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Automatic Model Order Reduction

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Workshop on Finite Elementys for Microwave Engineering
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Adaptive Reduced Order Modeling

Subtitle: Reducing the Computational Cost of Electromagnetic Coupling Analysis

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Adaptive Reduced Order Model Generation Using Radial Basis Functions

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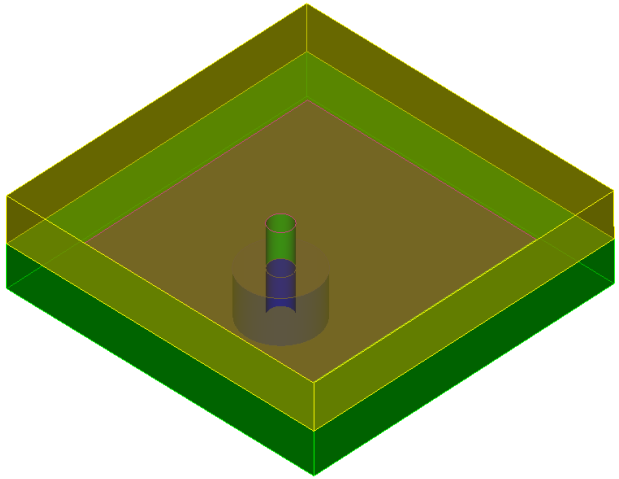
Workshop on Finite Elements for Microwave Engineering
2010

Outline

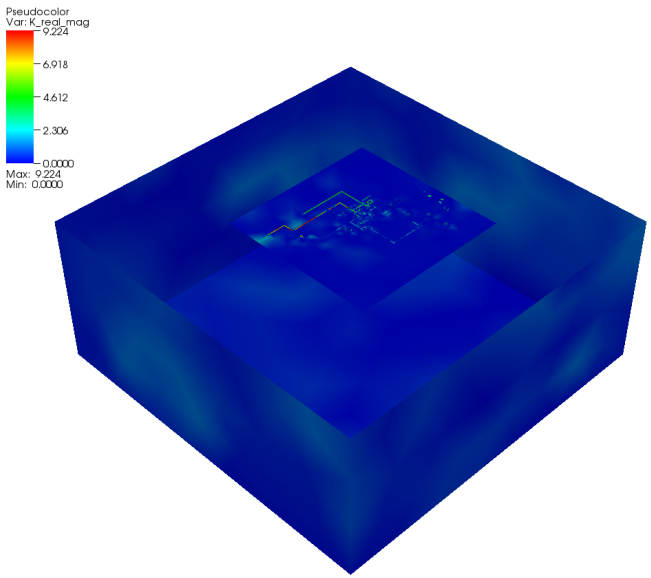
1. Model Order Reduction
   - Motivation
   - Classic MOR Techniques in FEM
2. Generalized MOR Algorithm
   - Adaptive Procedure
   - Black-Box Approach
3. Examples
   - Patch Antenna
   - Oscillator Circuit



Motivation

- General problem: solve matrix equation $A(\mathbf{s})\mathbf{x} = \mathbf{b}(\mathbf{s})$, with the system depending on a vector of parameters $\mathbf{s}$.
  - FEM, MoM, PO, ...
  - Parameters may be frequency, matrials, dimensions, ...
- Even if solution take a few minutes, thousands of such solutions may be needed
  - Uncertainty Quantification
  - Optimization
  - Parameter Exploration

Circuit board capable of moving inside enclosure



Advantages

- Find a reduced test matrix $V \in R^{N \times m}$, with $m \ll N$
  - Solve reduced order model

$$\tilde{A}\tilde{\mathbf{x}} = V^H A V \tilde{\mathbf{x}} = V^H \mathbf{b}$$

  - Numerous strategies for finding V: matching Taylor polynomials up to a certin order at one or more points in parameter space
- Commonly applied to FEM , which gives numerous advantages
  - Sparse matricies
  - Simple form for $A(\mathbf{s})$ and $\mathbf{b}(\mathbf{s})$, e.g.,

$$(S + \epsilon_1 \omega^2 T_1 + \omega^2 T)\mathbf{x} = \omega \mathbf{b}$$

  - Easy to precalculate $V^H S V$, $V^H T_1 V$, etc., so $\tilde{A}$ can be rapidly and exactly calculated for any parameter values

Disadvantages

- Moment-matching order and points typically chosen beforehand, based on user knowledge and experience. Meeting desired error tolerance is combination of luck and skill.
- Difficult to incrementally increase dimension of reduced order system.

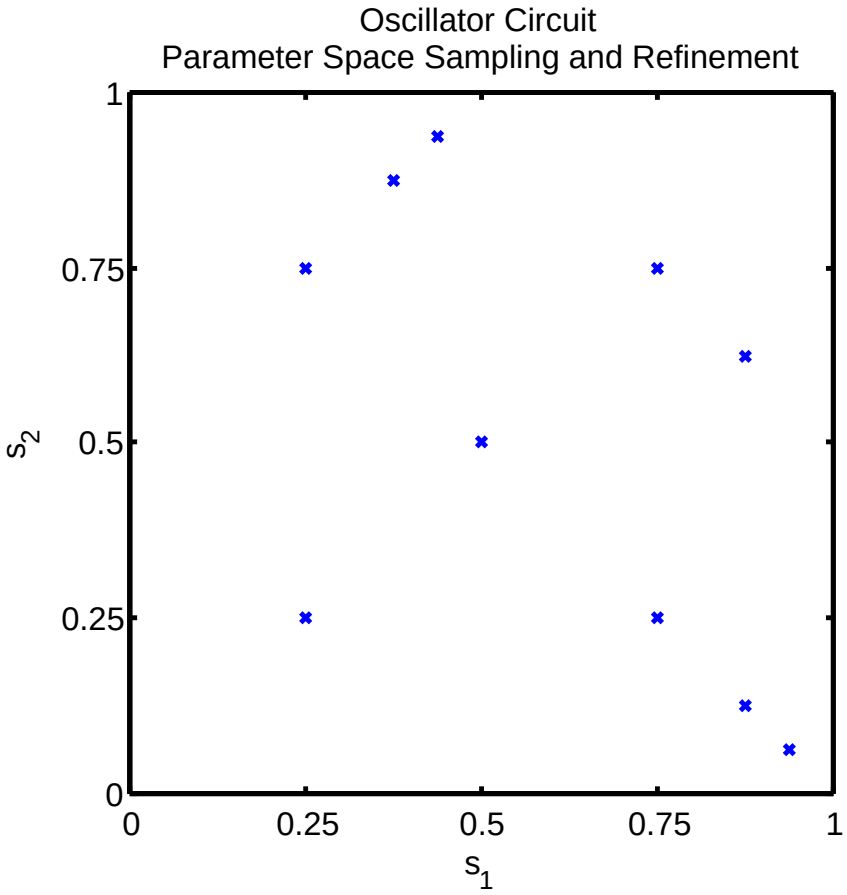
$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & x & & y & & \\
 & x^2 & & xy & & y^2 & \\
x^3 & & x^2y & & xy^2 & & y^3
\end{array}
$$

