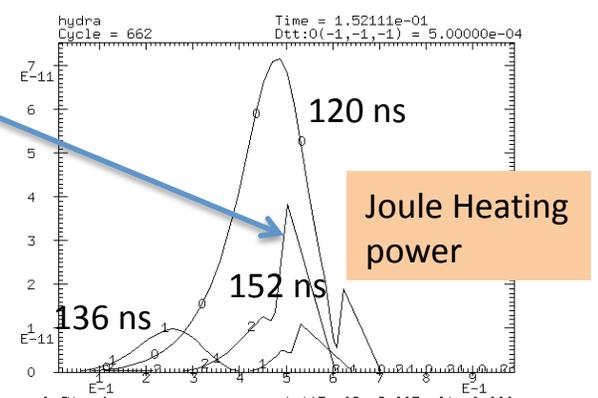
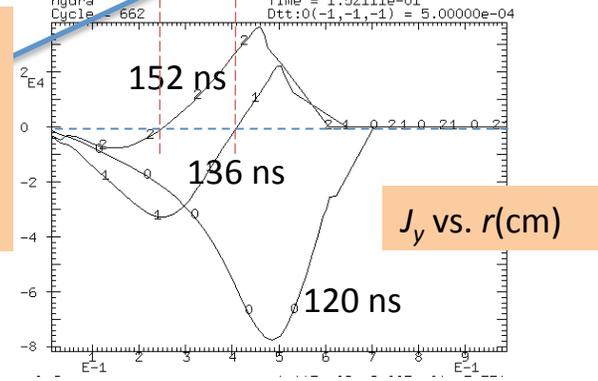
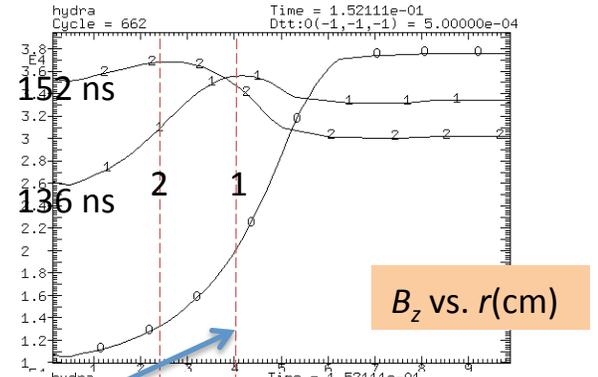
- LIFE minichamber theta pinch
- Xe radius: 0.75 cm, Inner coil radius: 1 cm
- Xe density: 4 $\mu\text{g}/\text{cc}$
- Drive: 100 ns rise, 30 kA/cm,
- HYDRA xnltemn 0.100, DCA LTE

(1) lag:
The wire loop version of Faraday's Law predicts $I_{\text{plasma}} = 0$ when $dI_{\text{coil}}/dt = 0$, but I_{plasma} is actually still near its peak



(2) reversal:
Drop in $|I_{\text{coil}}|$ produces maximum in B and reversal in J ($J \sim \text{gradient of } B$)

