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A MODIFIED TREATMENT OF SOURCES IN IMPLICIT MONTE CARLO RADIATION TRANSPORT

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We describe a modification of the treatment of photon sources in the IMC algorithm [1]. We describe this modified algorithm in the context of thermal emission in an infinite medium test problem at equilibrium and show that it completely eliminates statistical noise.

Let us examine the case of an infinite medium problem with gray, constant opacities with matter and radiation at equilibrium at temperature T, simulated with the IMC method. The census photons representing the initial radiation all initially have time $t = 0$ and have a total energy aT^4. During a time step of size Δt, the energy of each census photon will decrease by a factor $\exp[-\sigma c \Delta t]$. The total energy in census photons at the end of the time step will therefore be

$$E_{census}(t = \Delta t) = aT^4 \exp[-\sigma c \Delta t]. \tag{1}$$

To simulate thermal emission, we will make N_s thermal source photons, each with a different initial time $t_{i,p}$ in $[0, \Delta t]$. We will assume all N_s photons have the same initial energy $ac\sigma_P T_0^4 V \Delta t / N_s$. Since $t_{i,p}$ is different for each thermal source photon, they will all reach time Δt with different energies $E_p(t = \Delta t) = E_p(t = 0) \exp[-\sigma c(\Delta t - t_{i,p})]$. The sum of these energies will be

$$E_{thermal} \equiv \sum_{p=1}^{N_s} E_p(t = \Delta t) = aT_0^4 V c\sigma \Delta t \frac{1}{N_s} \sum_{p=1}^{N_s} \exp[-\sigma c(\Delta t - t_{i,p})] \ . \tag{2}$$

Since

$$\lim_{N_s \to \infty} \frac{1}{N_s} \sum_{p=1}^{N_s} exp[-\sigma c(\Delta t - t_{i,p})] = \frac{1}{\Delta t} \int_0^{\Delta t} exp[-\sigma c(\Delta t - \tau)] \mathrm{d}\tau = \frac{1 - exp[-c\sigma \Delta t]}{c\sigma} \ , \tag{3}$$

the sum in Eq.(2) is a Monte Carlo estimate for an integral over all possible thermal emission times. Using Eq.(2) and Eq.(3), we find that, in the limit of a large N_s, the radiation energy due to thermally emitted photons at $t = \Delta t$ will be

$$E_{thermal}(t = \Delta t) = aT_0^4 V(1 - exp[-c\sigma\Delta t]), \tag{4}$$

so $E_{census} + E_{thermal} = aT_0^4 V$, which is the value necessary to maintain thermal equilibrium. The matter energy will also be the same as the initial value, by energy conservation. With a finite number of photons, we will not maintain thermal equilibrium exactly, because the sum in Eq.(2) will only approximate the integral, with an error that is proportional to $N_s^{\frac{1}{2}}$ [2].

This is illustrated in Fig. 1. This plot shows an IMC simulation using one zone, a cube with unit length in each direction. All faces have reflecting boundaries, making it effectively an infinite medium problem. The material and radiation temperatures were initialized to 1. The material has a heat capacity c_v = 1.0, and an absorption opacity $\sigma = 10$. The simulation used 100 photons per time step, and units were chosen so that $a = c = 1$. The simulation used $\Delta t = 0.001$ from $t = 0$ to $t = 1$, $\Delta t = 0.01$ from $t = 1$ to $t = 2$, and $\Delta t = 0.1$ for $t > 2$.