- Requires intrusive modifications of FEM code to isolate parts that depend on parameters.

Desired Features

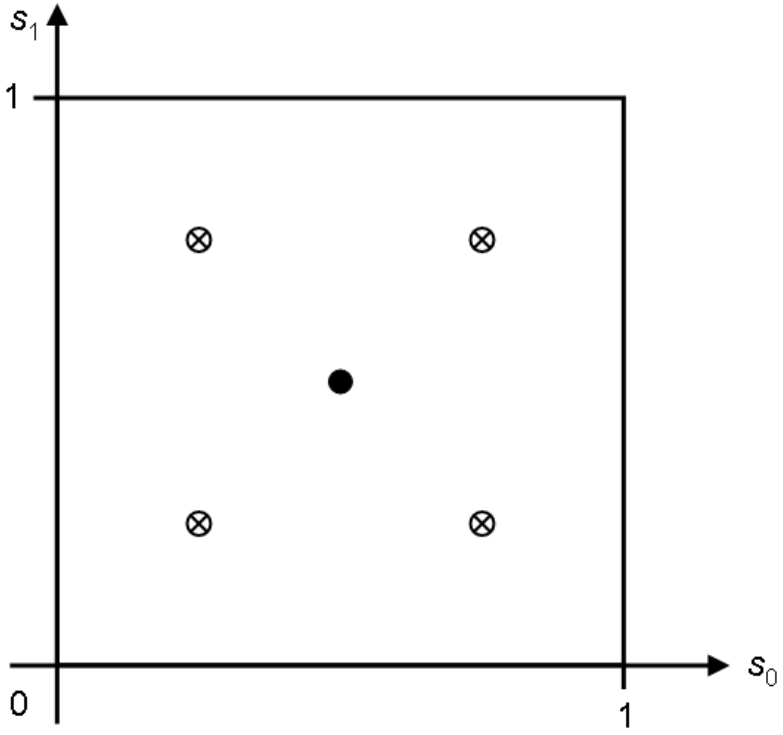
Ability to:

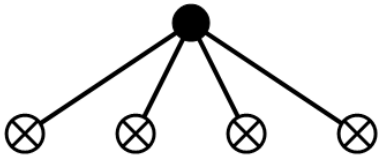
- update model until reaching user-specified accuracy
- easily adapt to an arbitrary number of parameters
- work with many different solvers and formulations



Adaptive Sampling

- Create model so as to match true solution at several samples points.
  - I.e., $\text{colspace}(V) = \text{span}\{\mathbf{x}(\mathbf{s}_1), \ldots, \mathbf{x}(\mathbf{s}_M)\}$
- Assume error estimator is given
  - distribute test points ($\otimes$) around sample points ($\bullet$)
  - choose next sample point where error is estimated to be largest





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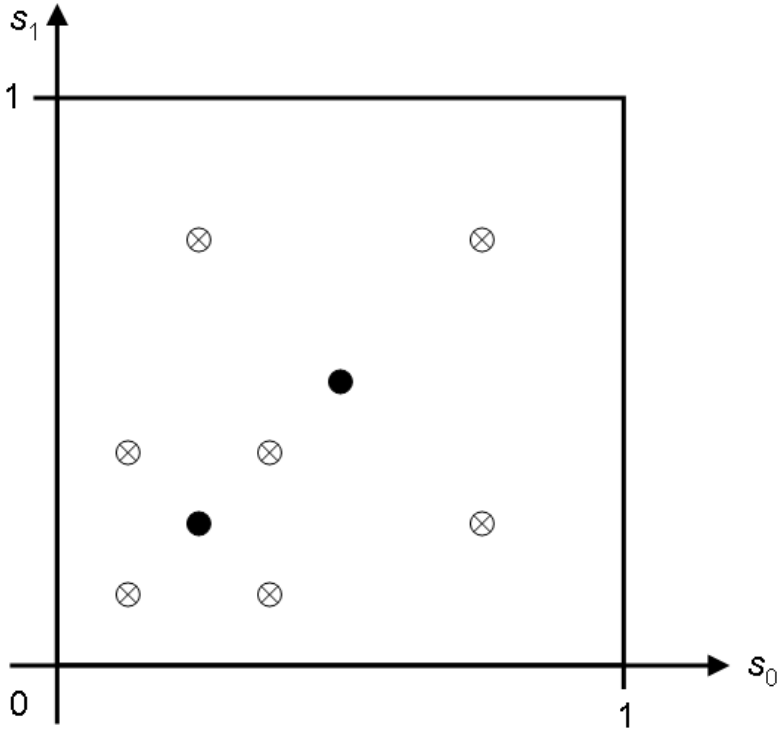

