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LLNL-TR-564554

Radiation Field Simulation and Estimation Algorithms for a Mobile Sensor and a Stationary Unknown Source

G. A. Clark

August 10, 2012

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LLNL-TR-564554

Radiation Field Simulation and Estimation Algorithms for a Mobile Sensor and a Stationary Unknown Source

June 5, 2009

Grace A. Clark, Ph.D., IEEE Fellow

LLNL/NSED/Systems and Intelligence Analysis Section

Lawrence Livermore National Laboratory, P. O. Box 808, Livermore, CA 94551

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Auspices and Disclaimer

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"Distributed Nuclear Detector Array (DNDA)" Project: "Data Fusion for High Sensitivity Tracking Using Distributed Mobile Radiation Detector Arrays," PI: Simon Labov, LLNL, Sponsor: Department of Homeland Security (DHS) Domestic Nuclear Detection Office (DNDO)

Clark_DNDA_Results_1

Radiation Field Simulation and Estimation Algorithms for a Mobile Sensor and a Stationary Unknown Source: Initial Results

May 13, 2009

Grace A. Clark

LLNL/NSED/Systems and Intelligence Analysis Section

Lawrence Livermore National Laboratory, P. O. Box 808, Livermore, CA 94551

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LLNL-XX-XXXXX

We Have an Interdisciplinary Team

- Simon Labov (LLNL/GHS) Principal Investigator
- Tom Edmunds (LLNL/NSED): Systems Engineering
- Yiming Yao (LLNL/NSED): Simulations
- Larry Hiller (LLNL, Physics): Simulations, Algorithms, systems
- Maya Gokhale (LLNL/ CS): Networks
- Gardar Johannesson (LLNL/NSED): Algorithms
- Dale Sloan (LLNL): Physics
- Richard Wheeler (LLNL): Physics
- Karl E. Nelson (LLNL/NSED): Algorithms, physics
- Grace Clark (LLNL/NSED: Estimation/Detection Algorithms

- Garrett Jernigan (UCB): Algorithms
- Adel Ganem (Zontrak Inc., San Ramon, CA): Networks
- K. Mani Chandy (Caltech): Algorithms
- Annie Liu (Caltech): Algorithms
- Ryan McLean (Caltech): Algorithms
- Matt Wu (Caltech): Algorithms

Agenda

- Introduction
- Algorithm R&D Plans
- Technical Approach
- Current Results
- Discussion and Plans

Algorithm R&D Plans in Priority Order

- Derive and Implement the Background and Source Simulation Algorithms
- Document the Background and Source Simulation Algorithms
- Derive and Document the Proposed Backpropagation Algorithm

- Implement the Proposed Backpropagation Algorithm and possibly some others

- R&D for a new full inversion algorithm with proximity and energy constraints
  - For a single block of measurements
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- Future Work:
  - Fold in attenuation away from the source: occlusions, shielding, etc.
  - Fold in asymmetric sources: occlusion near the source

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Problem Definition

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Problem Definition

Given:

- A Simple two-dimensional (planar) radiation field (no buildings, etc.)
  - A grid of radiation samples

- A single constant radiation point source
- A single mobile radiation sensor traveling along a planar trajectory one meter above the plane measuring counts per second at each grid point
- Measurements of the sensor position (GPS) along the trajectory

Goals:

- Estimate the radiation (counts/sec) at each point on the 2D grid based only upon the sensor measurements acquired along the sensor trajectory.
- Detect the source, estimate its location and estimate its radiation strength.

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Derivation of the Algorithms for Simulating Background and Source Radiation

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Simulations Algorithms Derivation p.1

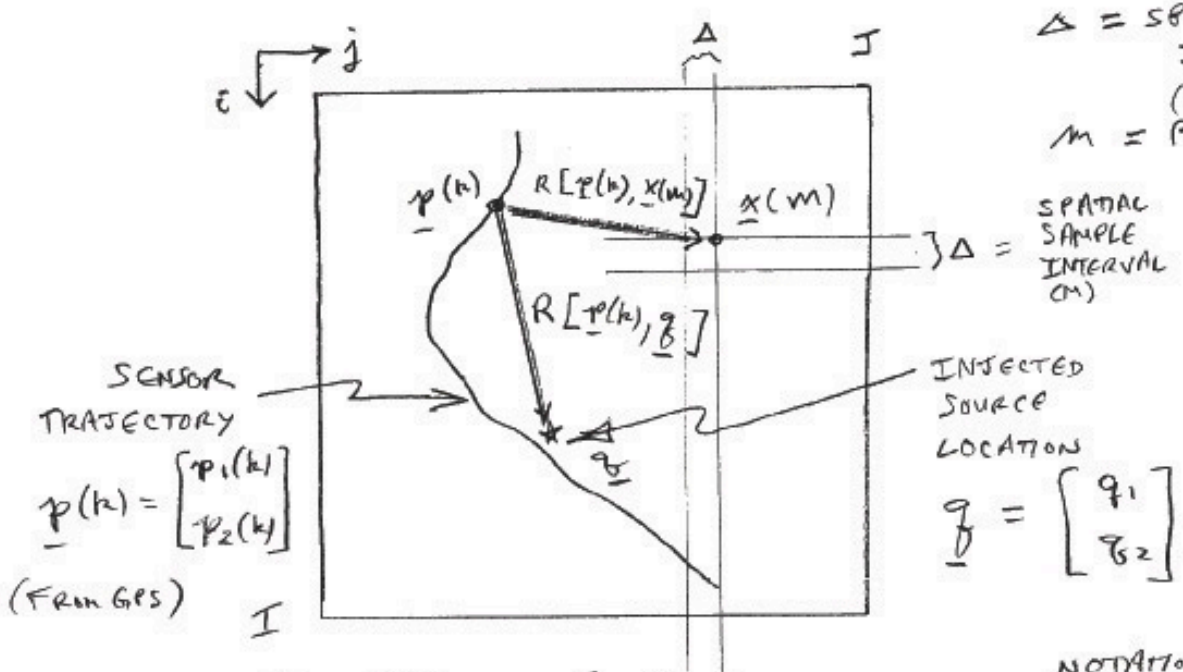
PROBLEM DESCRIPTION: THE SPATIAL GRID

k = TIME OR POSITION INDEX ON THE SENSOR TRAJECTORY

Δ = SPATIAL SAMPLE INTERVAL (M)

(ASSUME $\Delta_i = \Delta_j = \Delta$)

m = POSITION INDEX ON THE SPATIAL GRID



$$\underline{p}(k) = \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix}$$

(FROM GPS)

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} i\Delta \\ j\Delta \end{bmatrix}$$

$$\underline{x}(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix} = \begin{bmatrix} i\Delta \\ j\Delta \end{bmatrix}$$

= LOCATION VECTOR OF A POINT ON THE SPATIAL GRID

NOTATION: THE SPATIAL GRID HAS BEEN CONVERTED TO A VECTOR (SEE NEXT PAGE)

Simulations Algorithms Derivation p.2

5/12/09 GAC

NOTATION FOR POINTS ON THE SPATIAL GRID

- LET

$$\underset{I \times J}{\underset{\sim}{X}} = \begin{bmatrix} | & | & | & | \\ | & | & | & | \\ | & | & | & | \end{bmatrix} = [\, x(i,j) \,]$$

TREAT THE GRID AS A 2D ARRAY, THEN CONVERT IT TO A VECTOR

- DO A LEXICOGRAPHIC ORDERING AS IN MATLAB $X(:)$

$\Rightarrow$ A VECTOR FORMED BY RASTER-SCANNING BY COLUMNS

$$\underset{\substack{IJ \times 1 \\ \parallel \\ M \times 1}}{\underset{\sim}{X}(:)} = \begin{bmatrix} | \\ | \\ \vdots \\ | \end{bmatrix} \begin{matrix} \}\ \text{Col } 1 \\ \}\ \text{Col } 2 \\ \vdots \\ \}\ \text{Col } J \end{matrix} \doteq \begin{bmatrix} \underline{x}^T(1) \\ \underline{x}^T(2) \\ \vdots \\ \underline{x}^T(m) \\ \vdots \\ \underline{x}^T(M) \end{bmatrix} \begin{matrix} m=1 \\ m=2 \\ \vdots \\ \vdots \\ m=M \end{matrix}$$

$$\underline{x}(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix}$$

$x_1(m)$ = ROW COORD.

$x_2(m)$ = COLUMN COORD.