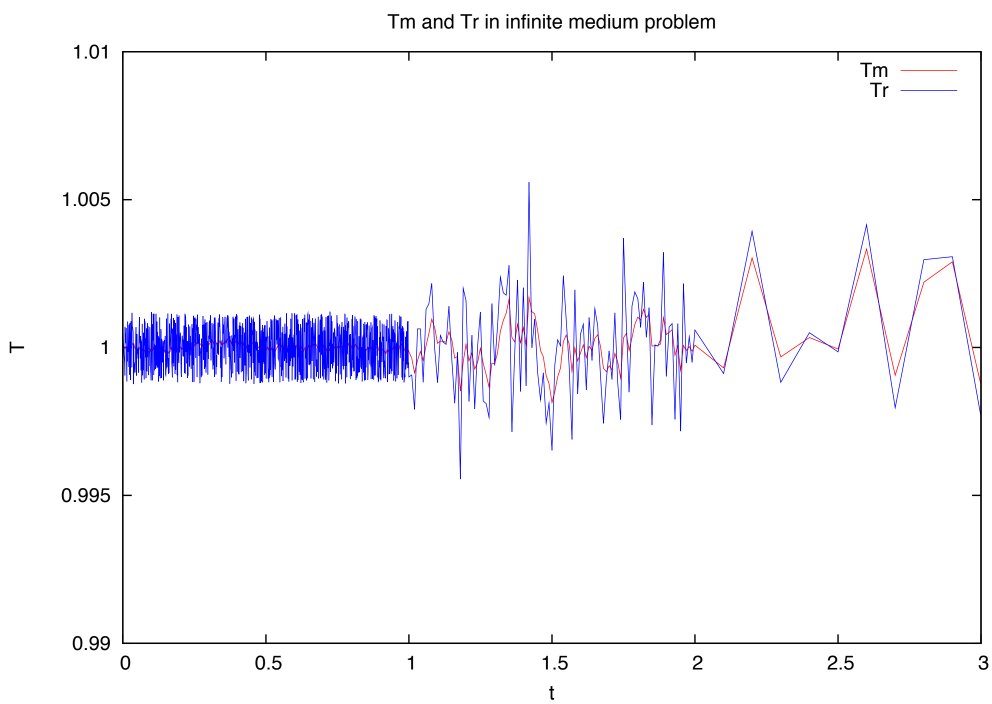

Figure 1. Matter and radiation temperature for an infinite medium test problem simulated with IMC with three different values for the time step Δt.

In the IMC algorithm, we regard physical quantities like opacity and heat capacity as constant throughout the time step. This means that the distance to scatter, the amount of absorption on a given photon path, etc., are independent of when they occur during the time step. We can therefore calculate the contribution that is made for every emission time in $[t^n, t^{n+1}]$ for each path taken by a source photon. This modified IMC algorithm is best illustrated by describing how we simulate the behavior of a source photon. As in IMC, we sample a position, direction, frequency, and energy E_p for each source photon. However, we do not sample an emission time. The energy emitted in a small period $[t_e, t_e + dt]$ will be $dE = \frac{E_p}{\Delta t} dt$. So we can think of a source photon as a large number of small photons with emission times spread out evenly between t^n to t^{n+1}, each with energy dE. Instead of the source photon having a time, it will have a distance traveled, s_p, with an initial value of 0. The photon will travel a total distance $c\Delta t$ in the time step, unless it exits the problem through a boundary. Some fraction of its energy will reach census on each path, and some fraction will be absorbed. These fractions will be functions of the initial and final value of s_p on the path. The total energy $E_p(s_p = 0)$ of the source photon will be either absorbed, leave the problem through a boundary, or reach census as the photon reaches $s_p = c\Delta t$. That is, $E_p(s_p = c\Delta t) = 0$.

First, we will calculate the amount of energy that reaches census on a given path from s_0 to $s_1 = s_0 + d_p$. Because of absorption, $dE(s_0 + d_p)$ = $dE(s_0)[1 - exp(-\sigma_a d_p)]$. The energy emitted in $[t^{n+1} - s_0/c, t^{n+1} - s_1/c]$ will reach census during the path. A photon that has moved a distance s_0 has a total energy consisting of emission that occurred in $[t^n, t^{n+1} - s_0/c]$. This range has a size of $t^{n+1} - s_0/c - t^n = \Delta t - s_0/c$, so

$$dE(s_0) = \frac{E_p(s_0)}{[\Delta t - s_0/c]} \tag{5}$$

and

$$dE(s) = dE(s_0) exp[-\sigma(s - s_0)]. \tag{6}$$

The total energy reaching census is the integral of Eq.(6) over the range $[s_0, s_1]$:

$$E_c(s_0, s_1) = \int_{s_0}^{s_1} dE(s) = \frac{E_p(s_0)}{[\Delta t - s_0/c]} \int_{s_0}^{s_1} exp[-\sigma(s - s_0)] = \frac{E_p(s_0)}{\sigma[\Delta t - s_0/c]} \left[1 - exp[-\sigma(s_1 - s_0)]\right]. \tag{7}$$