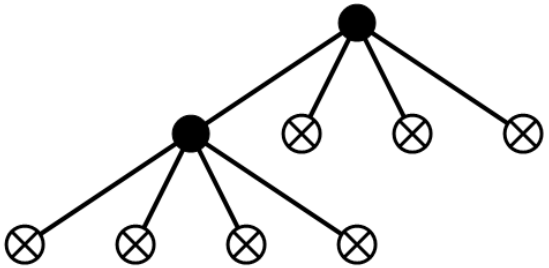

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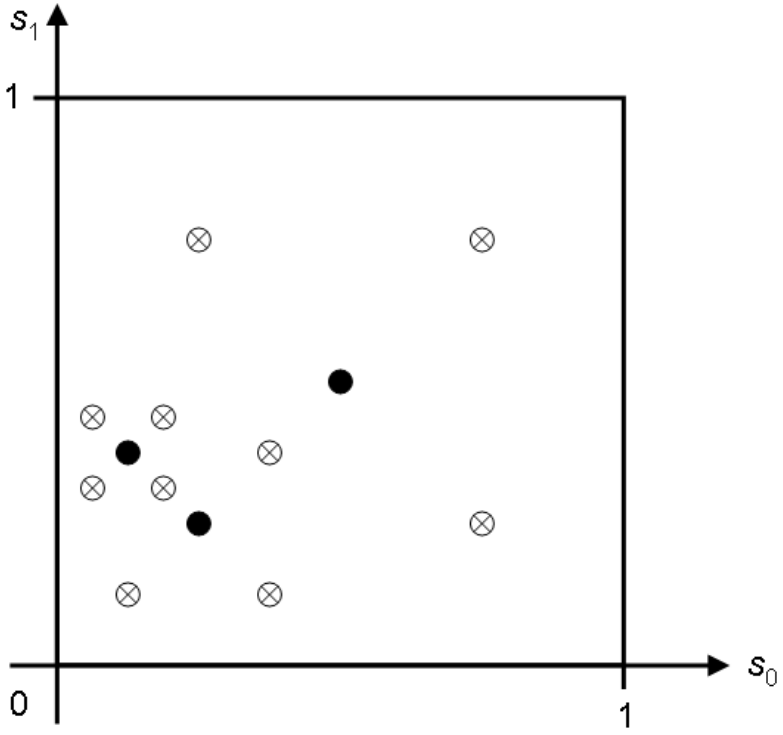
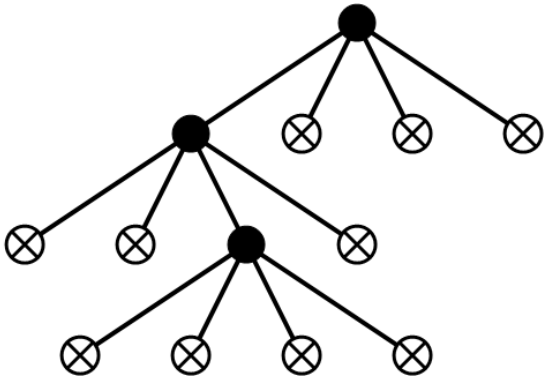




Error Estimation

Error estimator based on change in model solution:

1. Starting with original trial matrix $\tilde{V}$, add sample point and update to get V
2. Solve

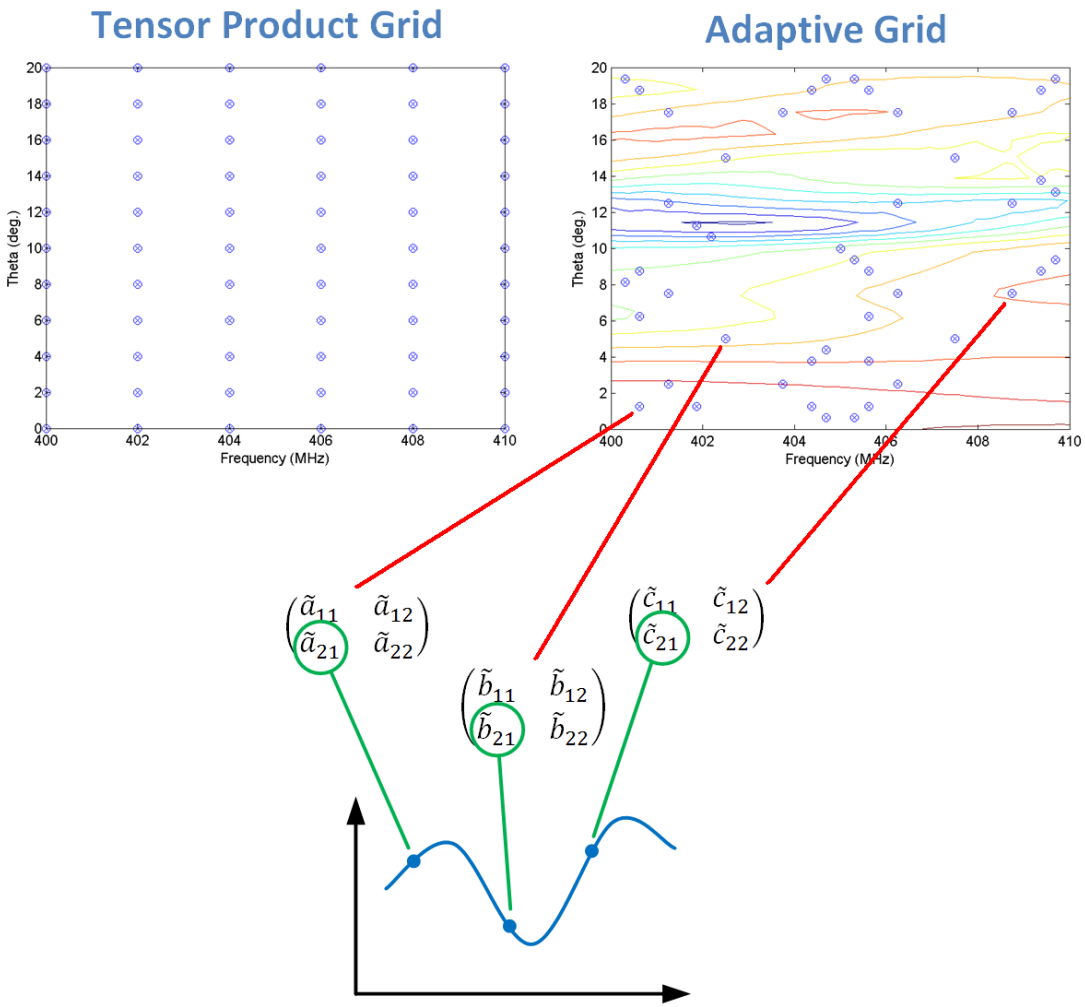
$$\tilde{V}^H A \tilde{V} \tilde{\mathbf{x}} = \tilde{V}^H \mathbf{b}$$
$$V^H A V \mathbf{x} = V^H \mathbf{b}$$

   for model solutions at each test point
3. Calculate

$$est.\ error = \frac{\|\tilde{V}\tilde{\mathbf{x}} - V\mathbf{x}\|}{\|V\mathbf{x}\|}$$