$M = IJ$

Simulations Algorithms Derivation p.3

- Let $\underline{x}(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix}$, for $m = 1, 2, \ldots, M$ $\quad M = IJ$

$x_1(m)$ = ROW COORDINATE $= i_m \Delta$

$x_2(m)$ = COLUMN COORDINATE $= j_m \Delta$

i_m = ROW INDEX

j_m = COLUMN INDEX

NOTE:
$\underline{x}^T(m) = [x_1(m) \;\; x_2(m)]$

- $\underset{M \times 1}{\underset{\sim}{X}} = \begin{bmatrix} \underline{x}^T(1) \\ \underline{x}^T(2) \\ \vdots \\ \underline{x}^T(m) \\ \vdots \\ \underline{x}^T(M) \end{bmatrix} = \begin{bmatrix} x_1(1) & x_2(1) \\ x_1(2) & x_2(2) \\ \vdots & \vdots \\ x_1(m) & x_2(m) \\ \vdots & \vdots \\ x_1(M) & x_2(M) \end{bmatrix}$

$M = IJ$

Simulations Algorithms Derivation p.4

DEFINITIONS

Δ = SPATIAL SAMPLING INTERVAL (METERS)

$i = 1, 2, \ldots, I$ = ROW INDEX ON THE SPATIAL GRID (SOUTH)

$j = 1, 2, \ldots, J$ = COLUMN INDEX " " " " (EAST)

$\omega(i,j)$ = RADIATION ($\frac{\text{COUNTS}}{\text{SEC}}$) ON THE SPATIAL GRID = TRUTH (SIMULATED)

$\Omega = [\omega(i,j)]$ = $I \times J$ MATRIX OF RADIATION VALUES ON THE SPATIAL GRID

$\mathbf{p}(k) = \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix}$ = 2x1 VECTOR OF LOCATION COORDINATES FOR THE SENSOR AT A GIVEN TIME AND POSITION ON ITS TRAJECTORY (FROM GPS)

= POSITION FOR MEASUREMENT y_i ON TRAJECTORY

$p_1(k)$ = SCALAR COORDINATE FOR SENSOR ALONG A ROW ON THE GRID

$p_2(k)$ = " " " " " " COLUMN " "

$p_1(k) = i\Delta$, i = ROW INDEX ON THE GRID

$p_2(k) = j\Delta$, j = COLUMN " " " "

Simulations Algorithms Derivation p.5

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

= 2x1 VECTOR OF THE LOCATION OF THE POINT OF INTEREST ON THE SPATIAL GRID AT A GIVEN POSITION. WE WISH TO ESTIMATE THE RADIATION AT THIS POINT OF INTEREST

$$q_1 = i\Delta$$
$$q_2 = j\Delta$$

$\bar{q}$ = THE POSITION OF A TRUE POINT SOURCE (e.g. INJECTED) "⊗"

k = POSITION OR TIME INDEX FOR A VECTOR POSITION ON THE SENSOR TRACTORY $k = 1, 2, \ldots, K$

USE THESE LATER

n = TIME INDEX = $1, 2, \ldots, N$ (Karl used "i")

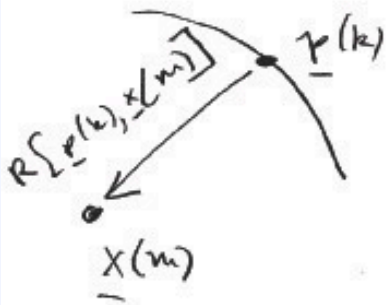
$t_n = n\tau$ = TIME (SECONDS) AT TIME INDEX n

τ = TIME SAMPLE INTERVAL (SECONDS)

Simulations Algorithms Derivation p.6

DEFINE

- $R[\underline{p}(k), \underline{x}(m)]$ = SCALAR EUCLIDEAN DISTANCE BETWEEN A POINT AT $\underline{p}(k)$ AT TIME OR POSITION k AND A POINT $\underline{x}(m)$ ON THE SPATIAL GRID



$$\underline{p}(k) = \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix}, \quad \underline{x}(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix}$$

$$R[\underline{p}(k), \underline{x}(m)] = \| \underline{p}(k) - \underline{x}(m) \|$$

$$= \left\{ [\underline{p}(k) - \underline{x}(m)]^T [\underline{p}(k) - \underline{x}(m)] \right\}^{1/2}$$

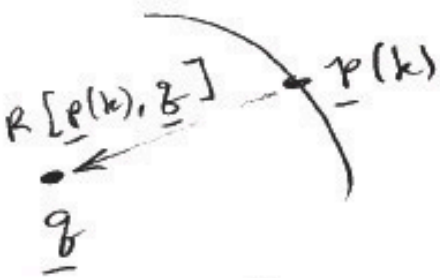
for $\underline{a} = [a_1, a_2]^T$

$$\|\underline{a}\| = \langle \underline{a}, \underline{a} \rangle^{1/2} = (\underline{a}^T \underline{a})^{1/2} = \sqrt{a_1^2 + a_2^2}$$

Simulations Algorithms Derivation p.7

DEFINE

- $R[\underline{p}(k), \underline{q}] = \| \underline{p}(k) - \underline{q} \|$



$$= \left\{ [\underline{p}(k) - \underline{q}]^T [\underline{p}(k) - \underline{q}] \right\}^{1/2}$$

- $s(m)$ = RADIATION VALUE (COUNTS/SEC) AT ONE METER ABOVE THE GRID PLANE

$$= \frac{\text{COUNTS}}{\text{SEC}}$$

= THIS IS WHAT WE WANT TO ESTIMATE

Simulations Algorithms Derivation p.8

GAC's NOTES ON HOW TO SIMULATE THE MEAN OF THE POISSON PROCESS TO MAKE THE MEASUREMENTS $y(k)$

$y(k)$ = SCALAR SENSOR MEASUREMENT AT TIME OR POSITION k

$= \text{POISSON}\{\mu[\underline{p}(k)]\}$ = A POISSON DRAW AT POSITION $\underline{p}_k$

$$\underbrace{\mu[\underline{p}(k)]}_{\text{MEAN}} = \sum_{m=1}^{M} \frac{\omega(m)}{R[\underline{p}(k), \underline{x}(m)]^2 + 1} + \frac{S(m)}{R[\underline{p}(k) - \underline{q}]^2 + 1}$$

DUE TO ONE METER ABOVE GRID PLANE

$\omega(m)$ = BACKGROUND RADIATION (CTS/SEC) AT POSITION m

$$R[\underline{p}(k), \underline{x}(m)] = \| \underline{p}(k) - \underline{x}(m) \|$$

$$= \left[[\underline{p}(k) - \underline{x}(m)]^T [\underline{p}(k) - \underline{x}(m)] \right]^{1/2}$$

= DISTANCE BETWEEN THE SENSOR POSITION $\underline{p}(k)$ AND THE SPATIAL POSITION $\underline{x}(m)$

$S(m)$ = RADIATION (CTS/SEC) OF THE SOURCE AT POSITION m

$\underline{q}$ = POSITION OF THE SOURCE

Simulations Algorithms Derivation p.9

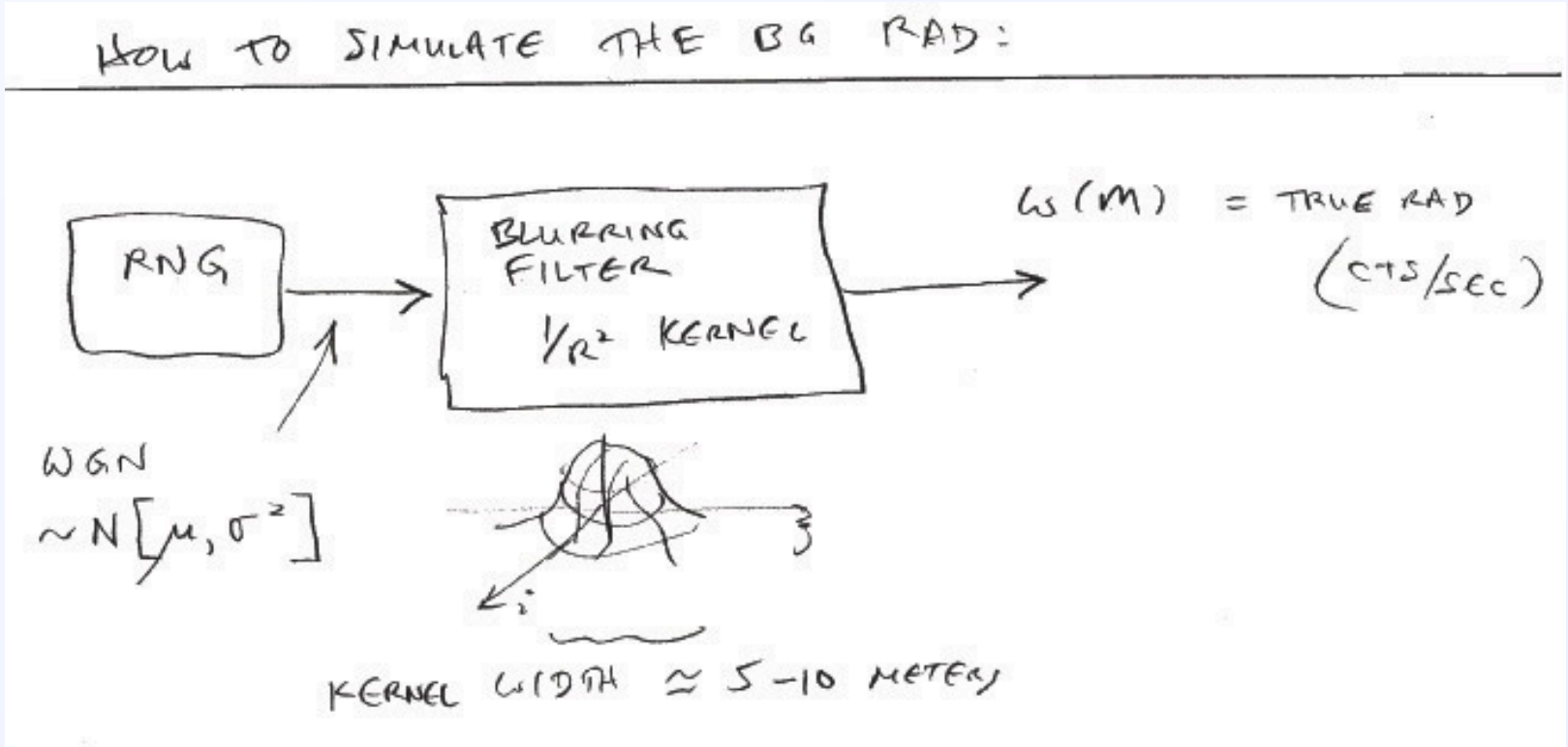
WE WISH TO SIMULATE THE RADIATION AS FOLLOWS:

- $S(m) = \begin{cases} 0, & \text{NO SOURCE} \\ \mu, & \text{INJECTED CONSTANT SOURCE} \end{cases}$

- $S(m) \geq 0$ (ALWAYS POSITIVE)

- A REALISTIC RANGE OF VALUES FOR SIMULATION:
  - → $\mu \approx 40$ COUNTS/SEC FOR BACKGROUND MEAN
  - → RANGE OF BG $\approx (+20, +80)$ OVER THE MAP

Simulations Algorithms Derivation p.10



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Simulations of Background Radiation, Source Radiation and Sensor Measurements

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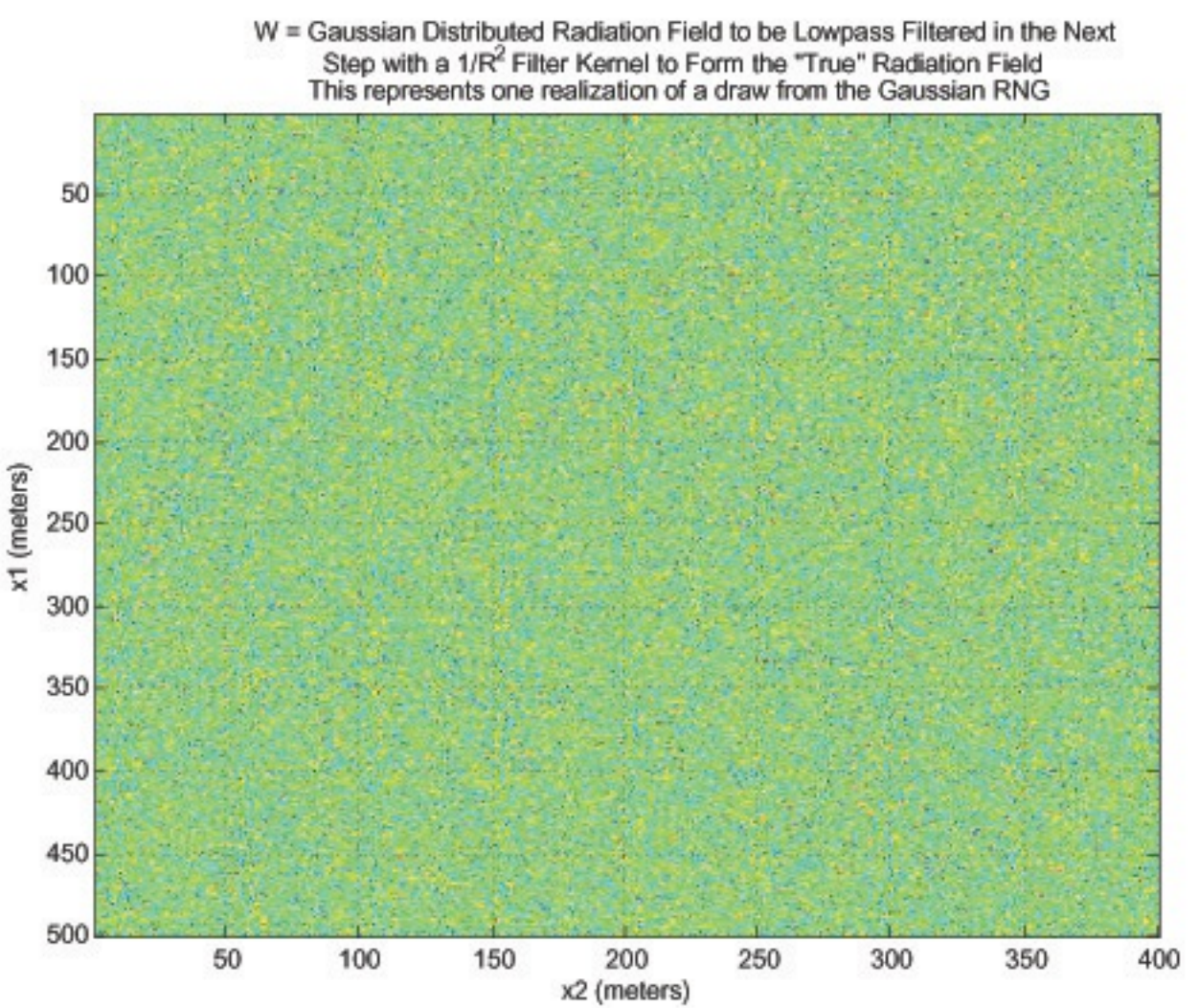
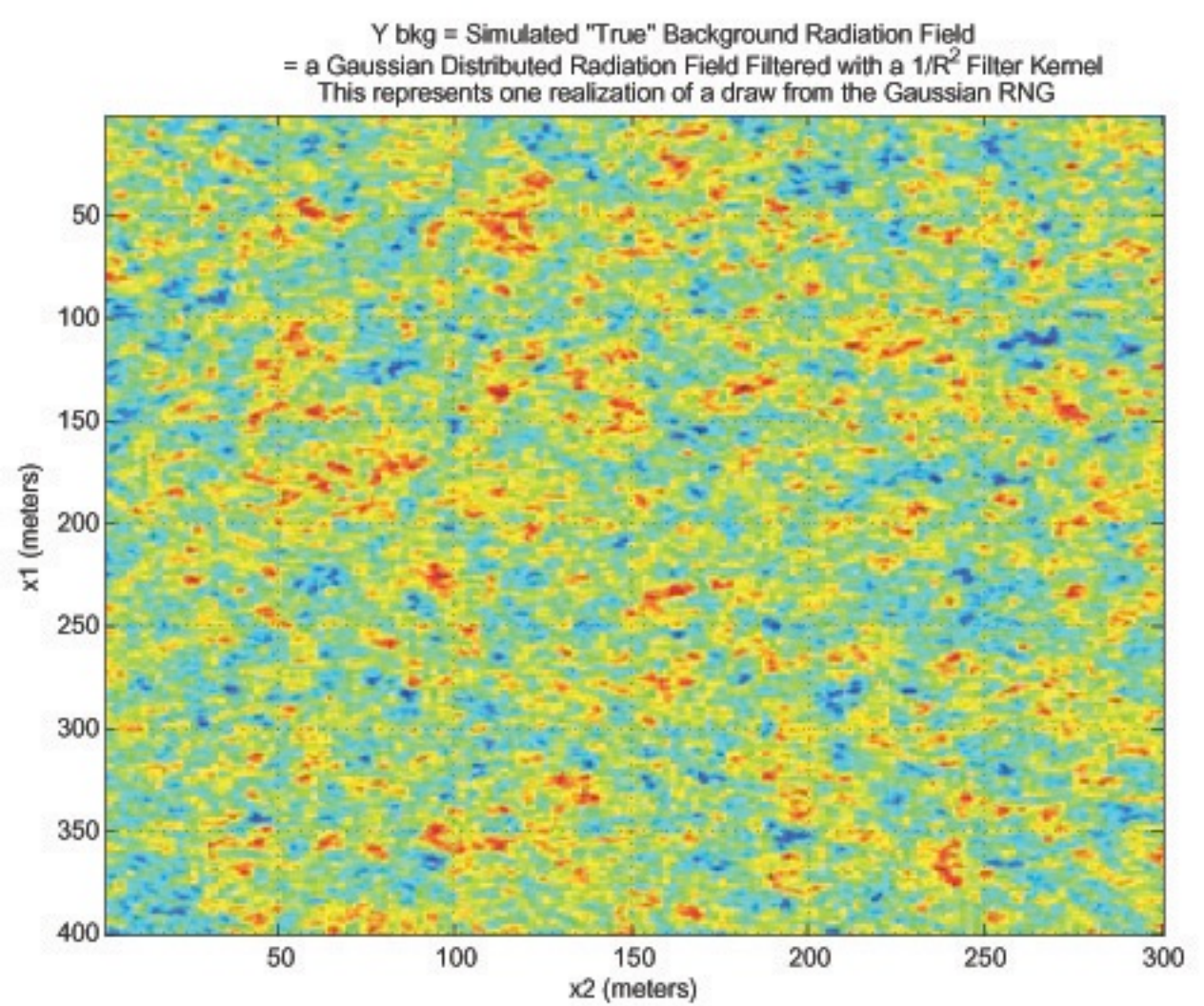
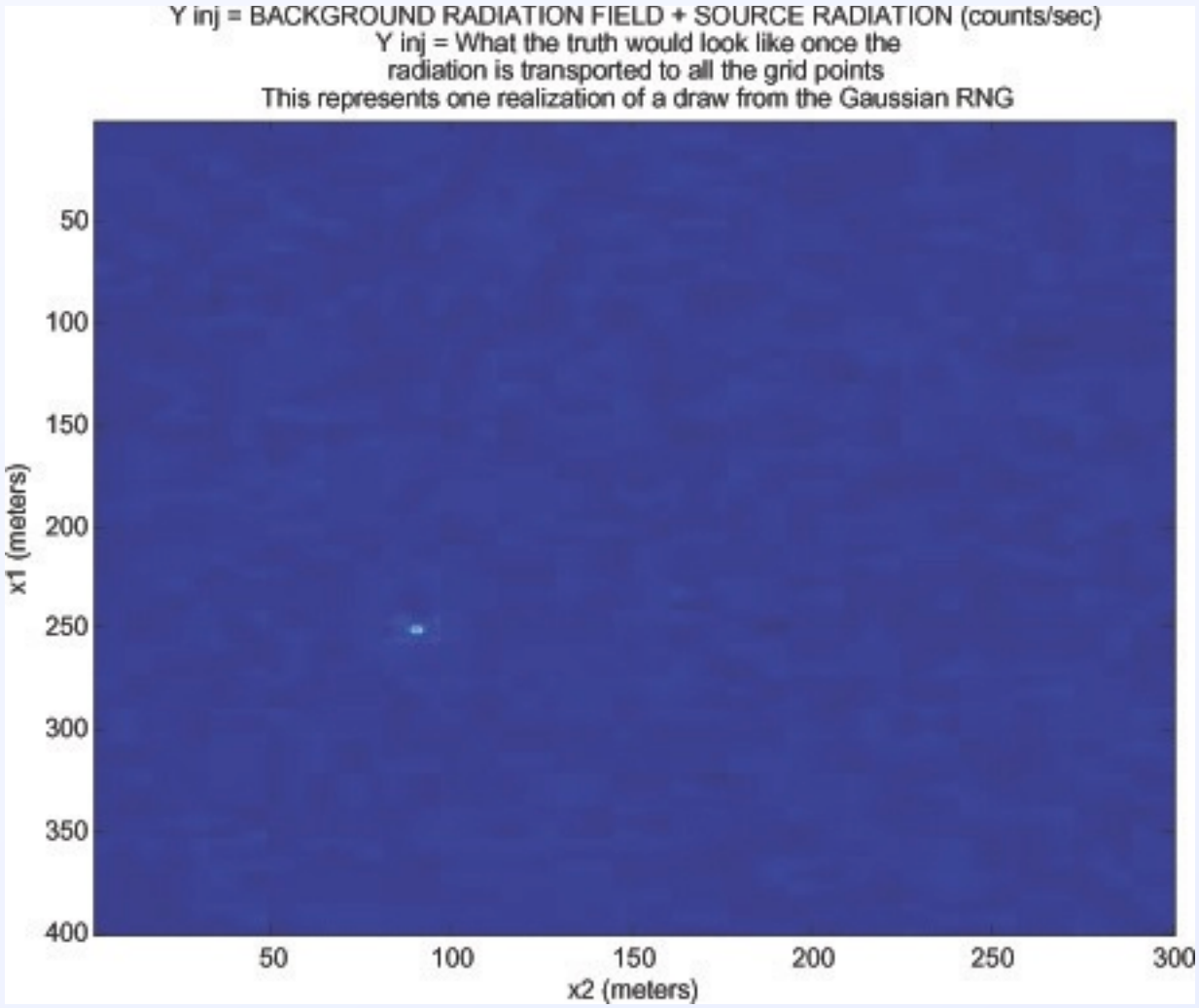
First Step in the Background Radiation Simulation:
Construct a 2D Gaussian Distributed Array