Next, we will calculate the amount of energy that is absorbed on the path from s_0 to s_1. This is done by conservation of energy. The photon energy at s_1 is related to the photon energy at s_0 by

$$E_a(s_0, s_1) + E_c(s_0, s_1) + E_p(s_1) = E_p(s_0). \tag{8}$$

$E_p(s_1)$ in Eq.(8) is given by Eq.(5). This yields

$$E_a(s_0, s_1) = E_p(s_0)(1 - exp[-\sigma(s_1 - s_0)]) \frac{\Delta t - s_1/c}{\Delta t - s_0/c} - E_c(s_0, s_1). \tag{9}$$

The modified IMC results for the test problem described earlier are show in Fig. 2

Fig. 2 shows that there is no statistical noise in the modified IMC simulation for any value of Δt. This happens because the source photons in the modified IMC algorithm each contribute exactly the amount of

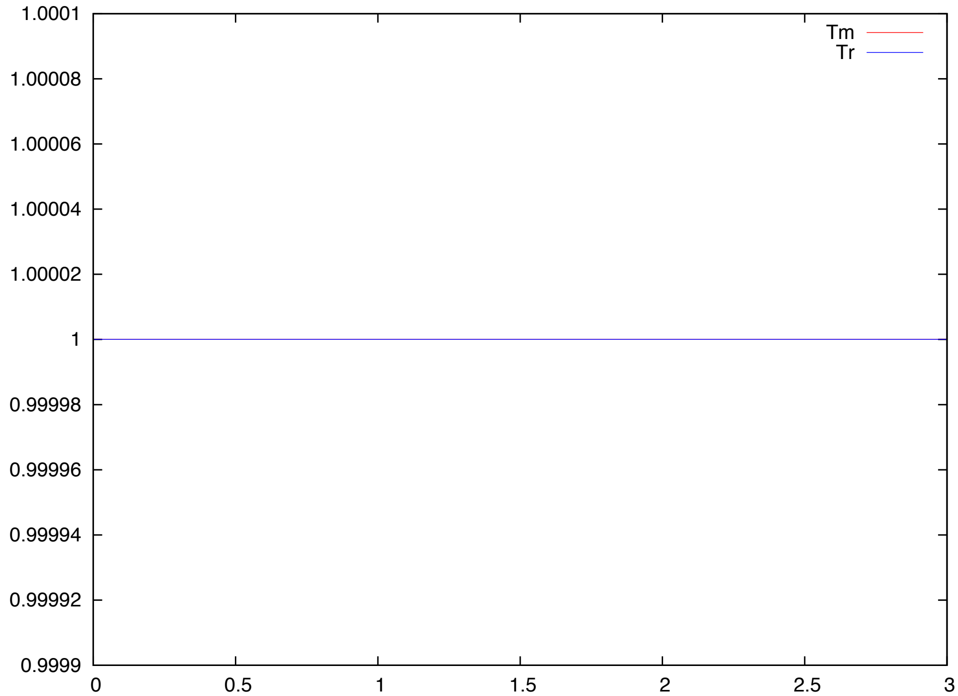

Figure 2. Matter and radiation temperature for infinite medium test problem using modified IMC with three different time steps

energy calculated in Eq.(2) to census. In effect, the integrand in Eq.(3) is evaluated exactly, not approximately as a sum over a finite number of emission times. The value of T_r at the end of the time step is 1.0 to roundoff. By conservation of energy, $T_m = 1.0$ to roundoff at the end of the time step also, and so these values are maintained in subsequent time steps of the calculation.

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