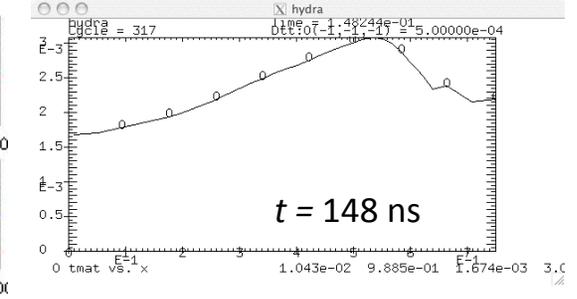
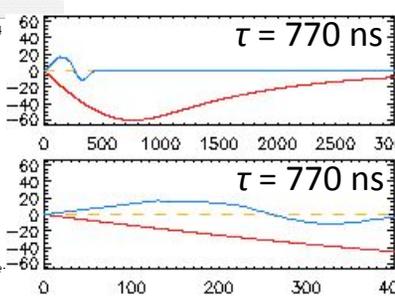
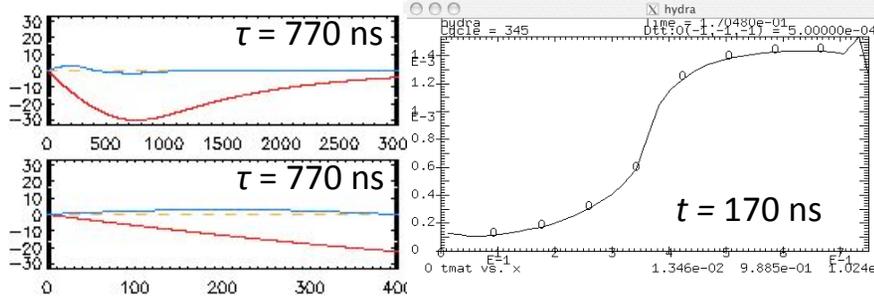
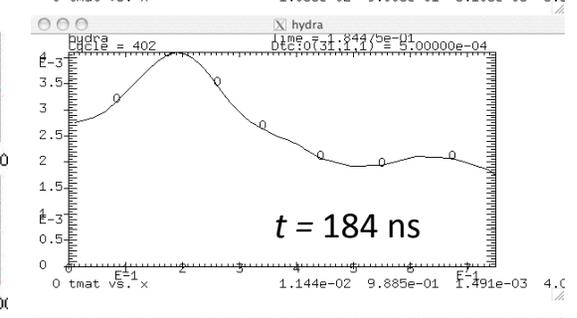
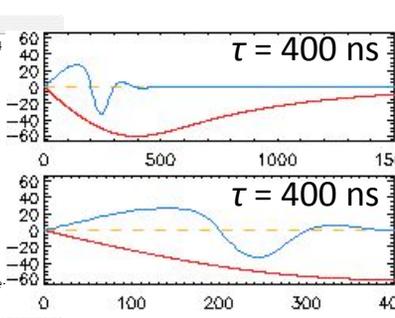
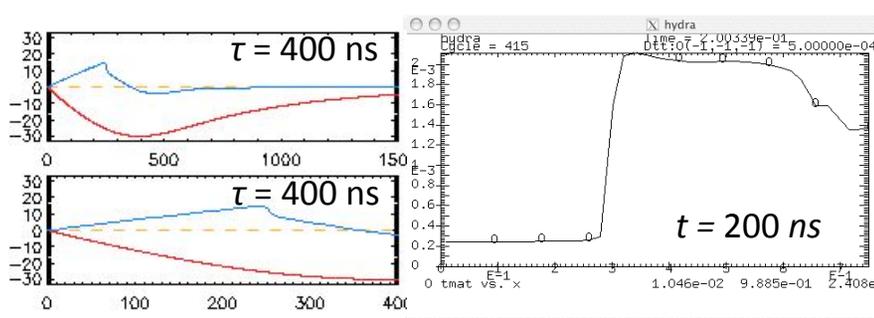
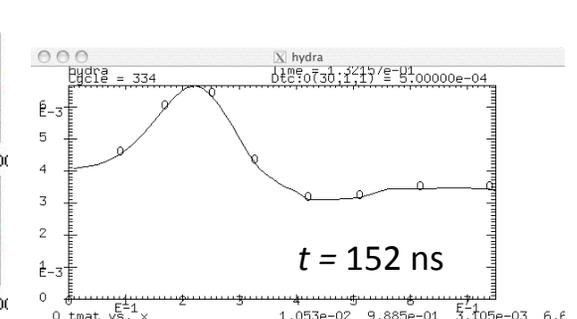
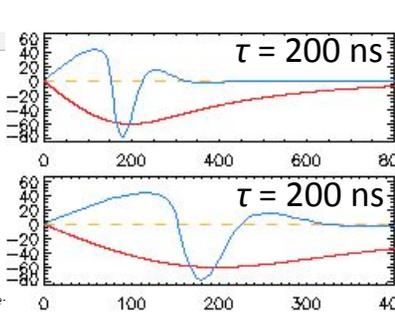
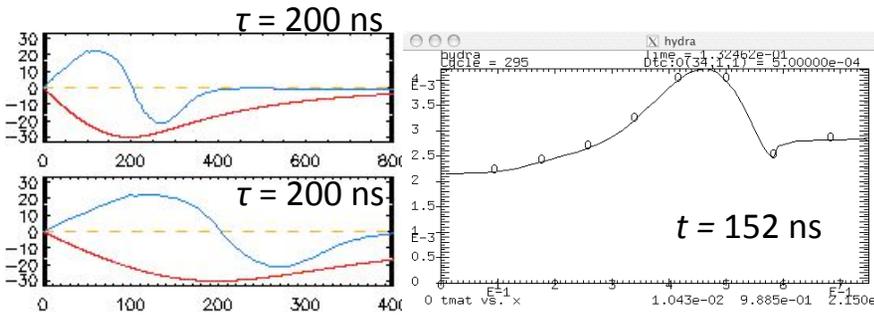
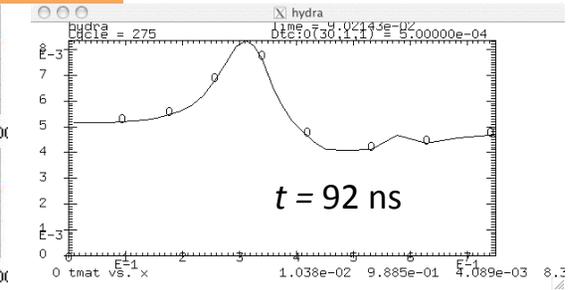
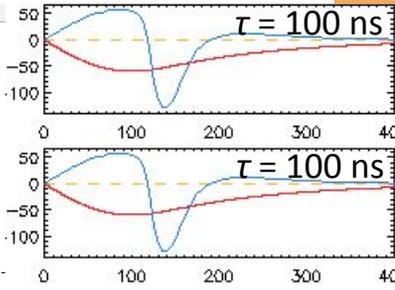
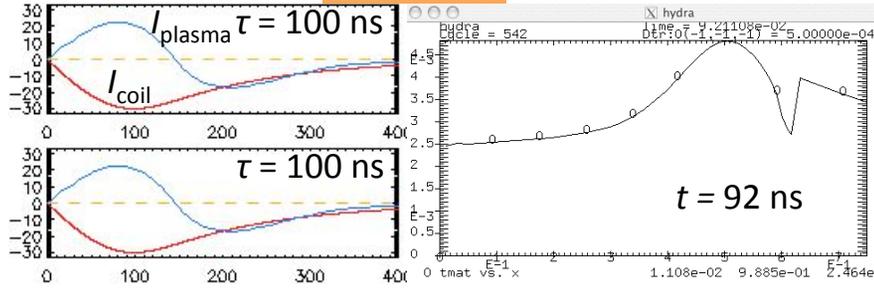
(3) The resulting new spike in J causes a new wave of Joule heating



For realistic longer pulses the total plasma current decouples from the coil current

I (kA) vs. t (ns) 30 kA/cm T_e (keV) vs. r (cm)

I (kA) vs. t (ns) 60 kA/cm T_e (keV) vs. r (cm)