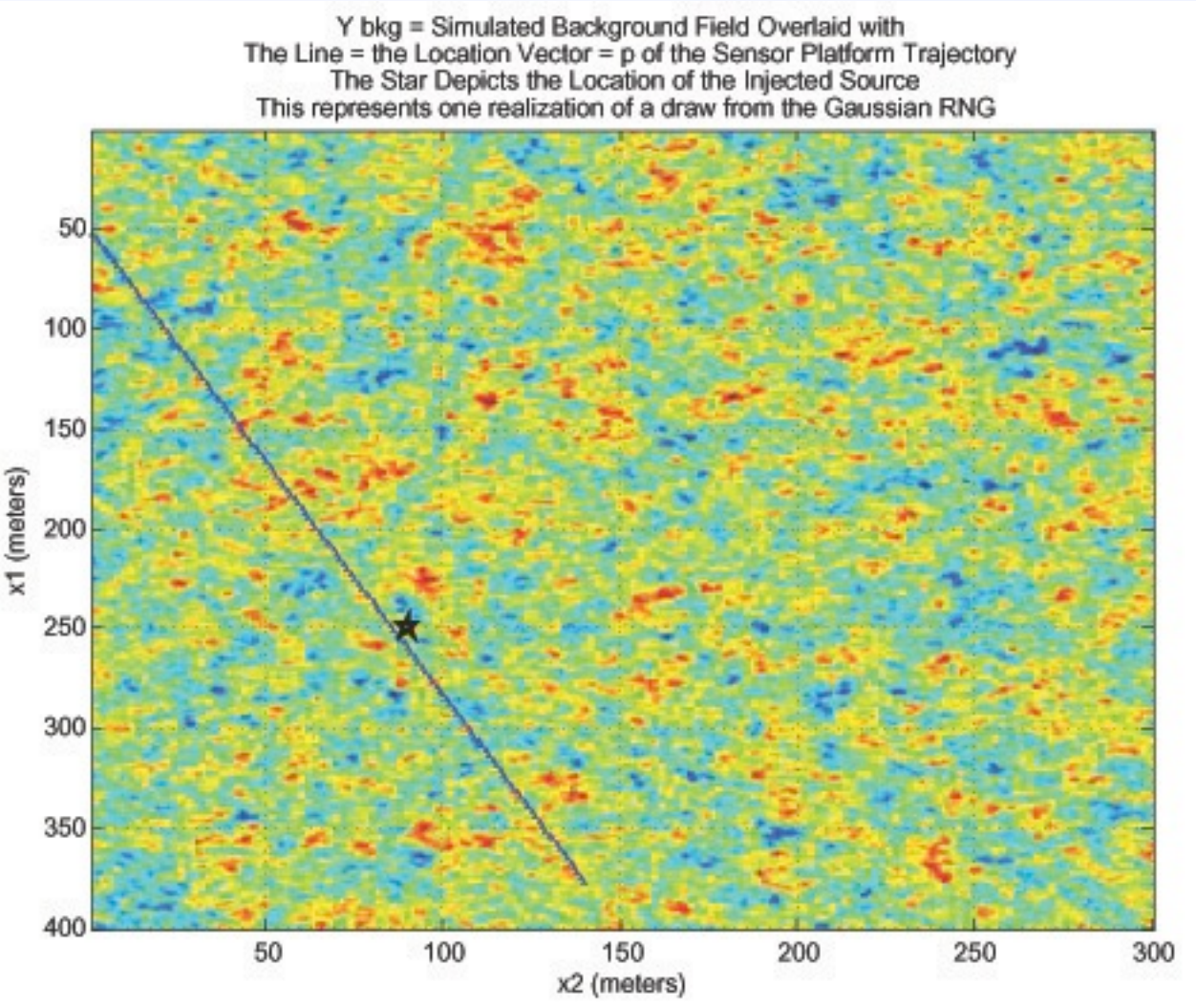
Simulated Background Radiation Field: *Gaussian Array Convolved with a "1/R² " 2D Filter Kernel:*



