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Open Boundary Conditions for Dissipative MHD

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Abstract

In modeling magnetic confinement, astrophysics, and plasma propulsion, representing the entire physical domain is often difficult or impossible, and artificial, or “open” boundaries are appropriate. A novel open boundary condition (BC) for dissipative MHD, called Lacuna-based open BC (LOBC), is presented. LOBC, based on the idea of lacuna-based truncation originally presented by V.S. Ryaben’kii and S.V. Tsynkov [1], provide truncation with low numerical noise and minimal reflections. For hyperbolic systems, characteristic-based BC (CBC) exist for separating the solution into outgoing and incoming parts. In the hyperbolic-parabolic dissipative MHD system, such separation is not possible, and CBC are numerically unstable. LOBC are applied in dissipative MHD test problems including a translating FRC, and coaxial-electrode plasma acceleration. Solution quality is compared to solutions using CBC and zero-normal derivative BC. LOBC are a promising new open BC option for dissipative MHD.

[1] V.S. Ryabenkii et al., J. Comput. Phys., 174 (2001) 712

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Motivation

- To model infinite (or very large) domains, open boundaries are often needed to limit the computational domain size without influencing the solution in the domain of interest.
- Dissipative MHD (a mixed hyperbolic-parabolic equation system) presents special challenges for open BC, including
  - high thermal and magnetic diffusion
  - flows and waves oblique to open boundaries
- Hyperbolic-based BC have proven inadequate. A general open BC is needed.

Outline

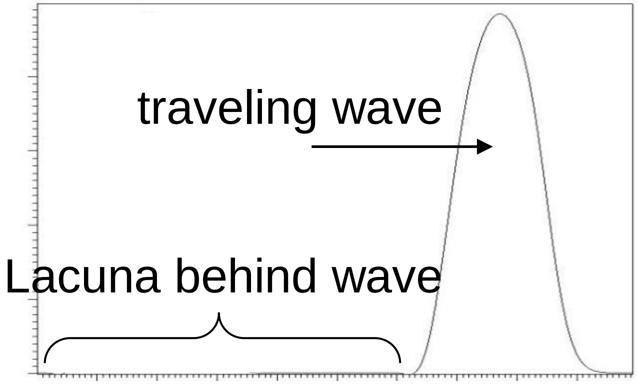
- Description of Lacuna-based open BC (LOBC)*
- Alternative open BCs
  - Approximate Riemann open BC (ARBC)
  - Thompson open BC (TBC)
  - Zero normal derivative (ZND)
- Test problems and results
  - Pressure pulse
  - FRC translation
  - Plasma acceleration
- Conclusions

* LOBC have been developed for single wave-speed hyperbolic systems by Ryaben'kii, Tsynkov *et al.* See V.S. Ryaben'kii, S.V. Tsynkov, V.I. Turchaninov, J. Comp. Phys. 174 (2001) 712.

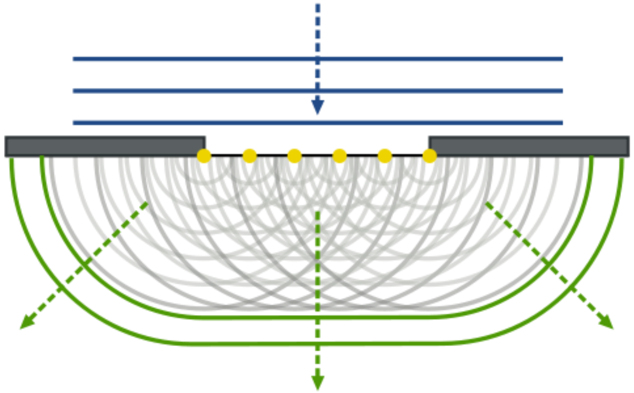
Lacuna-based open BC (LOBC)

Lacunae are still regions behind waves in hyperbolic systems

- Lacunae are easily observed in the 1D scalar wave equation.



- Huygens (1629-1695) used the concept of discrete propagation of individual wavelets to explain diffraction.



A transition region is used to generate sources for an auxiliary problem

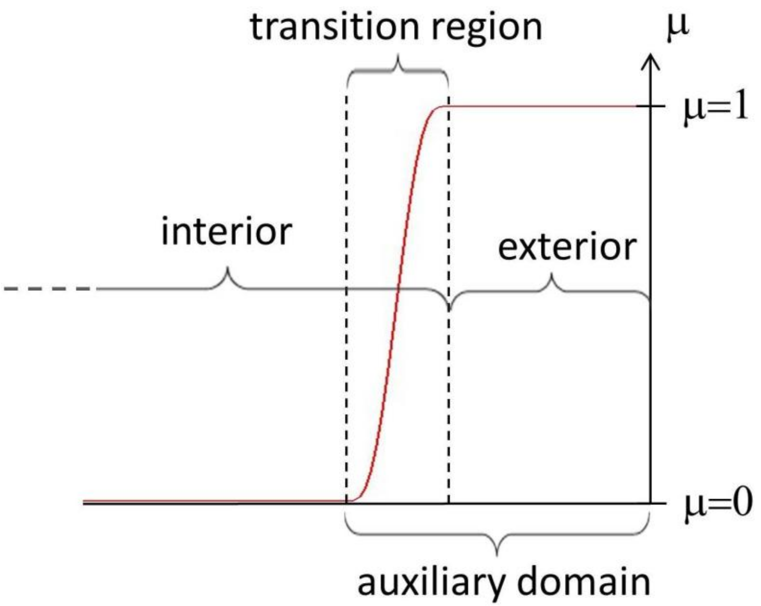
- Interior problem

$$\frac{\partial \mathbf{q}}{\partial t}+\frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{q})=\mathbf{S}(\mathbf{q})$$