HYDRA results observe Faraday's Law, and fluid velocity matters

Since the fluid is compressible, we need to make a Lorentz transformation of Ohm's Law to the lab frame



$$\eta \mathbf{J} = \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \quad \boxed{\text{Ohm's Law (lab frame)}}$$

$$\frac{1}{c} \dot{\Phi}(r) = -(2\pi r) \left(\eta J_\theta(r) - \boxed{v_r(r) B_z(r)} \right)$$

$$\boxed{\text{Faradays' Law (lab frame)}}$$

Terms in Faraday's Law from HYDRA:

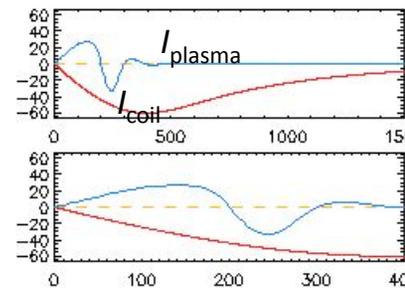
— $\frac{1}{c} \dot{\Phi}$ (Numerical derivative) LHS

— $-(2\pi r) \left(\eta J_\theta(r) - \boxed{v_r(r) B_z(r)} \right)$ RHS

— $-(2\pi r) \eta J_\theta(r)$ ← Separate terms in RHS

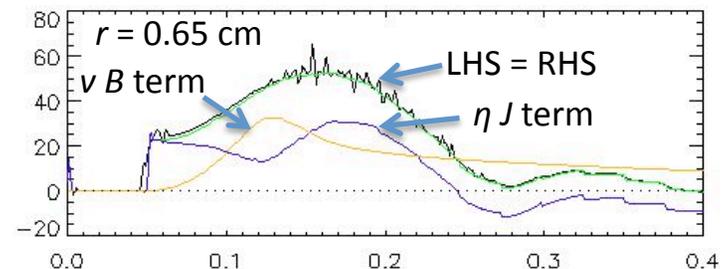
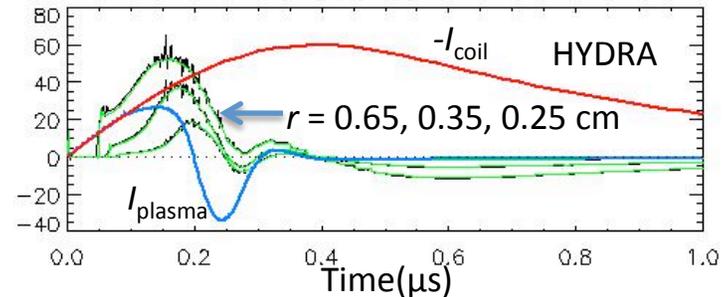
— $-(2\pi r) \boxed{v_r(r) B_z(r)}$ ← terms in RHS

HYDRA results reproduce Faraday's Law at three test radii r in the plasma, and the $v B$ term proves significant



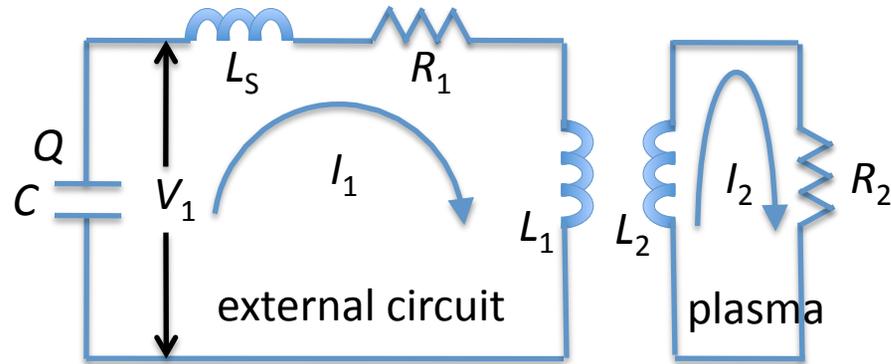
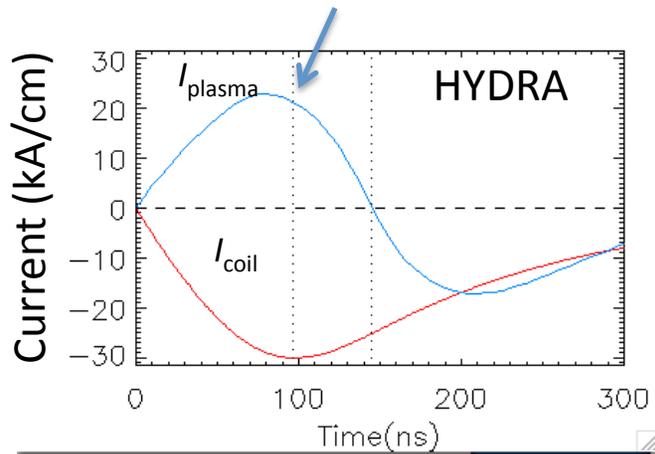
← **Model:**
60 kA/cm
 $\tau = 400$ ns
 I (kA) vs. t (ns)

Results



A simplified circuit model predicts a lag in plasma current similar to what HYDRA predicts for a fast rise; this simplified analysis can be generalized

(1) HYDRA predicts $I_{\text{plasma}} = 0$ well after $dI_{\text{coil}}/dt = 0$ → (2) Simplified circuit model with fixed inductances



