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INFLUENCE OF THE DENSITY LAW ON VARIOUS FISSILE SINGLE UNIT AND ARRAY STORAGE METHODS

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INTRODUCTION

The advancement of computational technology has resulted in the wide-spread availability of powerful radiation transport Monte Carlo codes. Prevailing practices today rely heavily on Monte Carlo codes to provide the basis for assessing the reactivity of various fissile systems for nuclear criticality safety (NCS). In 1958, Weinberg and Wigner expressed their concerns on a "deplorable trend in reactor design - the tendency to substitute a code for a theory" [1]. Unfortunately, their concerns have largely become a reality in many modern NCS practices. Lacking the time or information to understand the underlying neutron physics of the fissile system under consideration is indeed a deplorable trend.

The purpose of this paper is to demonstrate that many features of criticality hand calculation methods [2] are indeed based upon the fundamentals of the density law and that many correlations of important physics parameters can be more easily understood from such a perspective. Historically, the density law was recognized by many pioneers in this field [3, 4], including during the Manhattan Project. However, it was by and large an "oral tradition" in that bits and pieces of great physical insights of the pioneers were scattered in many earlier publications. This paper attempts to bring together some of the "jewels" of the pioneers which might have been lost or forgotten.

INFLUENCE OF THE DENSITY LAW

Stratton said it well, "This is the density law in criticality physics which is simultaneously exact, simple, and useful. In a critical system, if the densities are increased everywhere to x times their initial value and all the linear dimensions are reduced 1/x times their initial value, the system will remain critical" [5]. This paper first summarizes the intrinsic facts about the density law including invariance of the diffusion equation and also the neutron transport equation under the density law transformation. Under the density law and k eigenvalue formulation, the system k_{eff} depends on the optical thickness of the system, i.e., $r \cdot \rho$, where r is the critical dimension at the density ρ. For a simple illustration, the density law can be expressed at the critical or near critical conditions for a bare sphere as Equation 1 and k_{eff} as Equation 2:

$$\frac{r}{r_o} = \frac{\rho_o}{\rho} \qquad (1)$$

$$k_{eff} = c \cdot r \cdot \rho \qquad (2)$$

where r is the radius of a sphere; r_o is the critical radius; ρ is the density of the fissile unit; ρ_o is the density at critical; and c is a constant depending on fissile material. Strictly speaking, the above formulation is valid only at a fixed $r \cdot \rho$ value. Although the constant c of Equation 2 was found to be fairly insensitive to conditions away from the critical condition, caution should be used in its range of applications [6].

The neutron characteristics of the two systems (the original system and the second system after the density law transformation) are the same. Our assessment indicates that many system parameters important to criticality hand calculation methods such as number of mean free paths, system criticality, and the surface mass density are conserved under the density law transformation. By assessing dependence of the system parameters including buckling, migration area, and fraction of neutron leakage on ρ or r, this provides great insight to realize that many parameters important to criticality assessment are closely influenced by the density law.

RESULTS

Single Unit

For a single unit fissile system, it is noted that the core density conversion is a direct application of the density law. The core density formulas can be directly derived from Equation 1. Furthermore, the material buckling is seen to be proportional to ρ^2, while the geometrical buckling is inversely proportional to r^2. The critical condition of equating material buckling to geometrical buckling is also the influence of the density law. It follows then that the buckling conversion method is indeed an application of the density law as well.

Even with these simple Equations 1 and 2 from the density law, it can be readily shown that k_{eff} of a subcritical bare spherical unit of the same density can be expressed simply as the ratio of r/r_o. Furthermore, k_{eff} can be derived to be $F^{1/3}$ for a bare sphere, where F is the fraction of critical mass [7]. There are many other benefits in understanding neutron leakage from the density law perspective. For example, the average escape probability is inherently related to the chord length distribution of a unit and the average number of the mean free paths from a neutron located in this unit to its boundaries. The conservation of the number of mean free paths under influence of the density law gives a better understanding of many such formulations and applications.

Arrays

The density-analog techniques may be seen as an extension of the application of the density law under the hypothesis that critical cubic arrays behave similarly to homogeneous systems. The two postulates [8] of the NB^2_N method, (a tremendous achievement and contribution from Joe Thomas, one of the pioneers in the criticality safety field), may be deduced and understood from influence of the density law. Furthermore, the conservation of NB^2_N, a measure of the array neutron leakage for array storage application, may be seen as built on the basis of the density law influence.

CONCLUSIONS

The density law played a great role in many of the basic correlations of key parameters important to nuclear criticality safety. Parameters such as leakage fraction, surface density, non-leakage probability, chord length distribution, migration area, array density, number of mean free paths, and k_{eff} can be better understood from the density law perspective. The "oral tradition" of the density law passed down by a cadre of unusually gifted pioneers in the field provides a great framework in understanding the underlying concepts of many correlations, guidance, and calculation methods. Indeed, the concept of the density law can provide criticality safety practitioners a key gateway to understand neutron characteristics of fissile systems on a more comprehensive basis.

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