- Auxiliary problem

$$\frac{\partial \mathbf{w}}{\partial t}+\frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{w})=\mathbf{S}(\mathbf{w})+\mathbf{\Omega}(\mathbf{q})$$

- To determine Ω, substitute μ**q** for **w** and solve.



$$\mu\frac{\partial \mathbf{q}}{\partial t}=-\mu\frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{q})+\mathbf{S}(\mathbf{q})$$

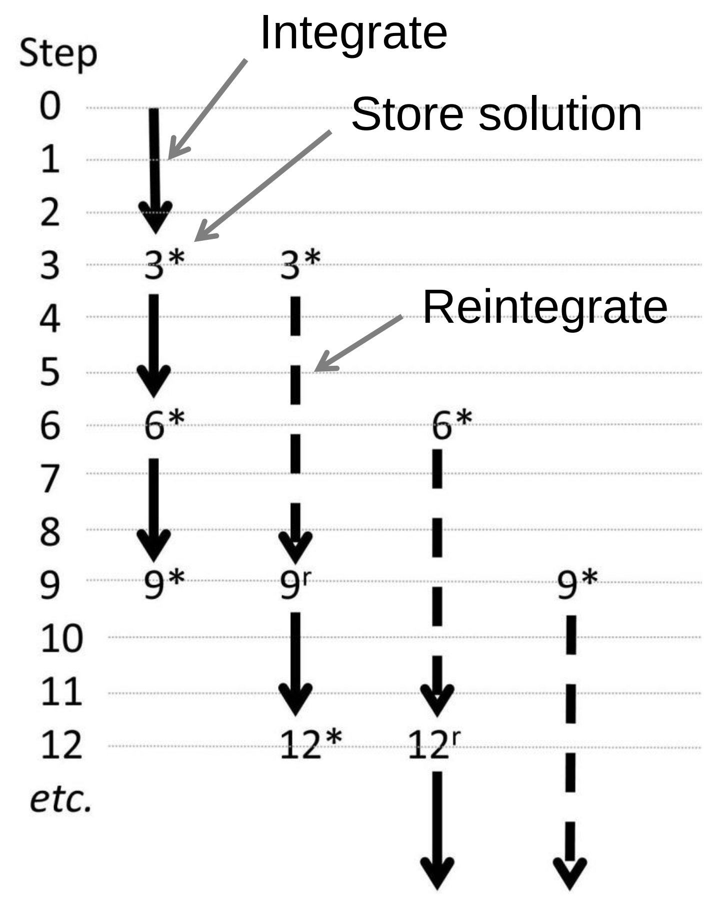
$$\Rightarrow \mathbf{\Omega}=\frac{\partial}{\partial x_i}\mathbf{F}^i(\mu\mathbf{q})-\mathbf{S}(\mu\mathbf{q})-\mu\frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{q})+\mathbf{S}(\mathbf{q})$$

Auxiliary problem is reintegrated; "lagging" source contributions are truncated

- Transition sources drive auxiliary problem solution to match interior solution at interior/exterior interface.
- Problem is reintegrated as required to prevent transition source terms from generating waves that reach the auxiliary problem boundary.

$$\mathbf{w}\Big|_{t_2} = \mathbf{w}\Big|_{t_0} + \int_{t_1}^{t_2} \frac{\partial \mathbf{w}}{\partial t}\, dt$$

$$= \int_{t_1}^{t_2} \left(-\frac{\partial}{\partial x_i} \mathbf{F}^i(\mathbf{w}) + \mathbf{S}(\mathbf{w}) + \mathbf{\Omega} \right) dt$$



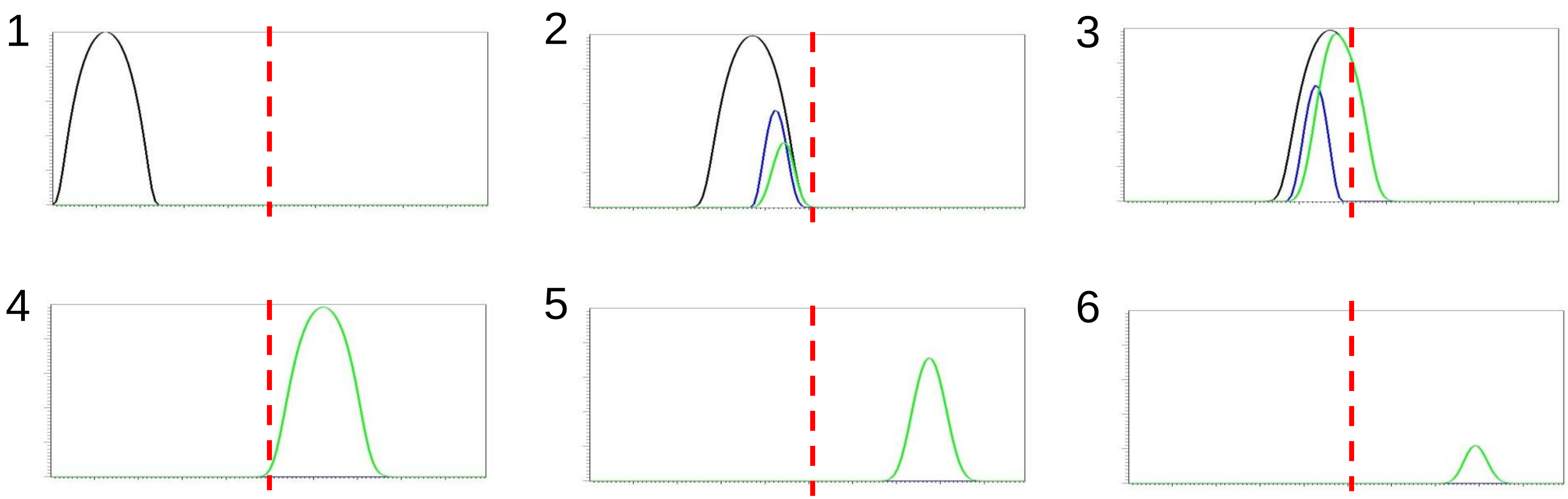
Reintegration damps hyperbolic features; parabolic physics is bounded by conventional BC