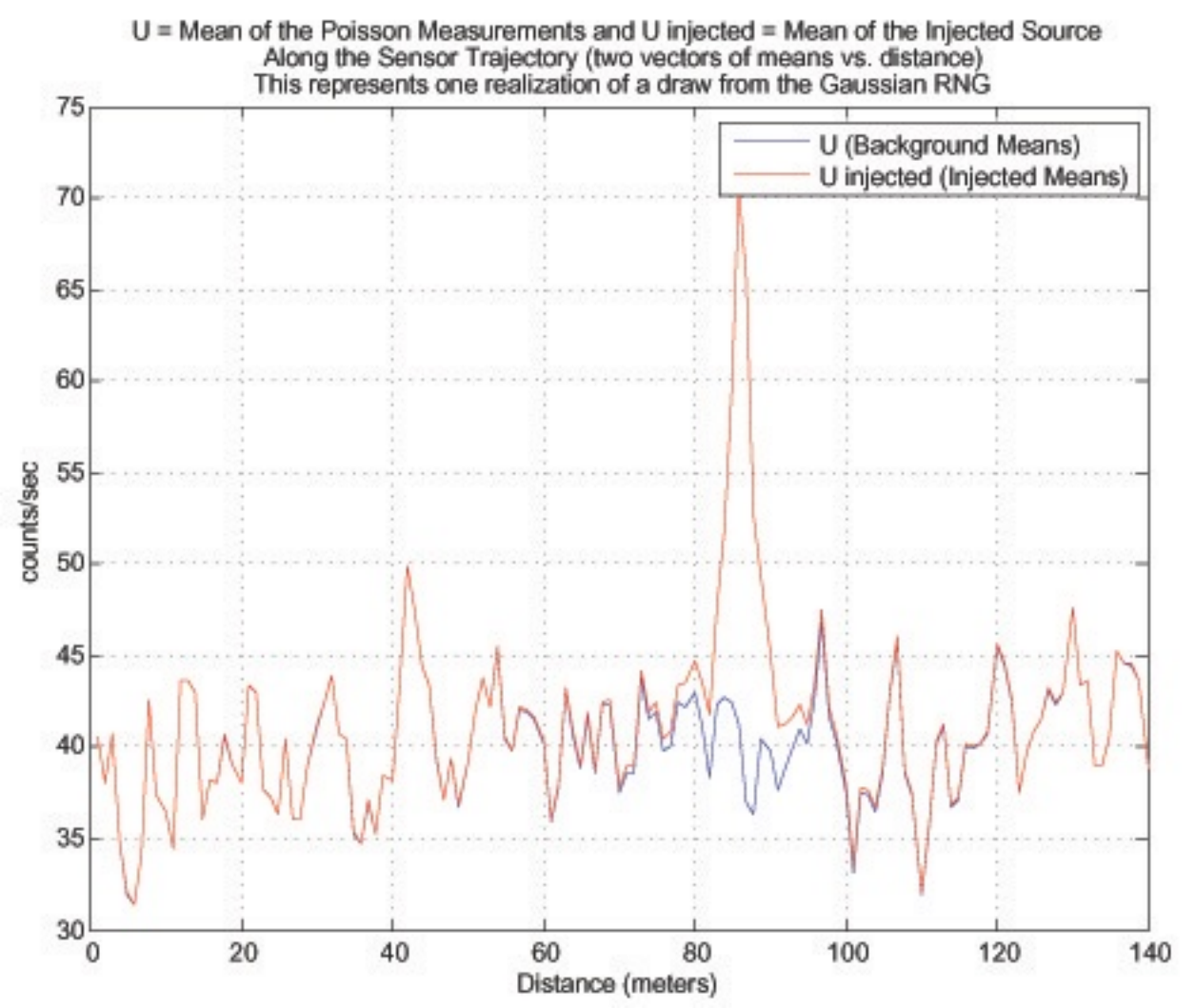
Background Radiation Field + Injected Poisson Point Source Radiation



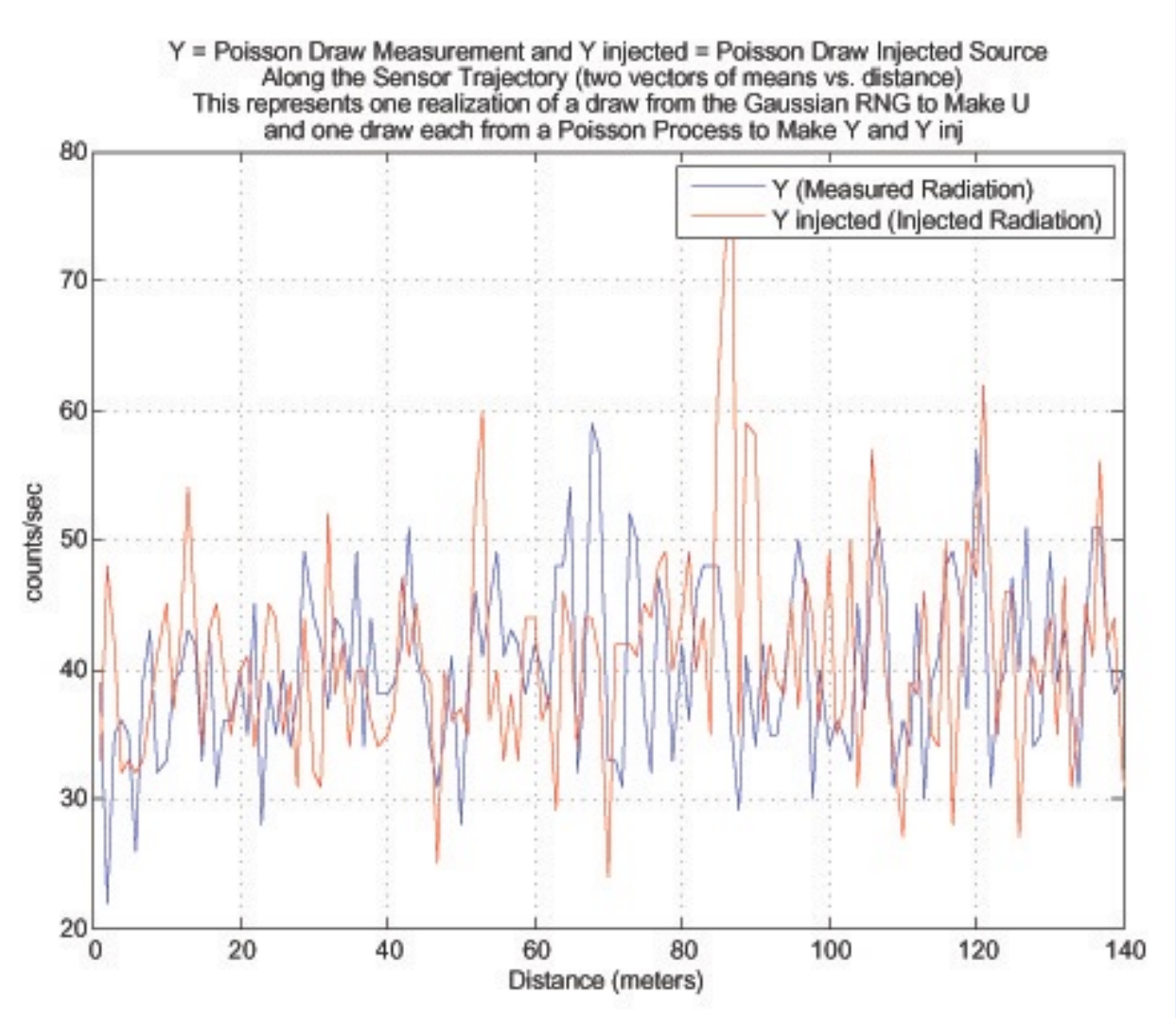
Background Radiation + Point Source Location + Sensor Trajectory



U = Mean of the Poisson BG Measurements and
Uinj = Mean of the Source Measurements Along the Sensor Trajectory



Y = Poisson BG Measurements and
Yinj = Poisson Source Measurements Along the Sensor Trajectory



Conclusions and Plans

- Derive and Implement the Background and Source Simulation Algorithms
- Document the Background and Source Simulation Algorithms
- Derive and Document the Proposed Backpropagation Algorithm

- Implement the Proposed Backpropagation Algorithm and possibly some others

- R&D for a new full inversion algorithm with proximity and energy constraints
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- Future Work:
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Clark_DNDA_Results_2
Preliminary Radiation Field Estimation Results Using the Back Propagation Algorithm
June 5, 2009

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Preliminary Results for the Backpropagation algorithm with Simulated Data

- This is the first radiation map result estimated using the Back Propagation algorithm
- I have not yet had time to validate the results
- The program was cancelled today, so I am documenting the results I have to date

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Experiment E2: Backpropagation Algorithms to Estimate the Radiation Field