Reduced Matrix Interpolation

- In black-box approach, assume only
  - Solutions of $A(\mathbf{s})\mathbf{x} = \mathbf{b}(\mathbf{s})$
  - Multiplication of $A(\mathbf{s})\mathbf{c}$
  - Calculation of $\mathbf{b}(\mathbf{s})$.
- Explicit calculation of $\tilde{A} = V^H A V$ would spoil ROM efficiency
- Instead, interpolate entries of $\tilde{A}$ at sample points



Interpolation Functions

- In general, polynomial disadventages as interpolating functions (curse of dimensionality, etc.)
- Radial basis functions

$$f_i(\mathbf{s}) = \Phi(\|\mathbf{s} - \mathbf{s}_i\|)$$

  - Depends only on distance from sample point $\mathbf{s}_i$
  - Many choices for $\Phi(r)$:

$\sqrt{r^2 + a^2}$	mulit-quadratic
e^{ar^2}	Gaussian
$r^{2p} \log r$	thin-plate spline

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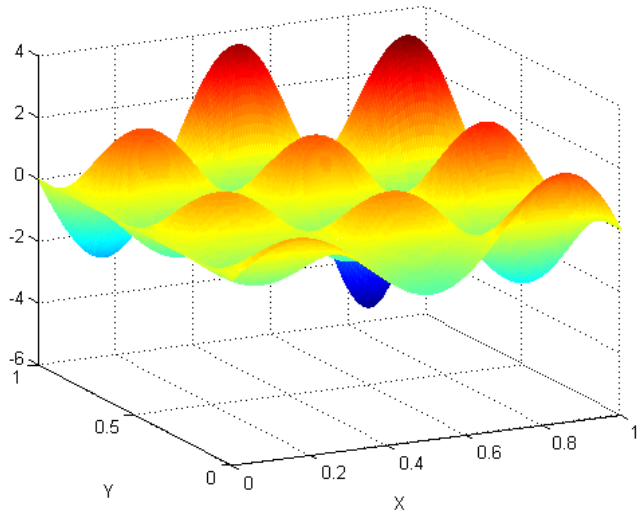
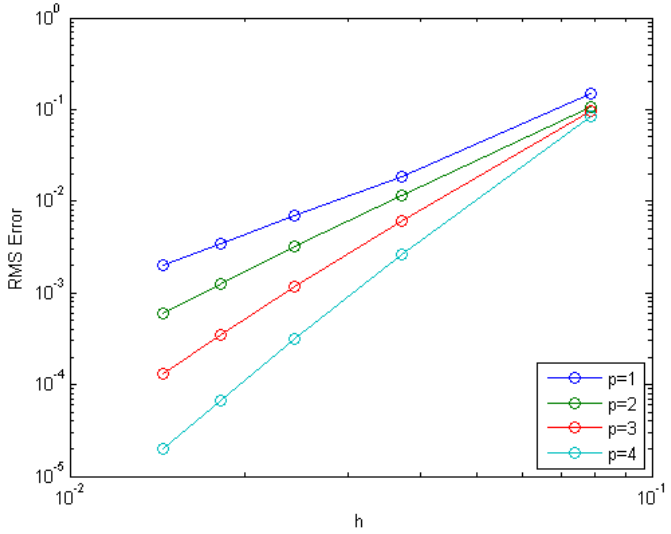
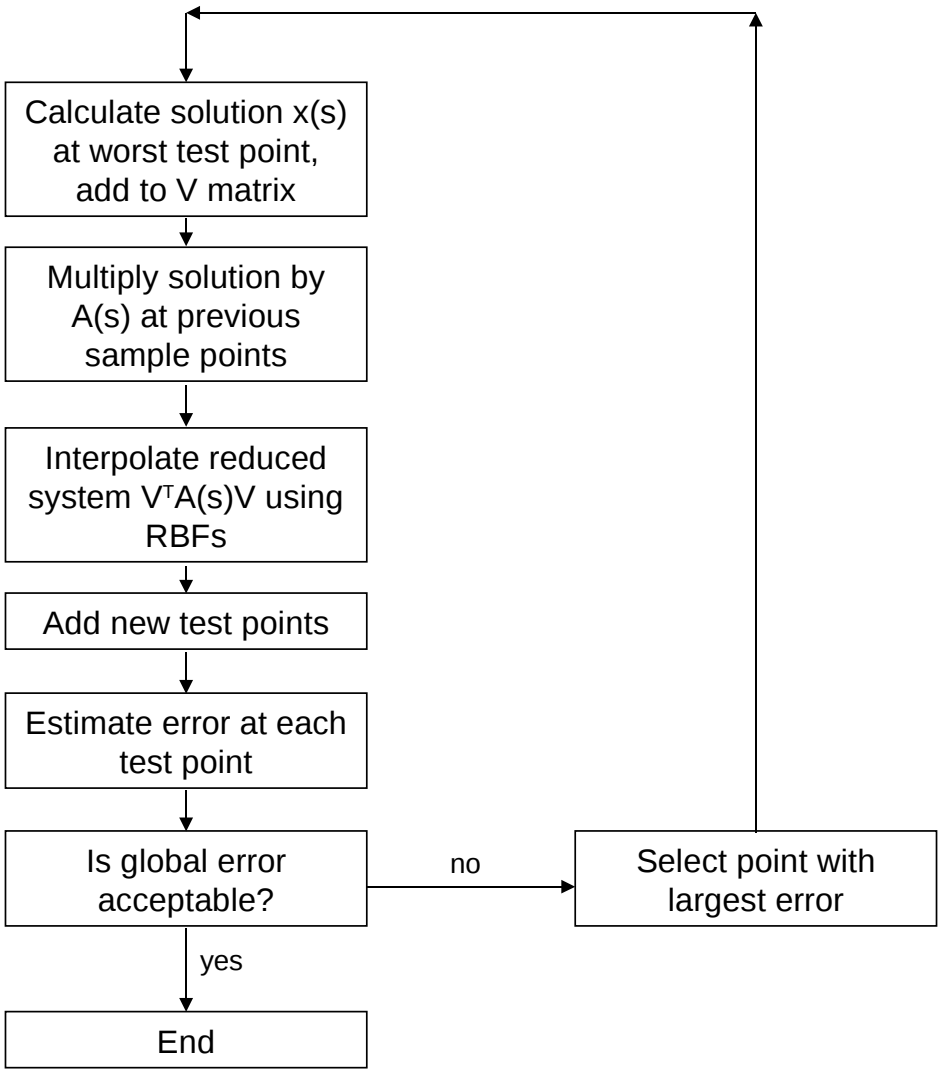