Kirchoff's law

$$Q = CV$$

$$I_1 = -\dot{Q}$$

$$V = R_1 I_1 + (L_s + L_1) \dot{I}_1 + M_{12} \dot{I}_2$$

$$0 = R_2 I_2 + L_2 \dot{I}_2 + M_{12} \dot{I}_1$$

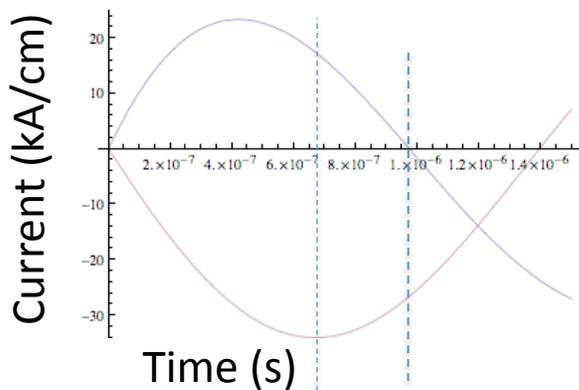
Initial conditions:

$$V(t=0) = V_0$$

$$Q(t=0) = CV_0$$

$$I_1(t=0) = I_2(t=0) = 0$$

(3) These ODE's are easily solved in Mathematica



Circuit parameters for 770 (not 100) ns rise time:

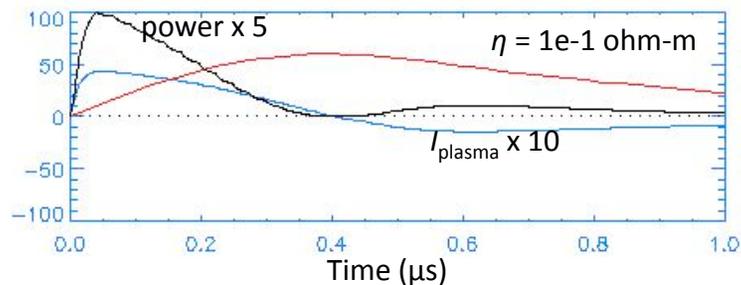
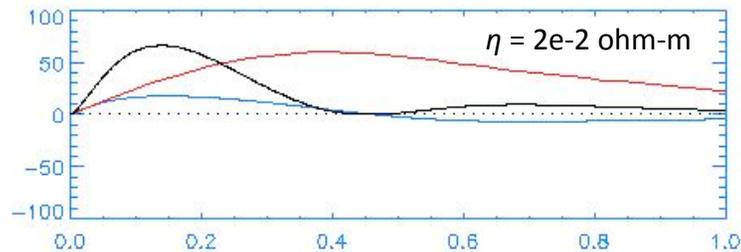
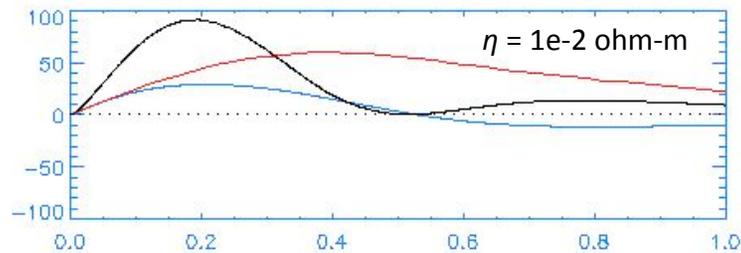
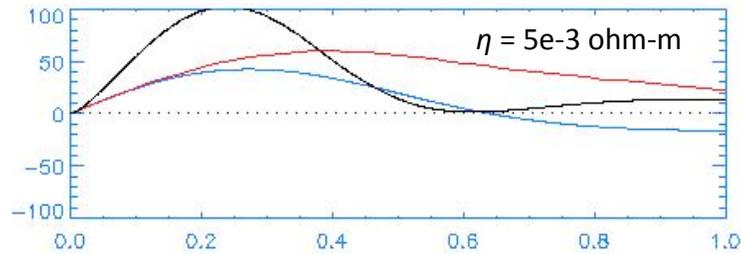
$$C = 1.6 \mu\text{F} \quad V_0 = 100 \text{ kV}$$

$$\{L_s, L_1, L_2\} = \{30, 105, 10\} \text{ nHk} = 0.43; M = k \sqrt{L_1 L_2}$$

$$\{R_1, R_2\} = \{0.005, 0.025\} \text{ ohm}$$

It is straightforward to generalize this analysis with multiple nested coils as the 'plasma'

For pure Cu model with a 400 ns drive rise time and constant resistivity, HYDRA suggests that the plasma current drops to zero only after the peak of coil current

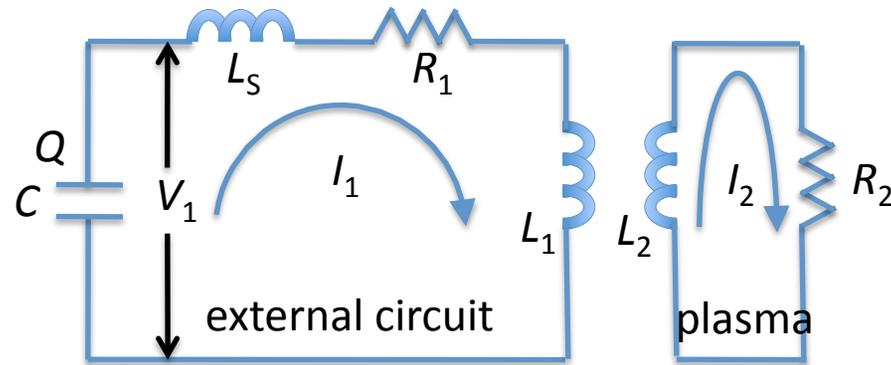


— $-1 \times$ Coil current (kA)
 — Plasma current (kA)
 — Joule heating power (arb. units)