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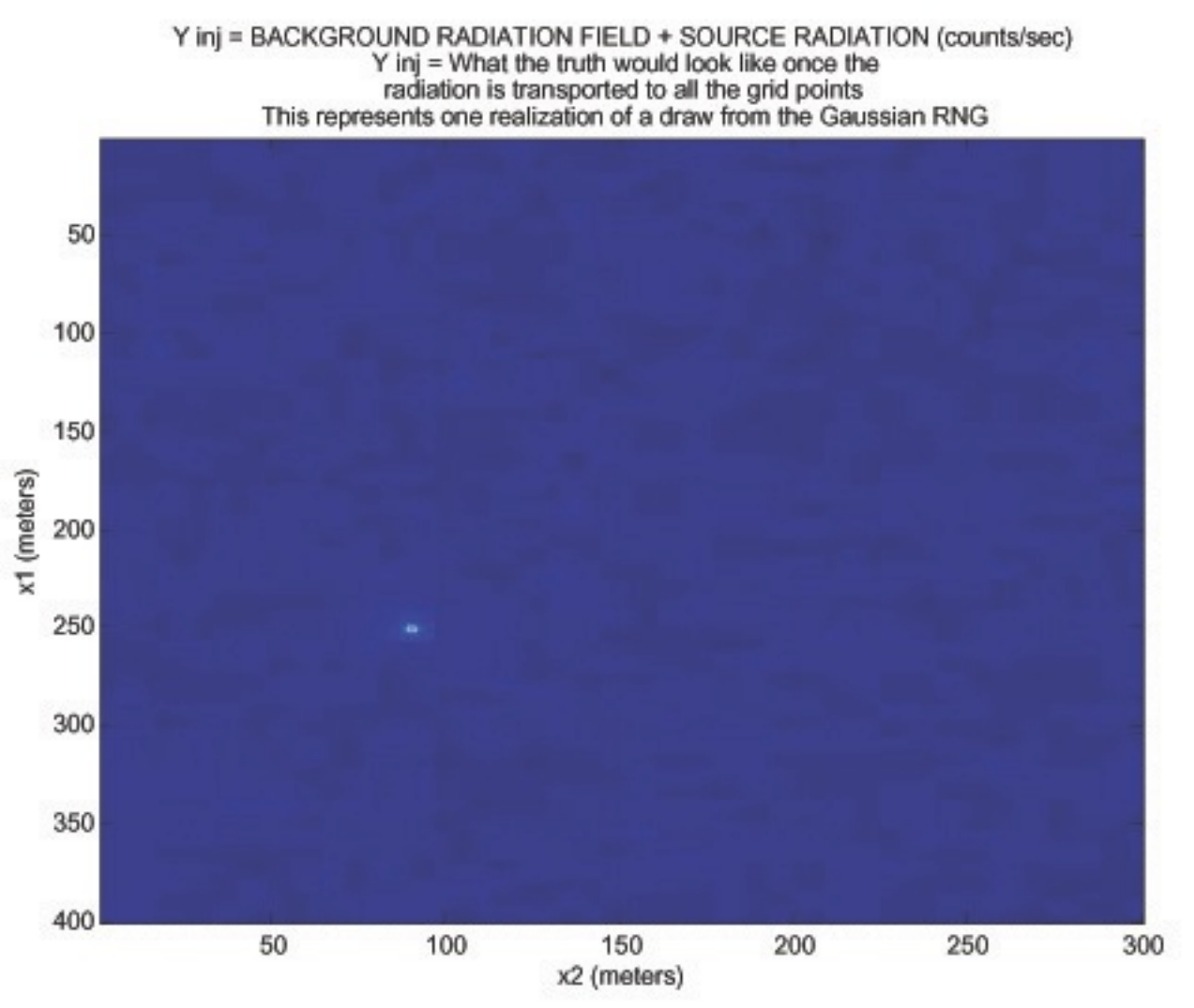
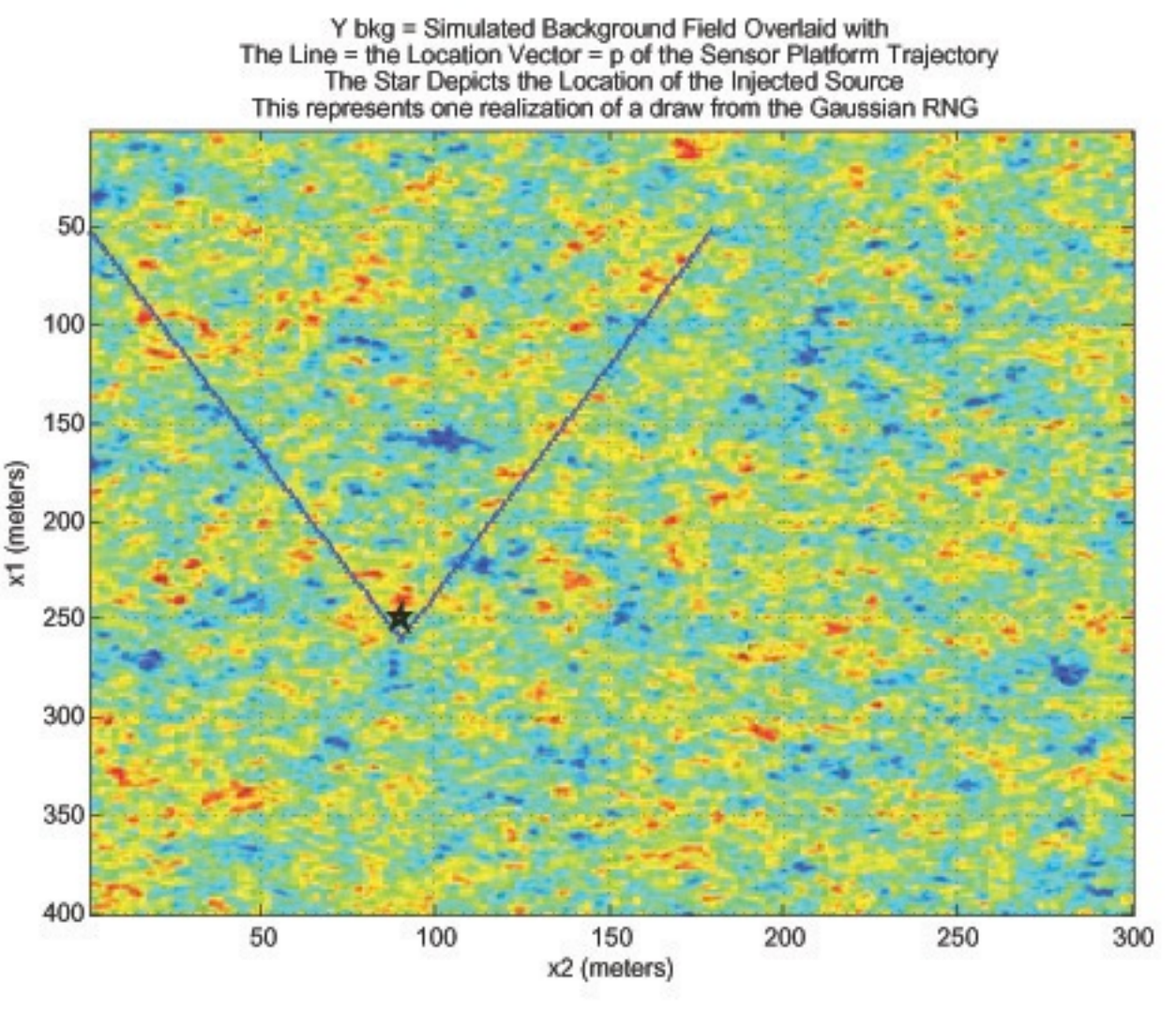
E2_Src_BGC.pdf