black = wave
green = auxiliary solution
blue = trans. source

open boundary:

1: Right-propagating wave is initialized.

2-3: Source terms drive auxiliary solution to match the main solution.



4-6: Solution is damped in exterior region as source terms are eliminated from the reintegration.

LOBC provide perfect non-reflection only under certain circumstances

- True lacunae are present only in systems with odd dimensionality (1D / 3D)*.
- True lacunae are present only if the system is purely hyperbolic.
- To capture lacunae when multiple wave speeds are present, the slowest wave must exit the transition region.

* R. Courant and D. Hilbert, Methods of Mathematical Physics, Volume II, Wiley, New York, 1962

Alternative open BCs

LOBC is compared to alternative techniques

- Approximate Riemann BC (ARBC)
  - Incoming flux ($\mathbf{F}_n{}^+$) and outgoing flux ($\mathbf{F}_n{}^-$) are found via characteristic analysis.
  - Prescribe exterior conditions to specify incoming flux.

- Thompson open BC (TBC)
  - Variations of incoming flux ($\frac{\partial}{\partial n}\mathbf{F}_n{}^+$) and outgoing flux ($\frac{\partial}{\partial n}\mathbf{F}_n{}^-$) are found via characteristic analysis.
  - Set variation of incoming flux to zero.

- Zero normal derivative BC (ZND)
  - Simply enforce zero normal derivative for all quantities.