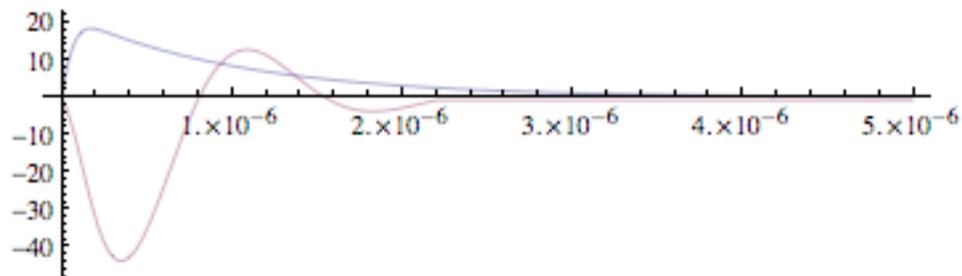
- No radiation
- The diffusion time varies with the resistivity
- Very little plasma current flows at higher resistivity
- The dense copper (8.93 g/cc) has very low velocity, so the $\mathbf{v} \times \mathbf{B}$ term in the resistivity should be small.
- It appears that diffusion time alone can not account for the fast oscillation of total current for Xe.
- Remaining possible causes include variations in resistivity, the decrease in plasma area with pinching and the resulting drop in captured flux, and the $\mathbf{v} \times \mathbf{B}$ term in Ohm's Law.

A simplified circuit model suggests a simple attenuation of the coupling to mock up pinching can not account for the fast oscillation

Simplified circuit model with fixed inductances

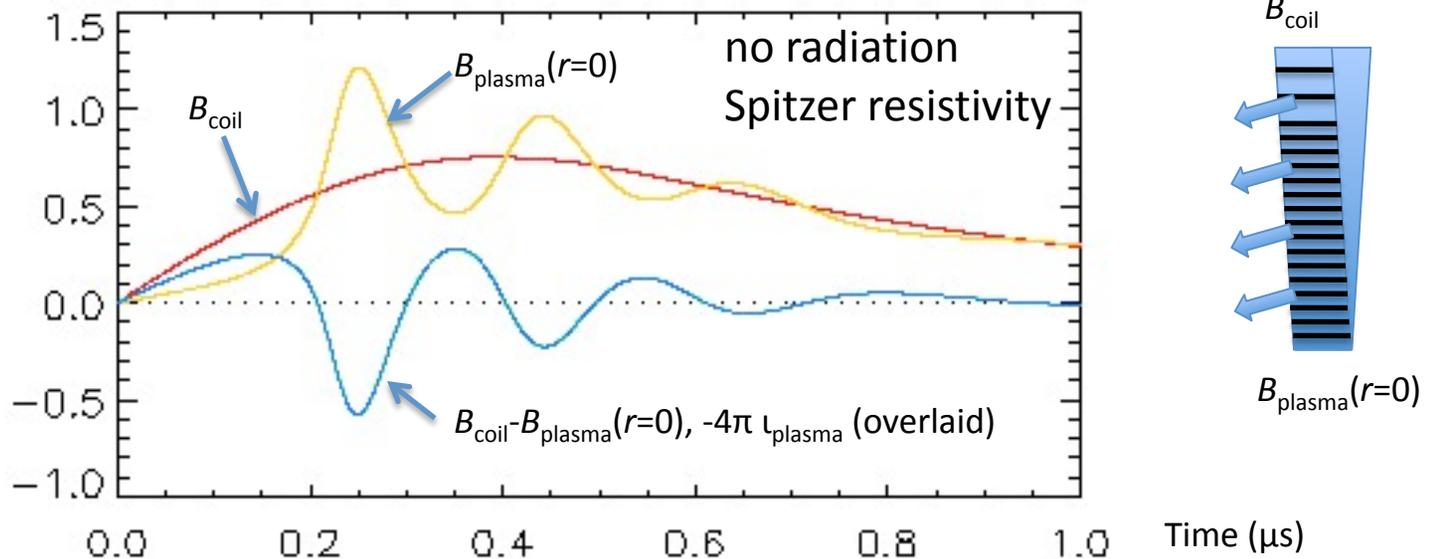


We multiply the mutual inductance M by a time-dependent factor $\exp(-t/100 \text{ ns})$



The plasma current peaks early but is only damped and does not oscillate

The total plasma current is proportional to the drop in B from the outside to the center of the plasma, and therefore oscillates with $B(r=0)$.