Map of Simulated Background and Source



E2_Ybkg_Traj_SrcC.pdf

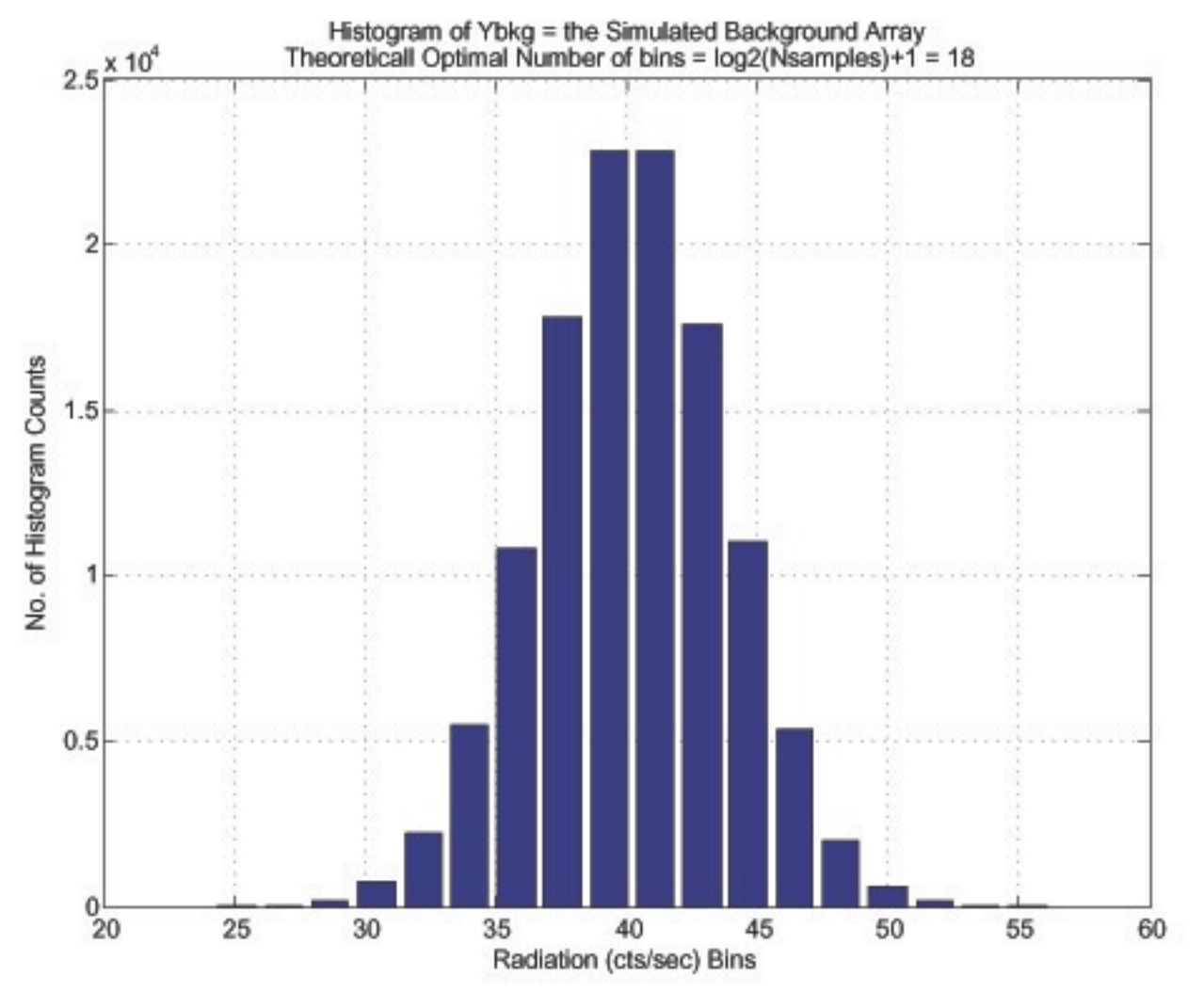
Background Plus Sensor Trajectory and Source