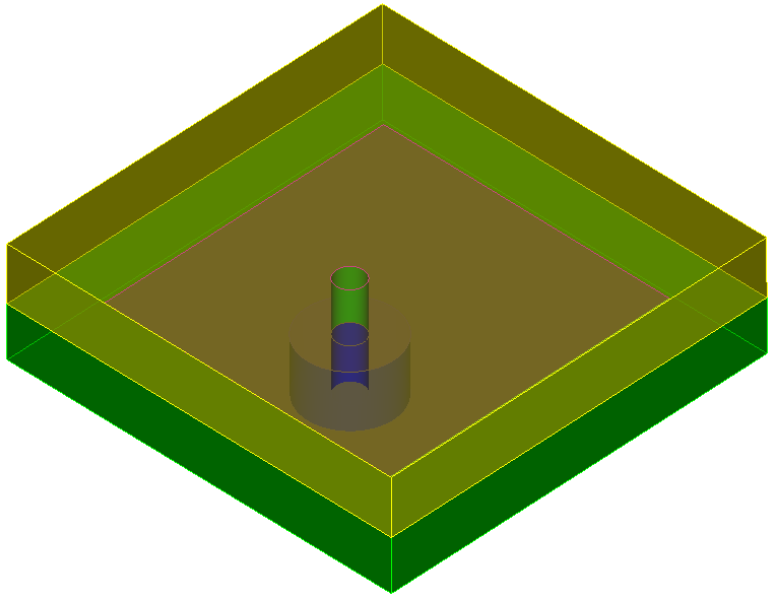


Summary


Calculate solution x(s) at worst test point, add to V matrix
Multiply solution by A(s) at previous sample points
Interpolate reduced system $V^T A(s) V$ using RBFs
Add new test points
Estimate error at each test point
Is global error acceptable?
no
Select point with largest error
yes
End

Patch Antenna

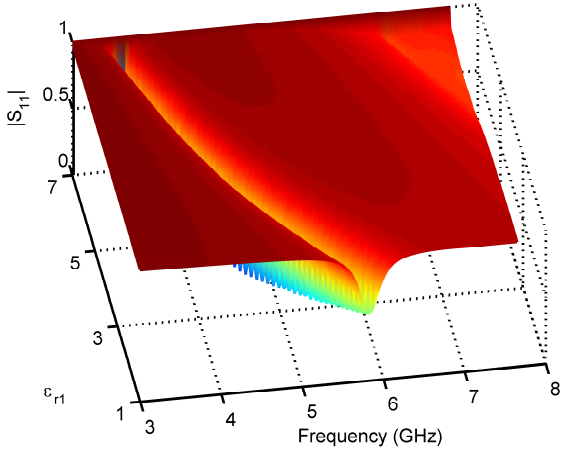
Parameter	Range
Frequency	3–8 GHz
ϵ_{r1} (bottom layer)	1–7
ϵ_{r2} (top layer)	1–7