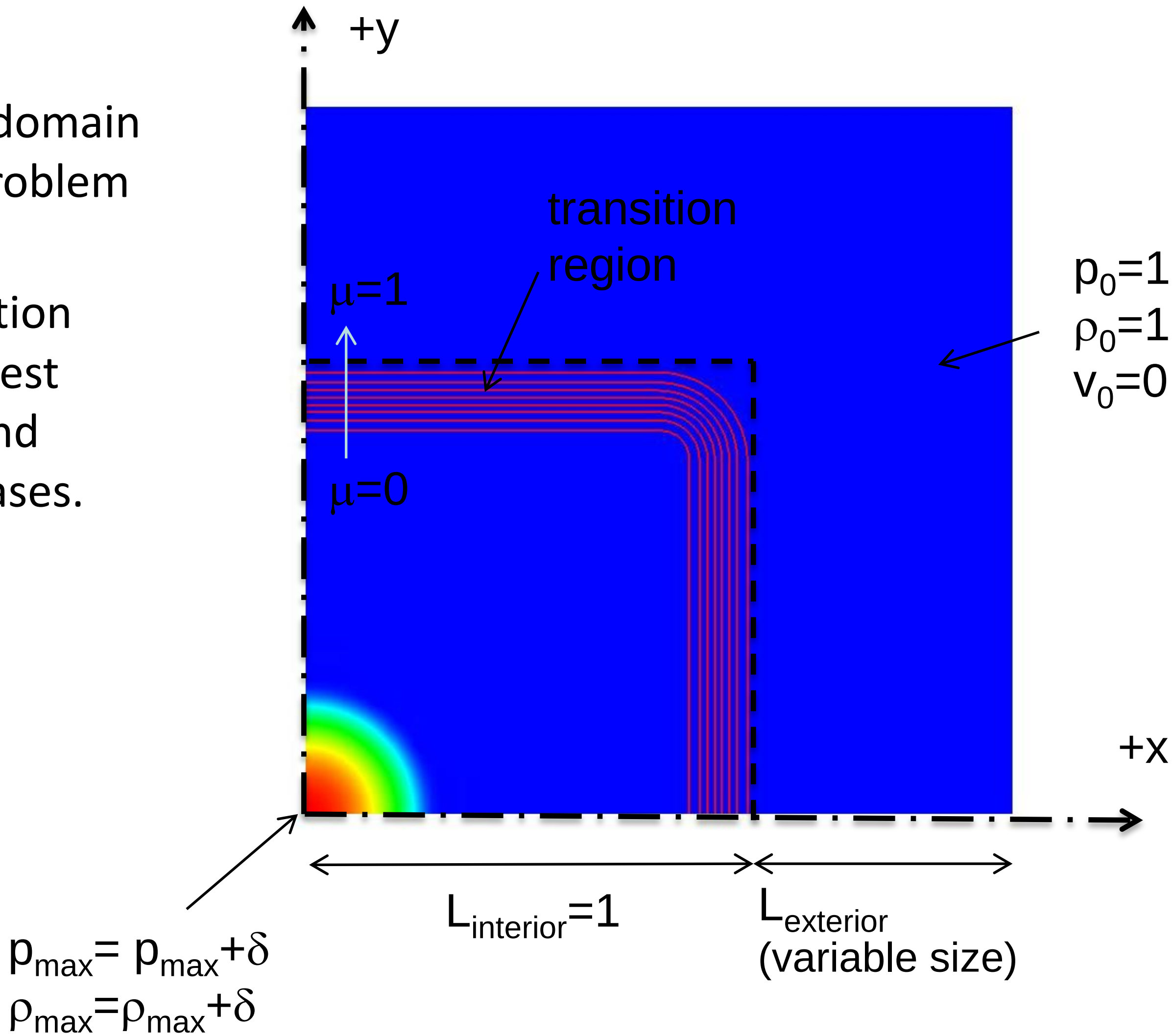
Test problems and results

Error evaluation: L_∞-norm of pressure error is found based on reference solution

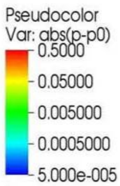
- In each of the problems presented, reference cases are computed using domains large enough to eliminate boundary effects.
- L_∞-norm of pressure error (i.e., the maximum error) is normalized by the maximum pressure in the reference simulation.

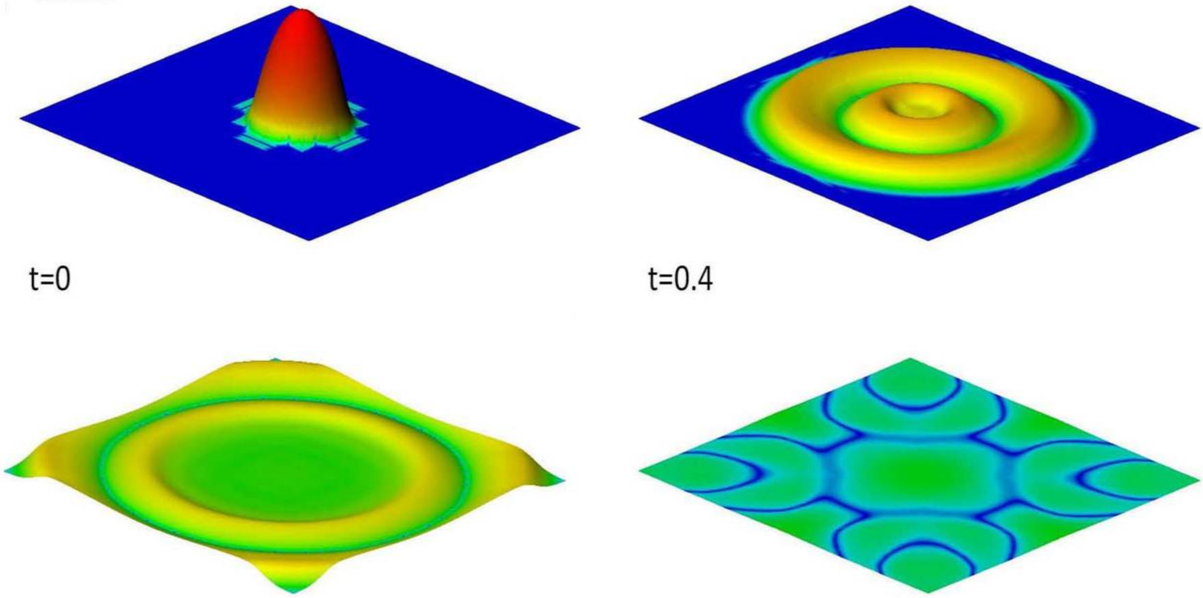
Pressure pulse problems explore LOBC performance

- 2D domain is shown; 1D domain is the restriction of the problem to the x-axis.
- Large and small perturbation sizes (δ=10^{-3} and δ=0.5) test (non-dissipative) linear and (dissipative) non-linear cases.