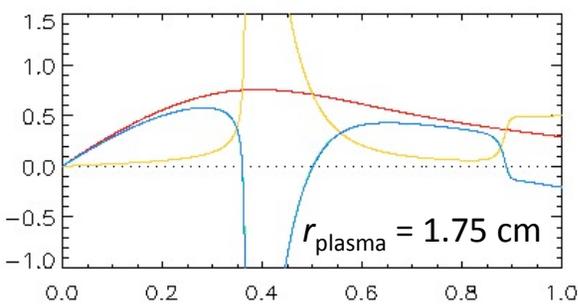
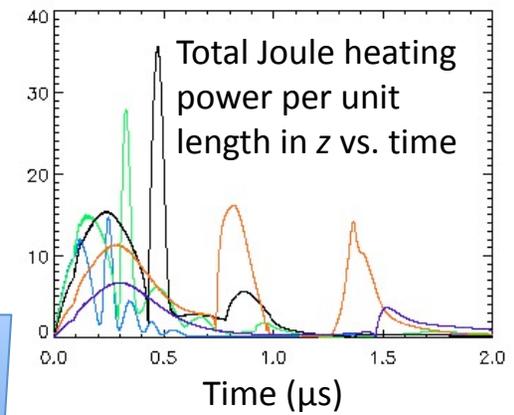
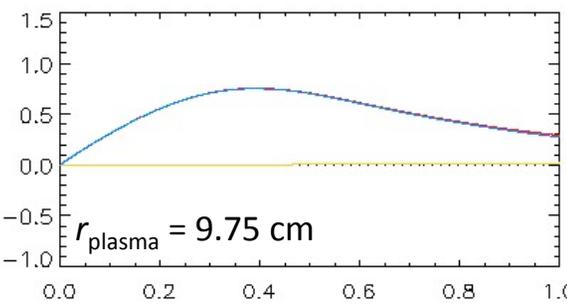
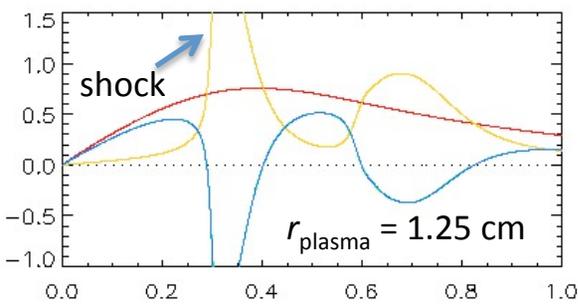
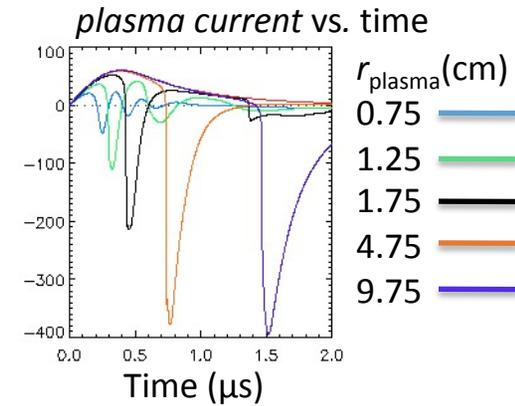
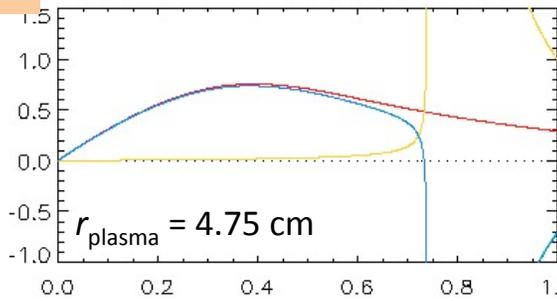
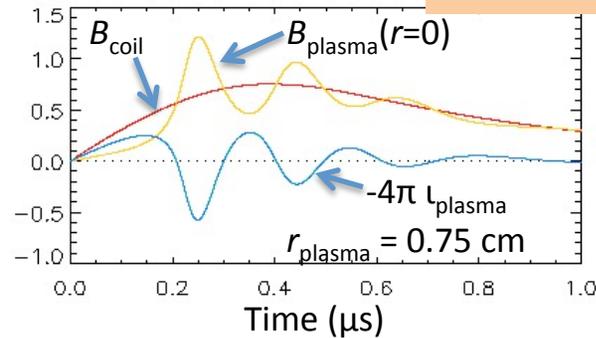
Proof that total plasma current is proportional to the drop in B :

$$\mathbf{B} = B\hat{z} \Rightarrow \mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B} \rightarrow J = J_{\Theta} = -\frac{c}{4\pi} \partial_r B,$$

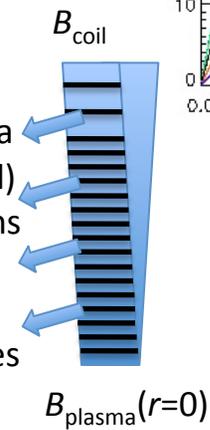
$$\iota \equiv \int_{r'=0}^r J(r') dr' = -\frac{c}{4\pi} [B(r) - B(0)] \quad \text{Plasma current per unit length in } z \text{ (cgs)}$$

The role of $B(r=0)$ is confirmed by modeling theta pinches of increasing radius with the same coil B field, so that $B(r=0)$ rises later in time

no radiation

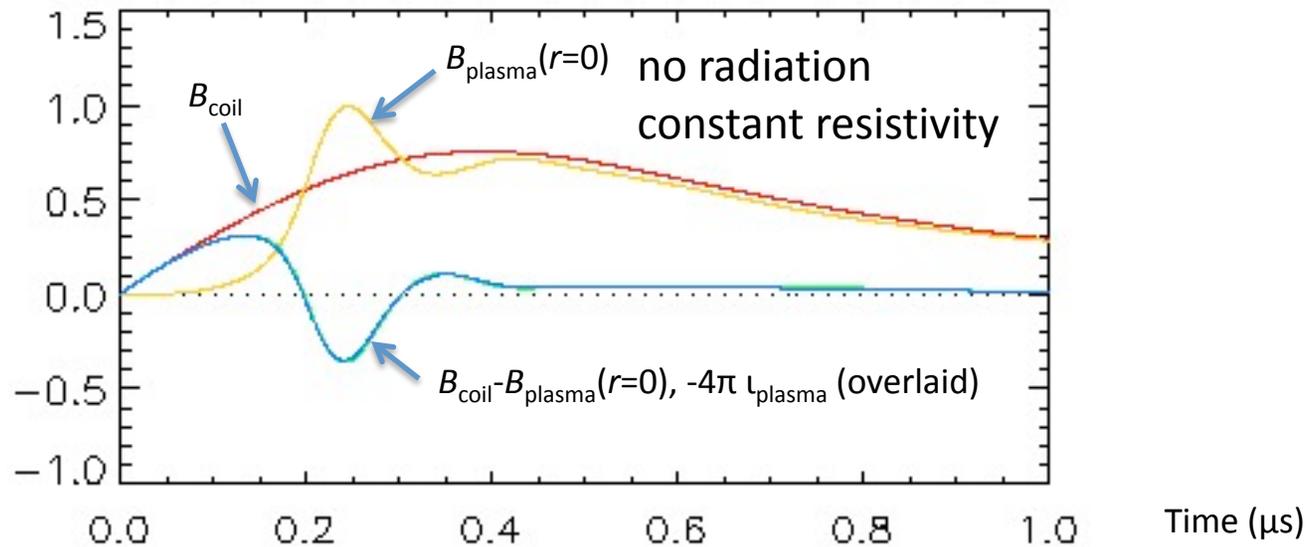


- MHD shocks form for larger radii, producing sudden increases in $B(r=0)$.
- The definition of total plasma current (not radially weighted) may overemphasize excursions at low r .
- The Joule heating power (radially weighted) emphasizes excursions less