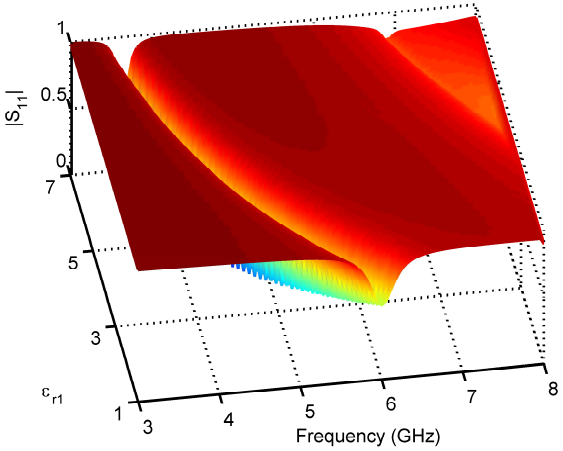
- 84,718 DOF
- Full solve time: 33 s

Patch Antenna Results

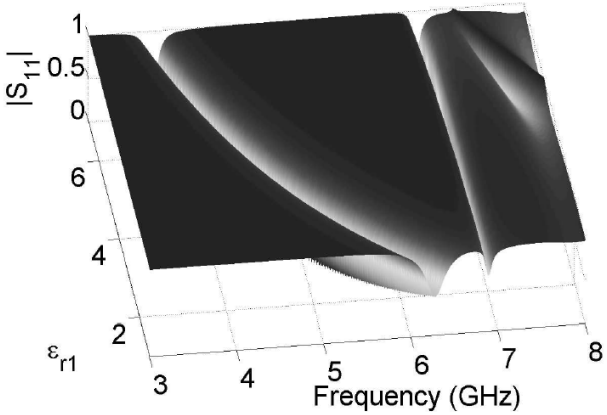
S_{11} with $\epsilon_{r2} = 3$



(a) $\tau = 0.1$



(b) $\tau = 0.01$

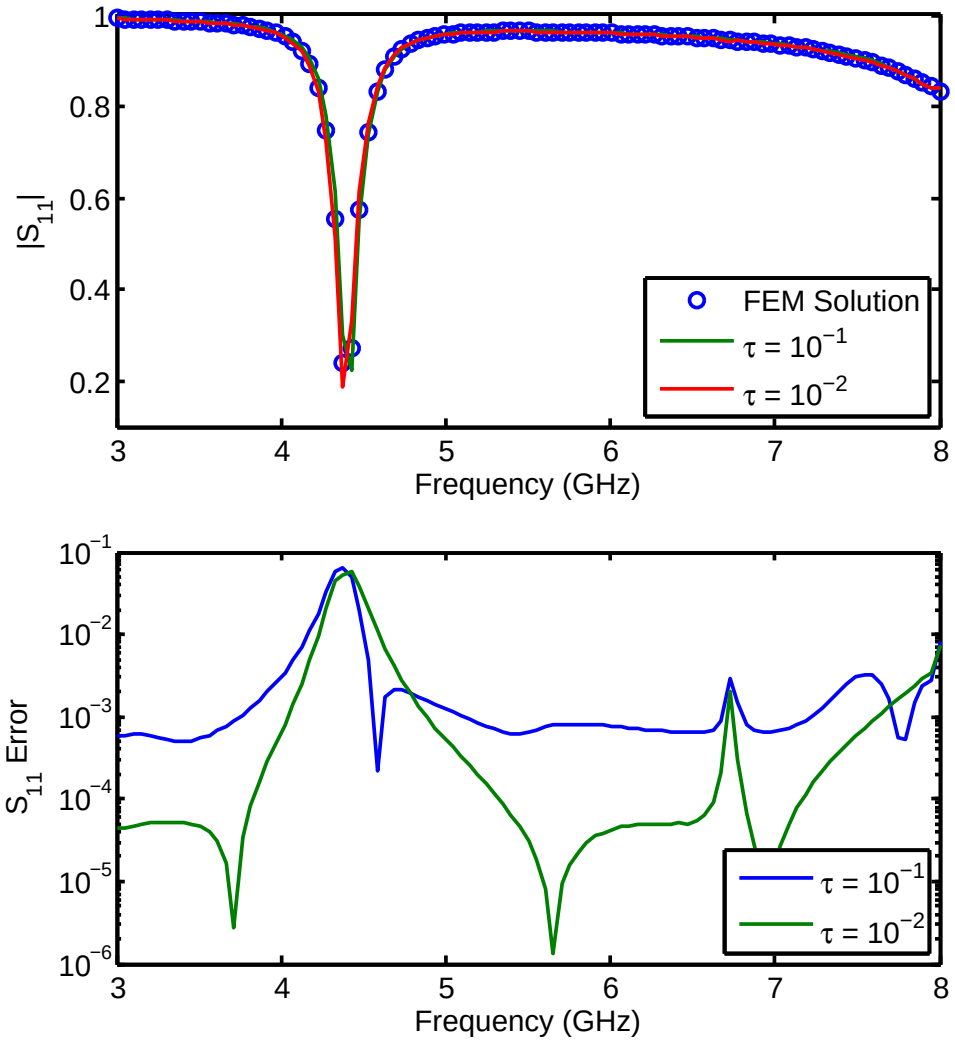


(c) Farle & Dyczij-Edlinger, TAP (in press)

	Tolerance $\tau = 0.1$	Tolerance $\tau = 0.01$	Farle & Dyczij-Edlinger
Model Dimension	22	39	165
Model Creation Time (hh:mm)	1:01	2:48	1:17
Model Evaluation Time (ms)	60.1	114	13.7

Patch Antenna ROM Error

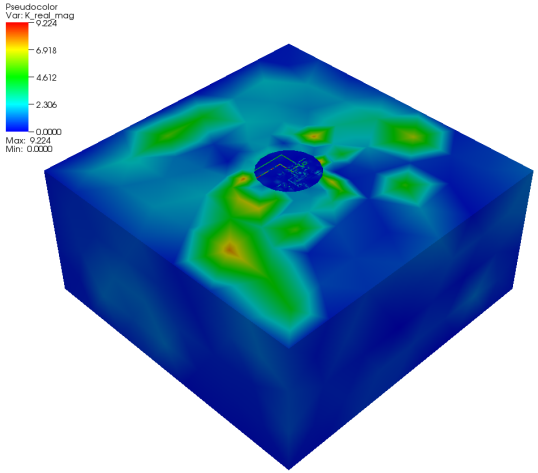
$$\epsilon_{r1} = 4, \ \epsilon_{r2} = 3$$