Nonlinear problem with ARBC





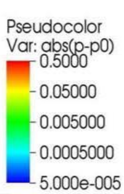
t=0

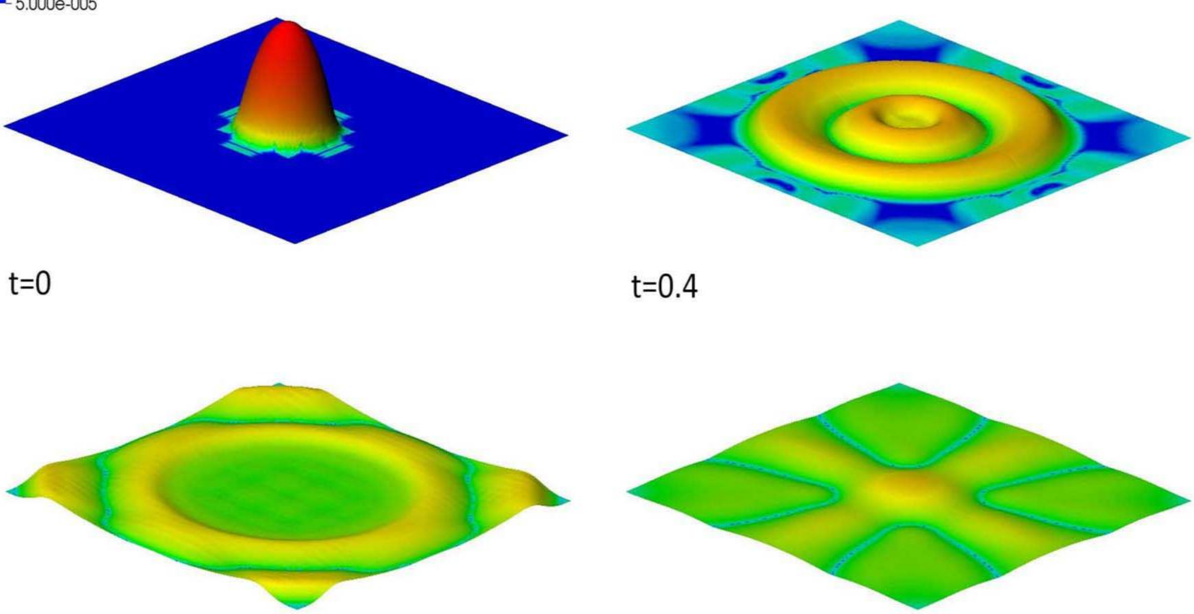
t=0.4

t=0.8

t=1.6

Nonlinear problem with ZND BC





t=0

t=0.4

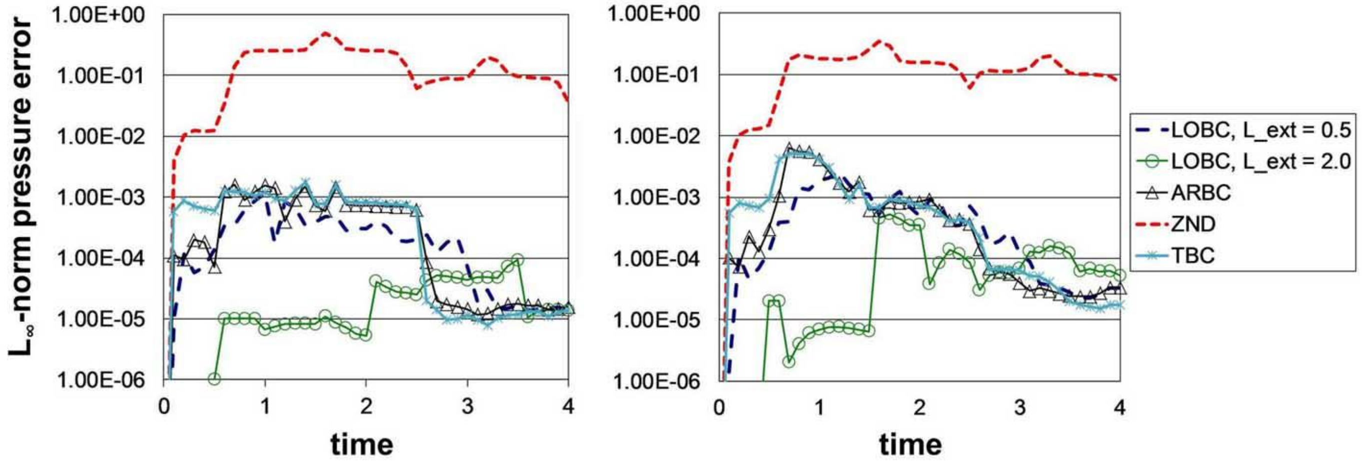
t=0.8

t=1.6

(linear) (nonlinear)

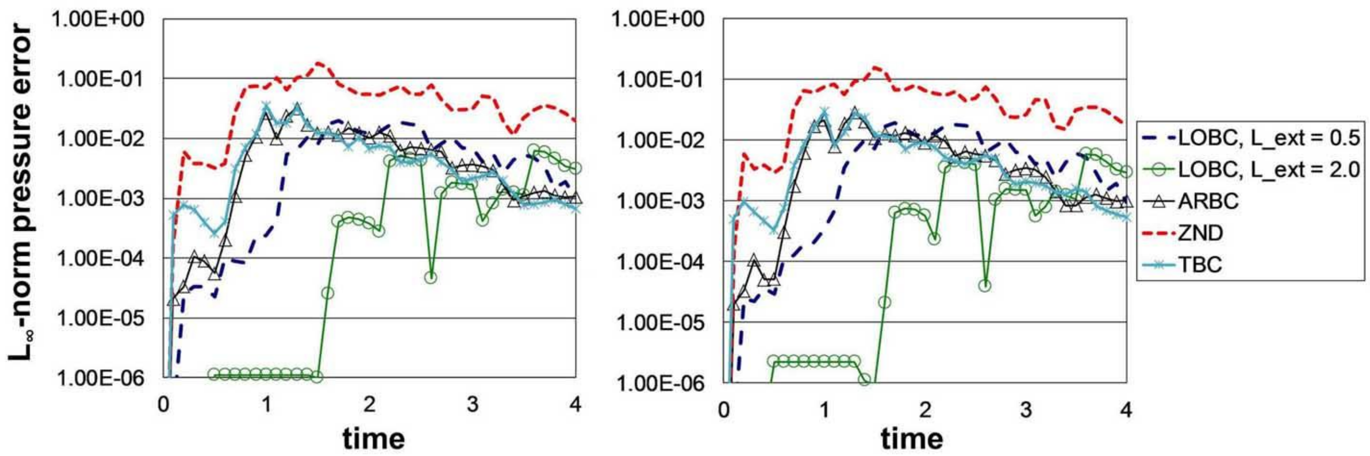
1D results:

$\delta = 10^{-3}$ $\delta = 0.5$

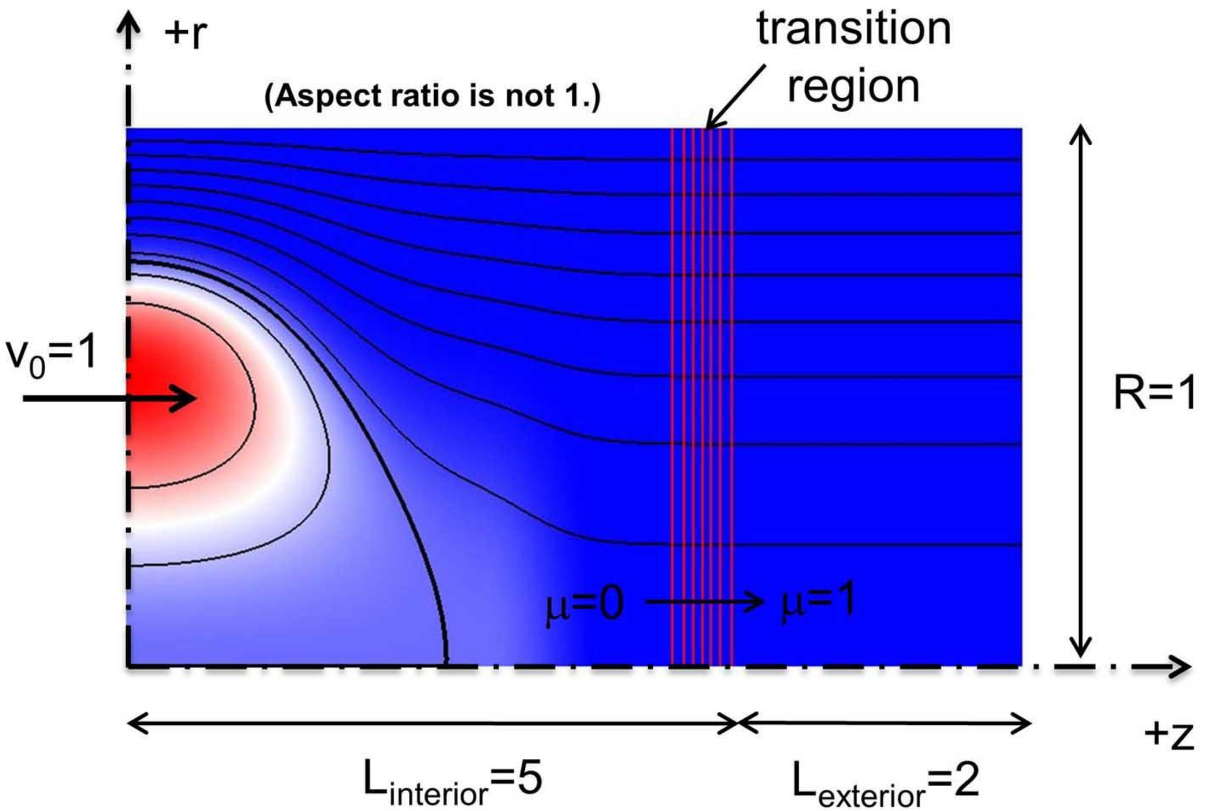


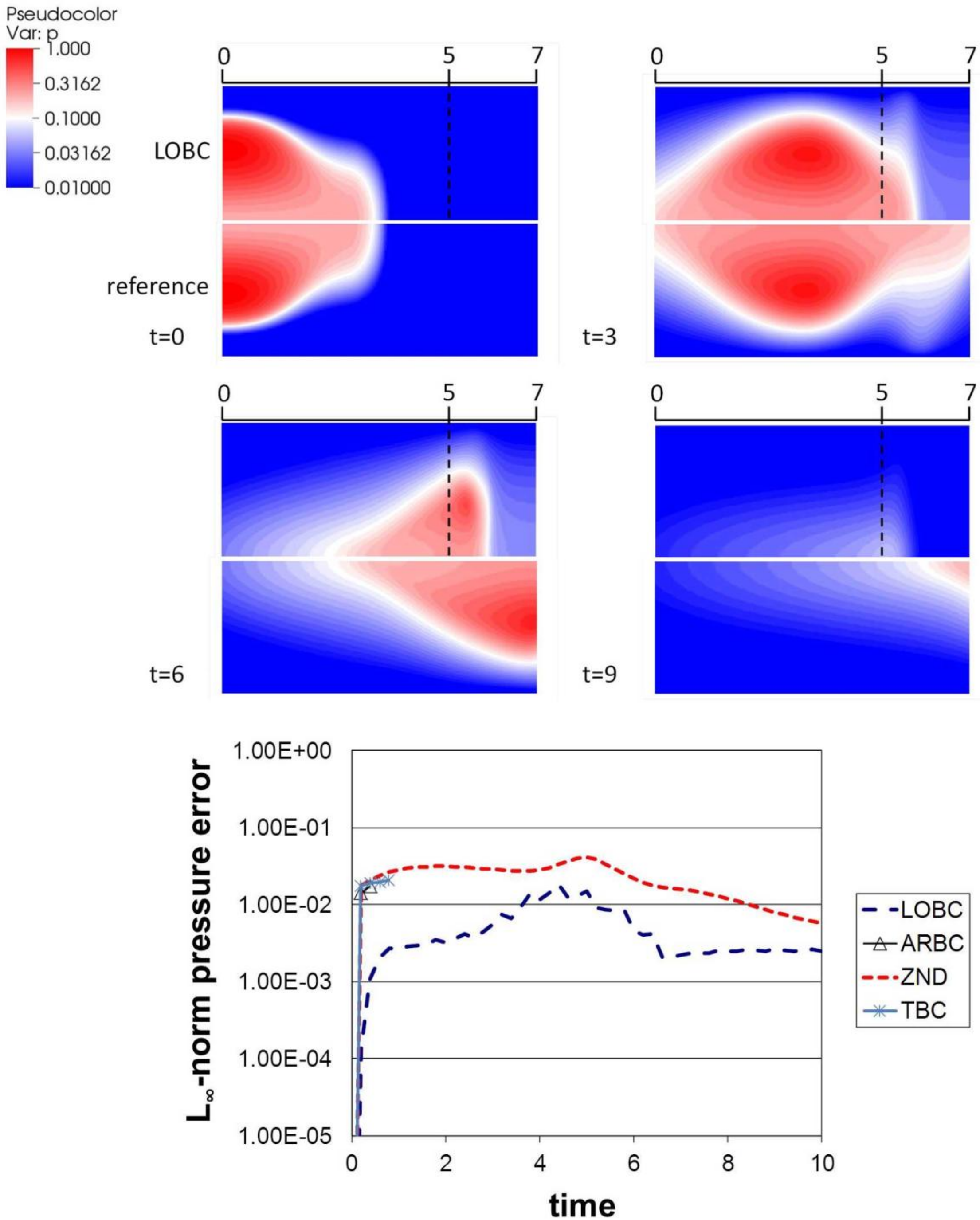
2D results:

$\delta = 10^{-3}$ $\delta = 0.5$

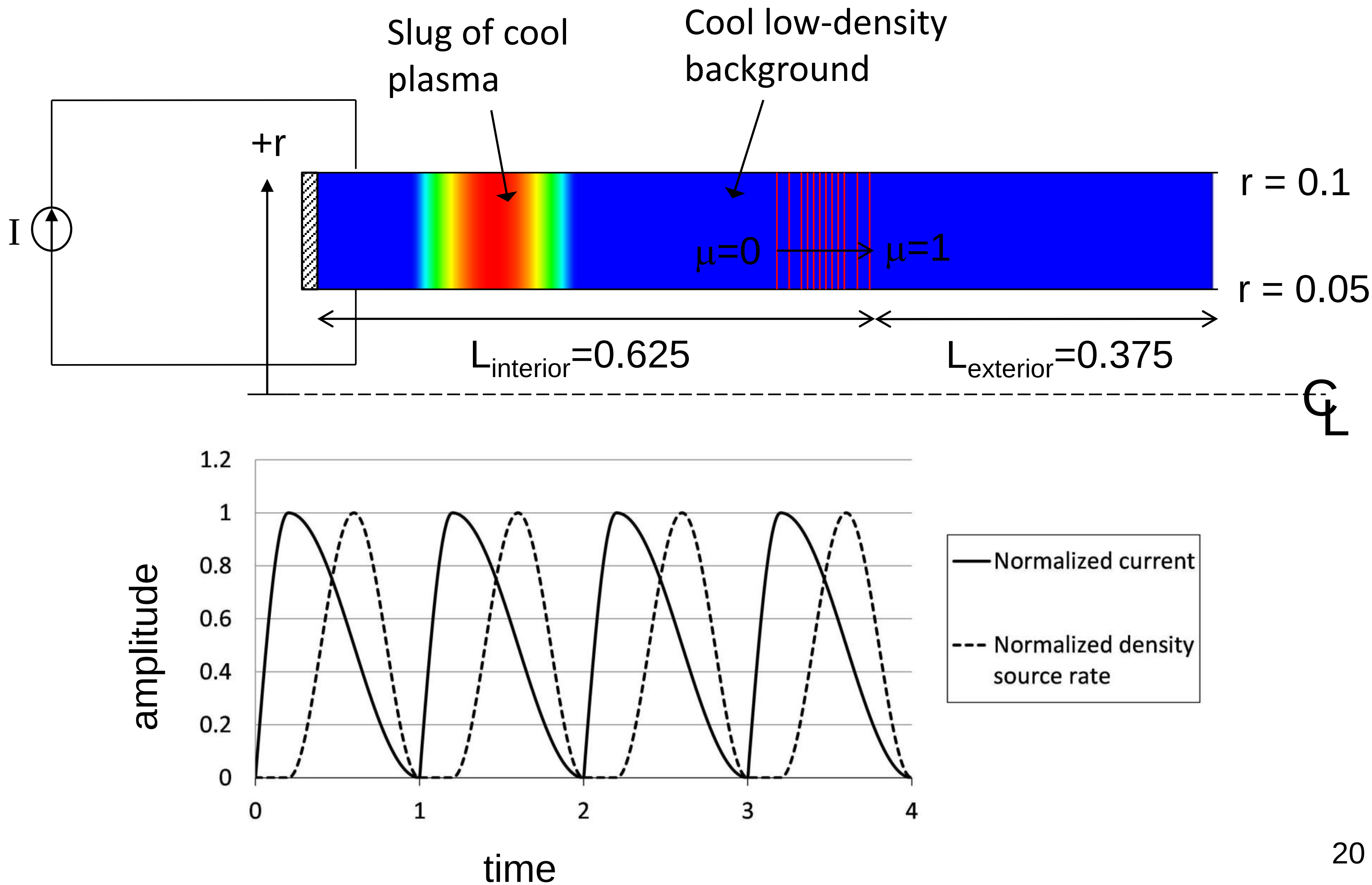


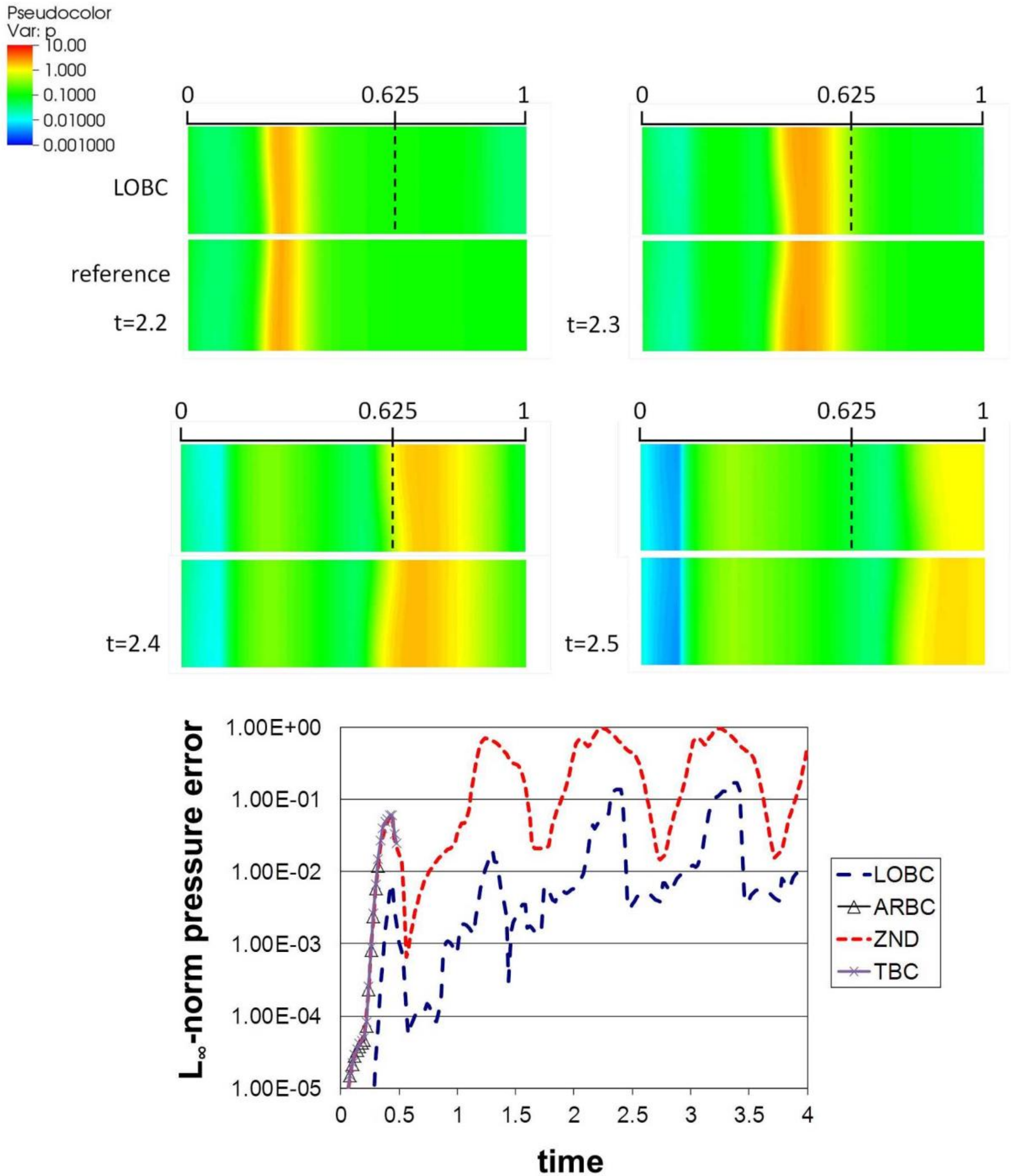
FRC translation challenges open BC with high parallel thermal conduction




Pseudocolor
Var: p
1.000
0.3162
0.1000
0.03162
0.01000
LOBC
reference
t=0
t=3
t=6
t=9
0
5
7
1.00E+00
1.00E-01
1.00E-02
1.00E-03
1.00E-04
1.00E-05
L_∞-norm pressure error
0
2
4
6
8
10
time
LOBC
ARBC
ZND
TBC

Cyclic plasma acceleration drives highly nonlinear current sheets through open BC




Pseudocolor
Var: p
10.00
1.000
0.1000
0.01000
0.001000
0
0.625
1
LOBC
reference
t=2.2
t=2.3
t=2.4
t=2.5
L∞-norm pressure error
1.00E+00
1.00E-01
1.00E-02
1.00E-03
1.00E-04
1.00E-05
0
0.5
1
1.5
2
2.5
3
3.5
4
time
LOBC
ARBC
ZND
TBC

Conclusions

Conclusions

- Several open BC possibilities for dissipative MHD are explored; LOBC are found to be effective where alternatives fail.
- In 1D LOBC applications, if dissipative scales are not matched to the buffer region size, reflections are minimal.
- In 2D LOBC applications, true lacunae are not present; increasing the buffer region size helps minimize reflections due to the lack of true lacunae.
- Further optimization of the LOBC implementation is possible; for details, see upcoming publication*.

* E.T. Meier, A.H. Glasser, V.S. Lukin, U. Shumlak, Modeling open boundaries in dissipative MHD simulation, J. Comput. Phys., 2012 (submitted).