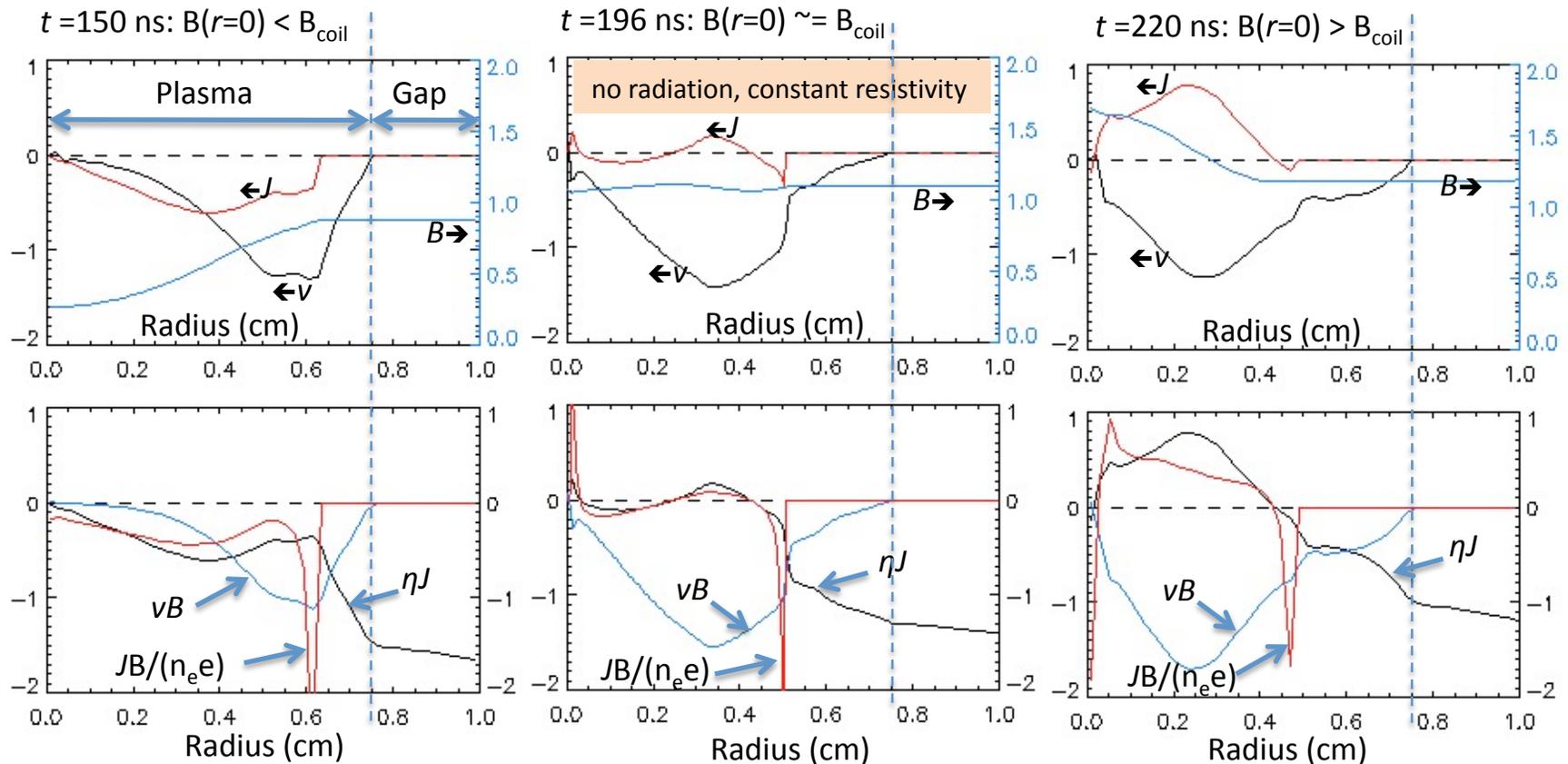
A larger radius could increase Joule heating, but the plasma may be less homogeneous, and may be shocked

HYDRA shows that spatially and temporally varying resistivity changes the oscillation but does not cause it

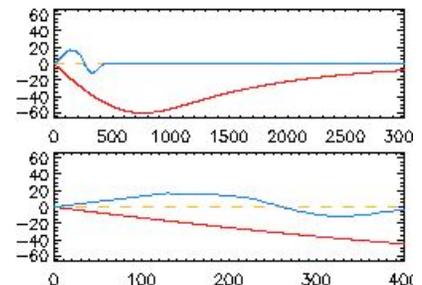


Remaining possible causes for the oscillation in HYDRA include decrease in plasma area with pinching and the resulting drop in captured flux, and the $v B$ term in Ohm's Law.