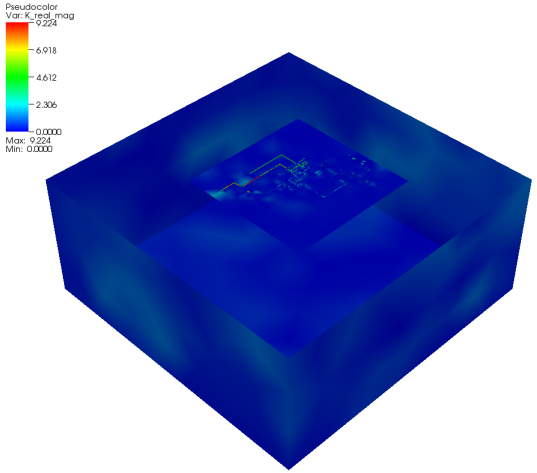


Oscillator Circuit

- Enclosure
  - 15 cm × 15 cm × 7 cm

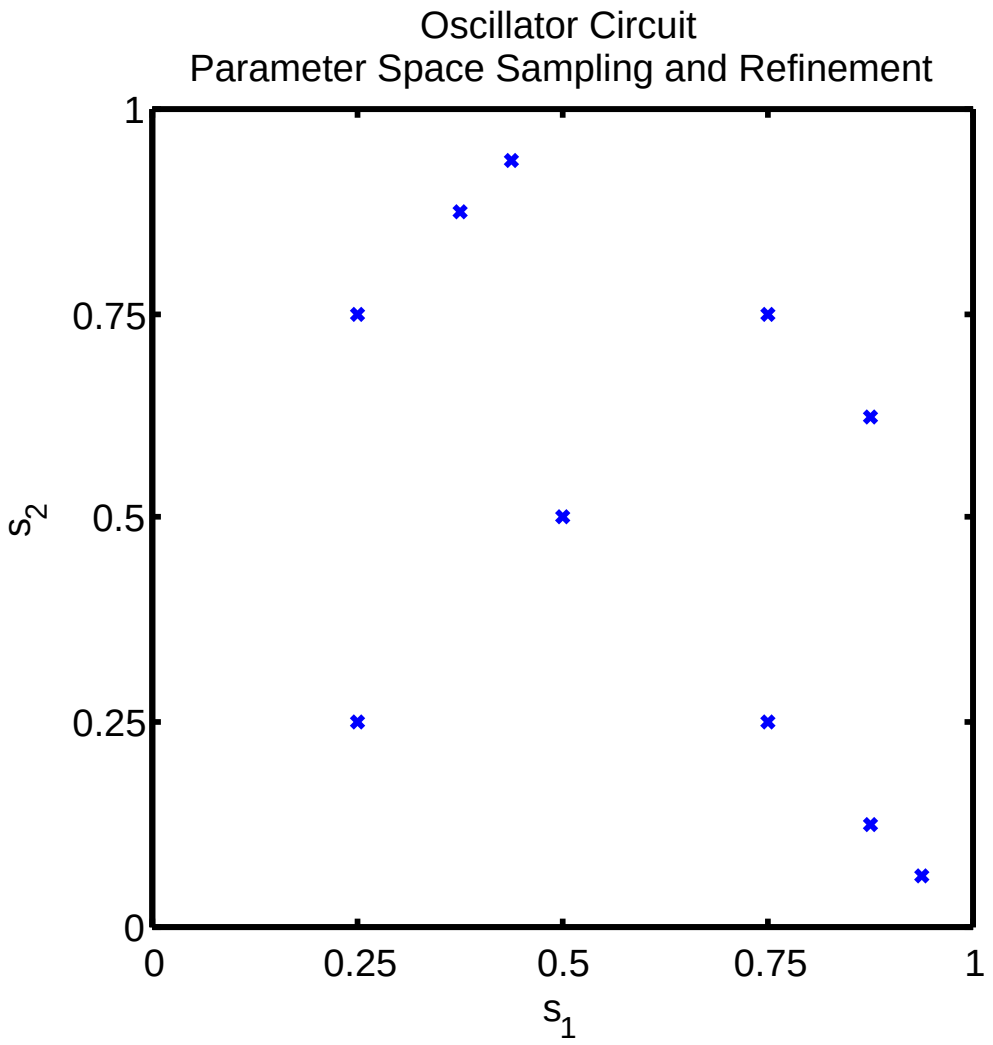
Frequency	2 GHz
Solution Method	EFIE
N	4122
MOR Tolerance	0.1
Samples	10
Time (mm:ss)[a]	27:15
Full Solution Time[a]	42 s
Model evaluation time	2.1 ms



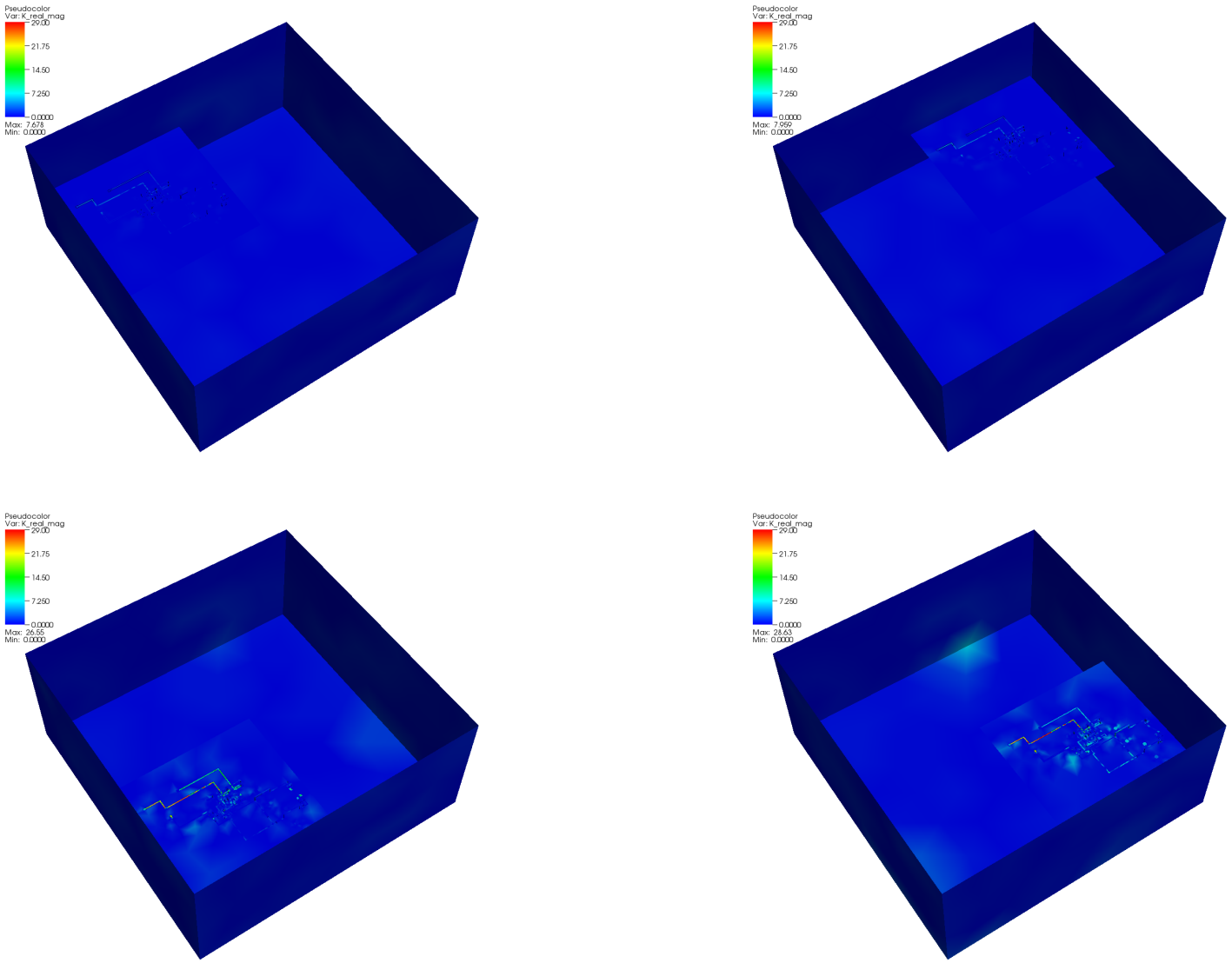


[a] Using two 8-core nodes

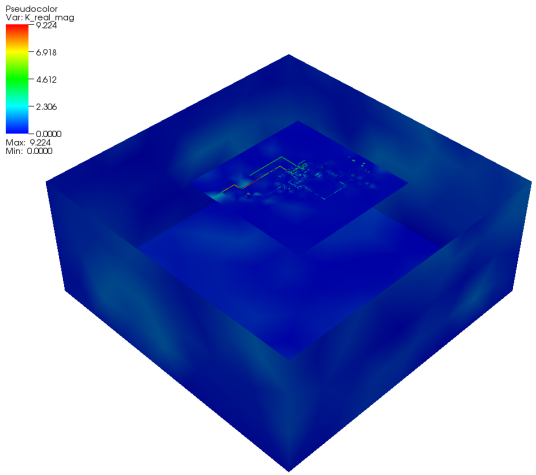
Sampling Locations

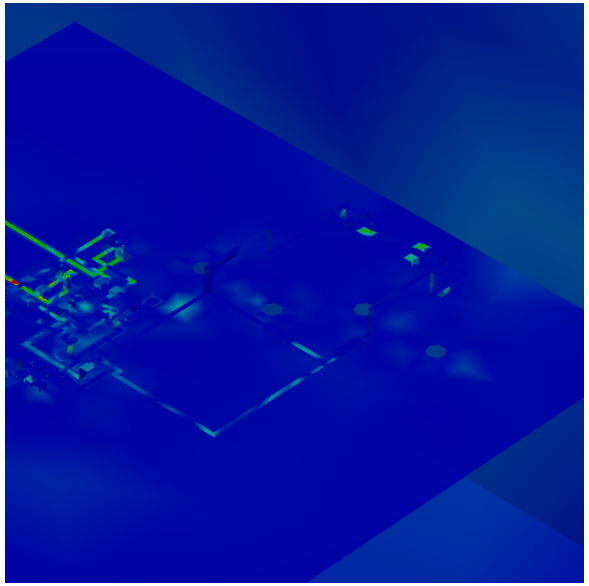


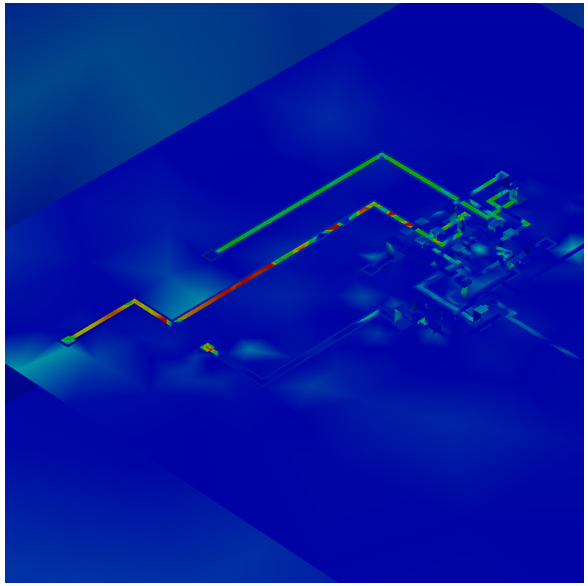
Changing CB Location

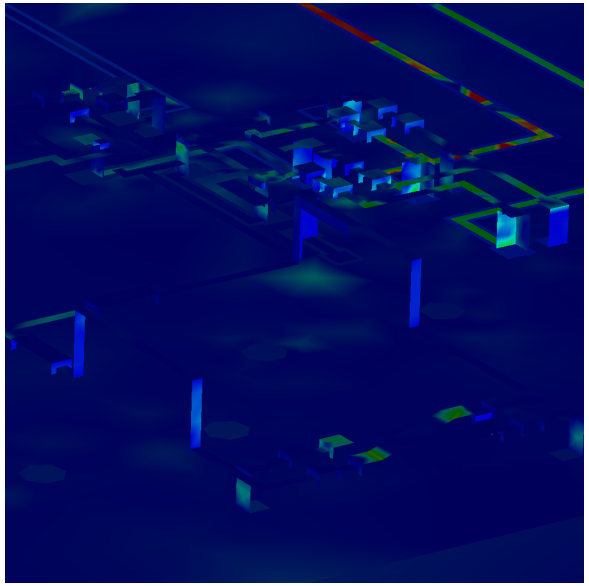


Solution Details









Conclusion

- Conventional MOR methods from FEM not easily applicable to other fomulations
  - requires black-box approach
- Error estimation allows adaptive sampling
- Radial basis function interpolation offers numerious advantages
  - easily to incrementally update model
  - generalizes well to multiple dimensions