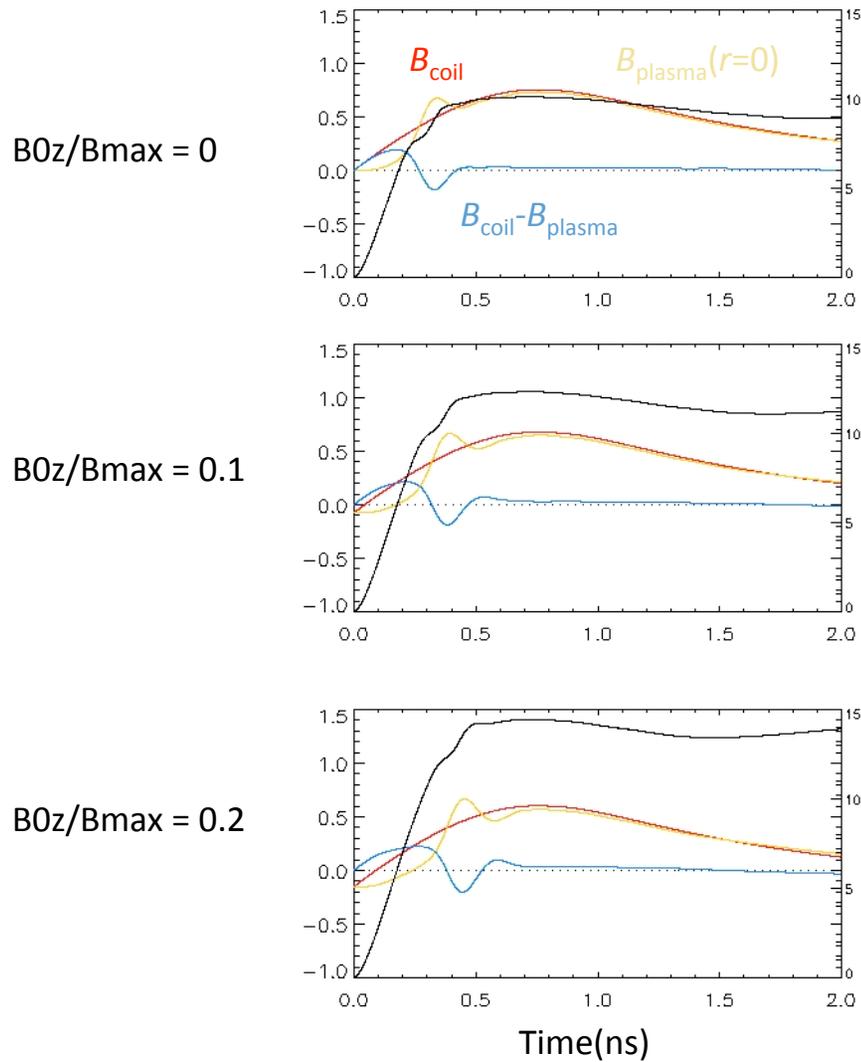
HYDRA spatial profiles near the time the plasma current first drops suggest the $\mathbf{v} \times \mathbf{B}$ and $\mathbf{J} \times \mathbf{B}$ terms may be significant



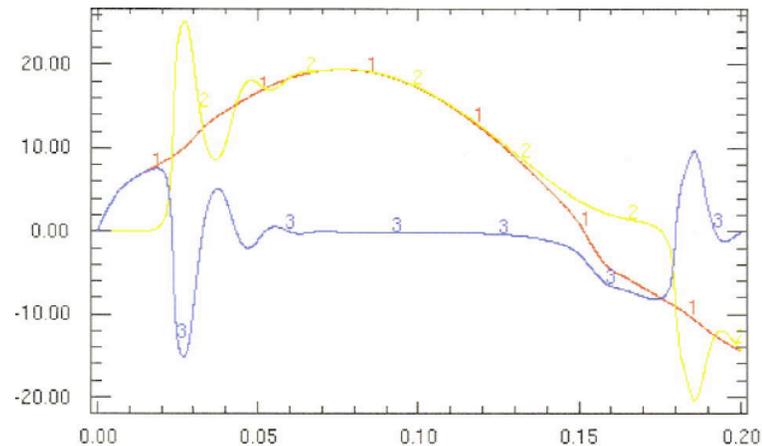
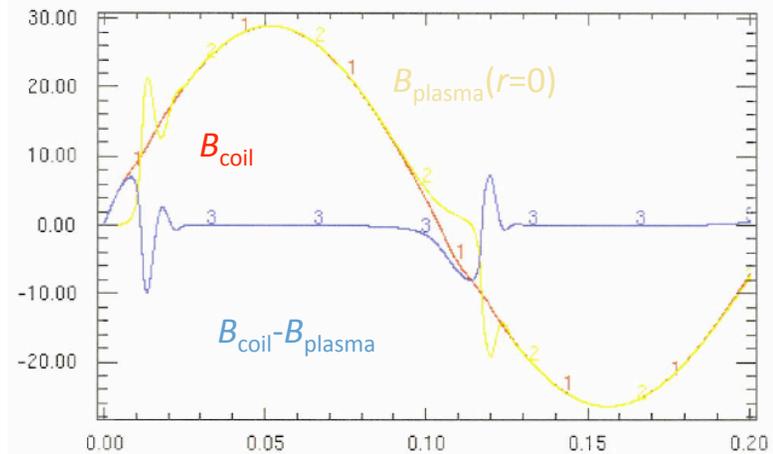
- The oscillation of $B(r=0)$ may be due to the significant $\mathbf{v} \times \mathbf{B}$ term in the \mathbf{E} field during the pinching of the plasma. Is this simply flux compression?
- The $\mathbf{J} \times \mathbf{B}$ term also appears to be large during the pinching. If it were included in the \mathbf{E} field, this term could modify the pinching. Can LASNEX help here?



HYDRA suggests a B_{0z} field can delay the drop in plasma current and increase the plasma temperature



Adding a circuit model to the simulation suggests inductive feedback from the plasma to the external circuit is significant



Time(10 μ s)