Source Location is Denoted by the Star

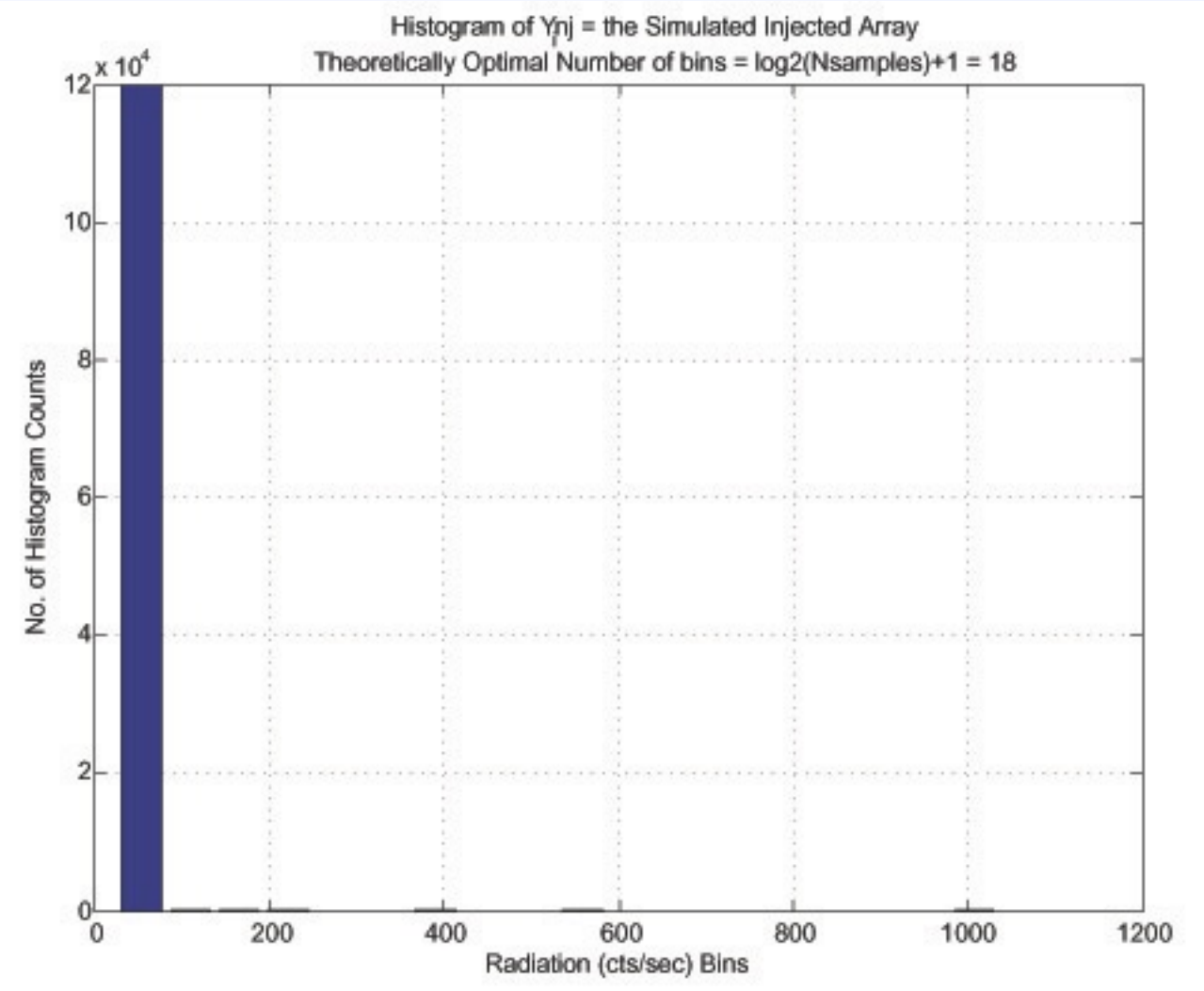
E2_Ybkg_HistoC.pdf

Histogram of the Simulated Background



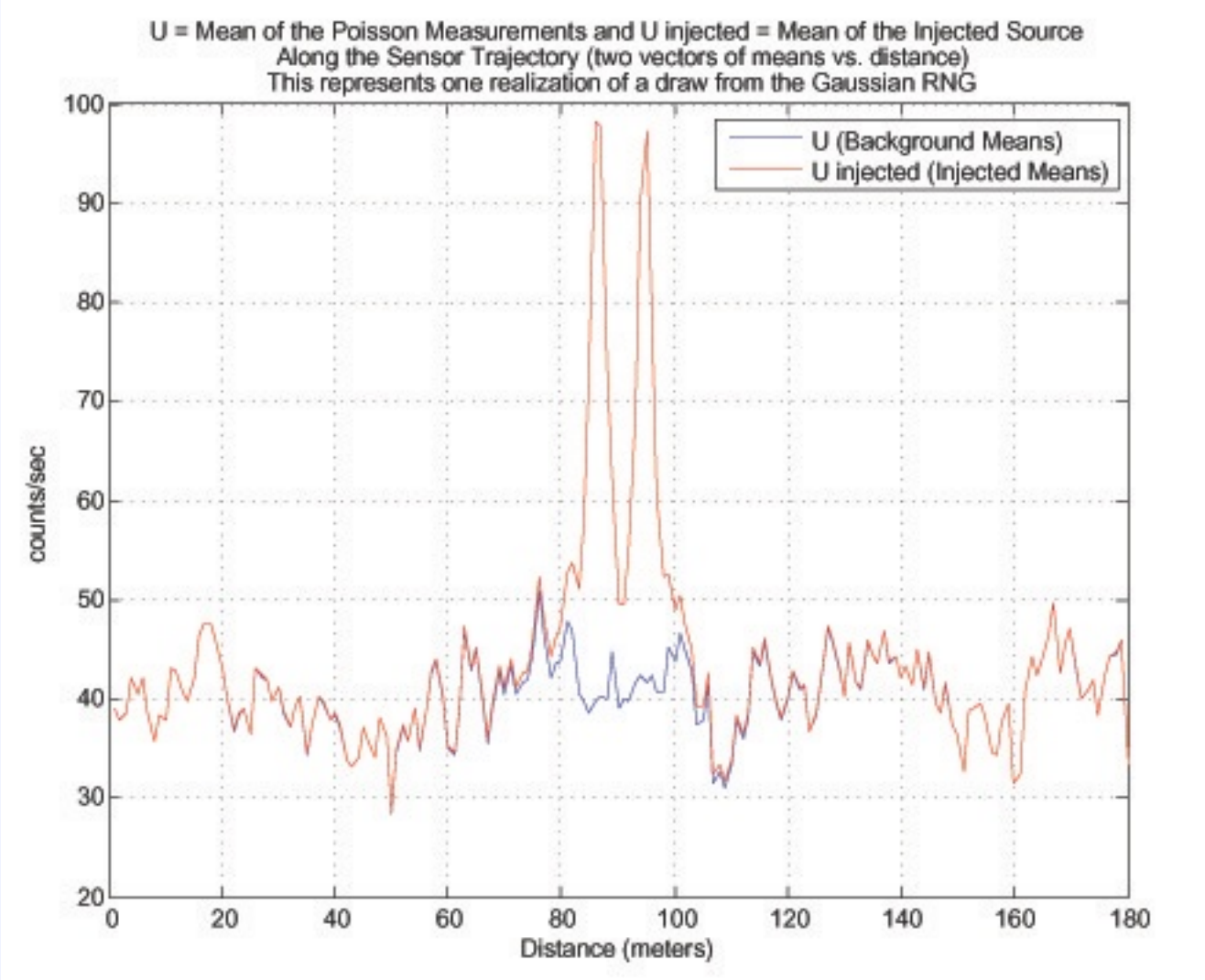
E2_Yinj_HistoC.pdf

Histogram of the Simulated Injected Source



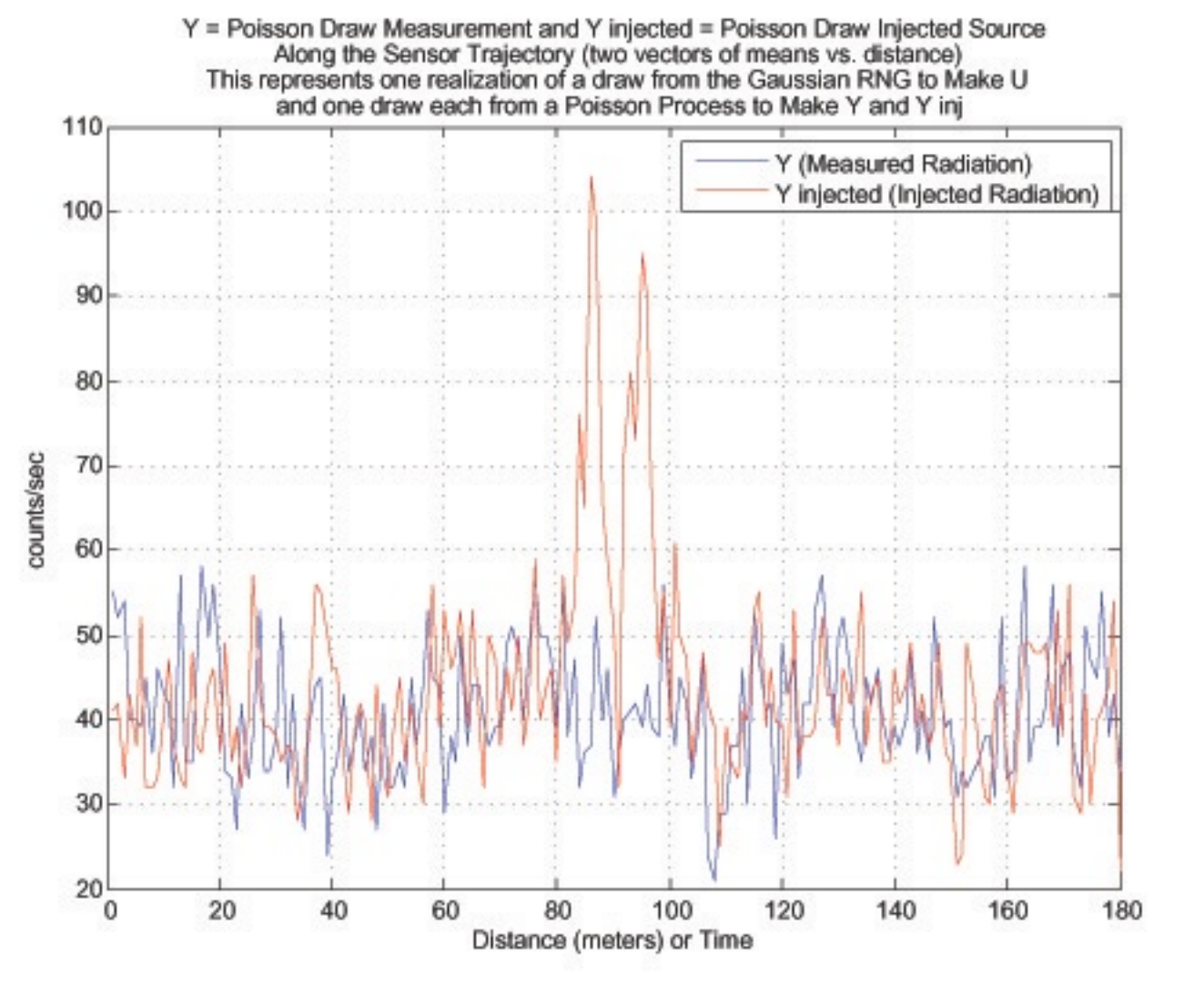
E2_U(k)C.pdf

Mean of the Simulated Poisson Measurements



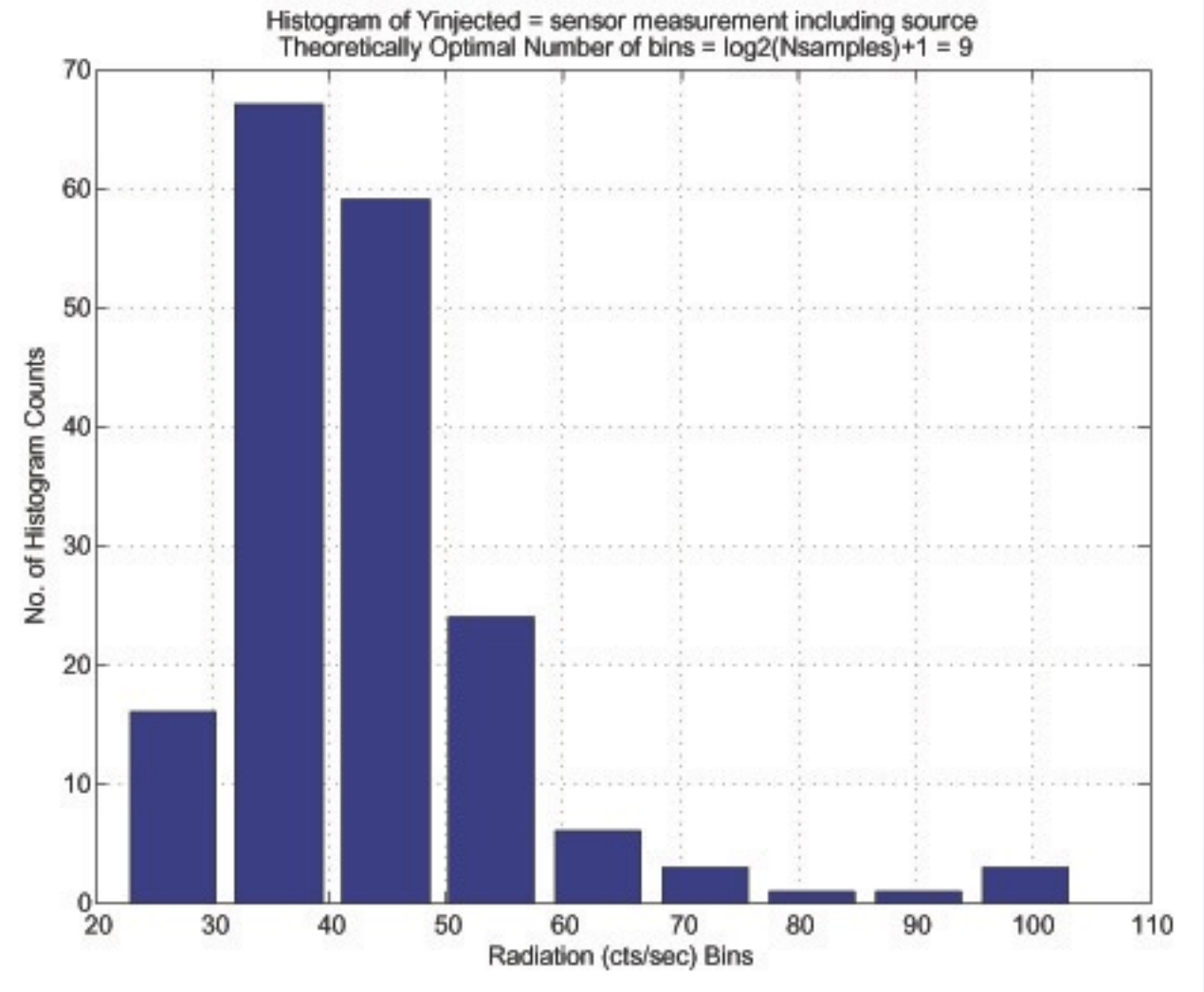
E2_Y(k)C.pdf

Simulated Measurements Along the Trajectory



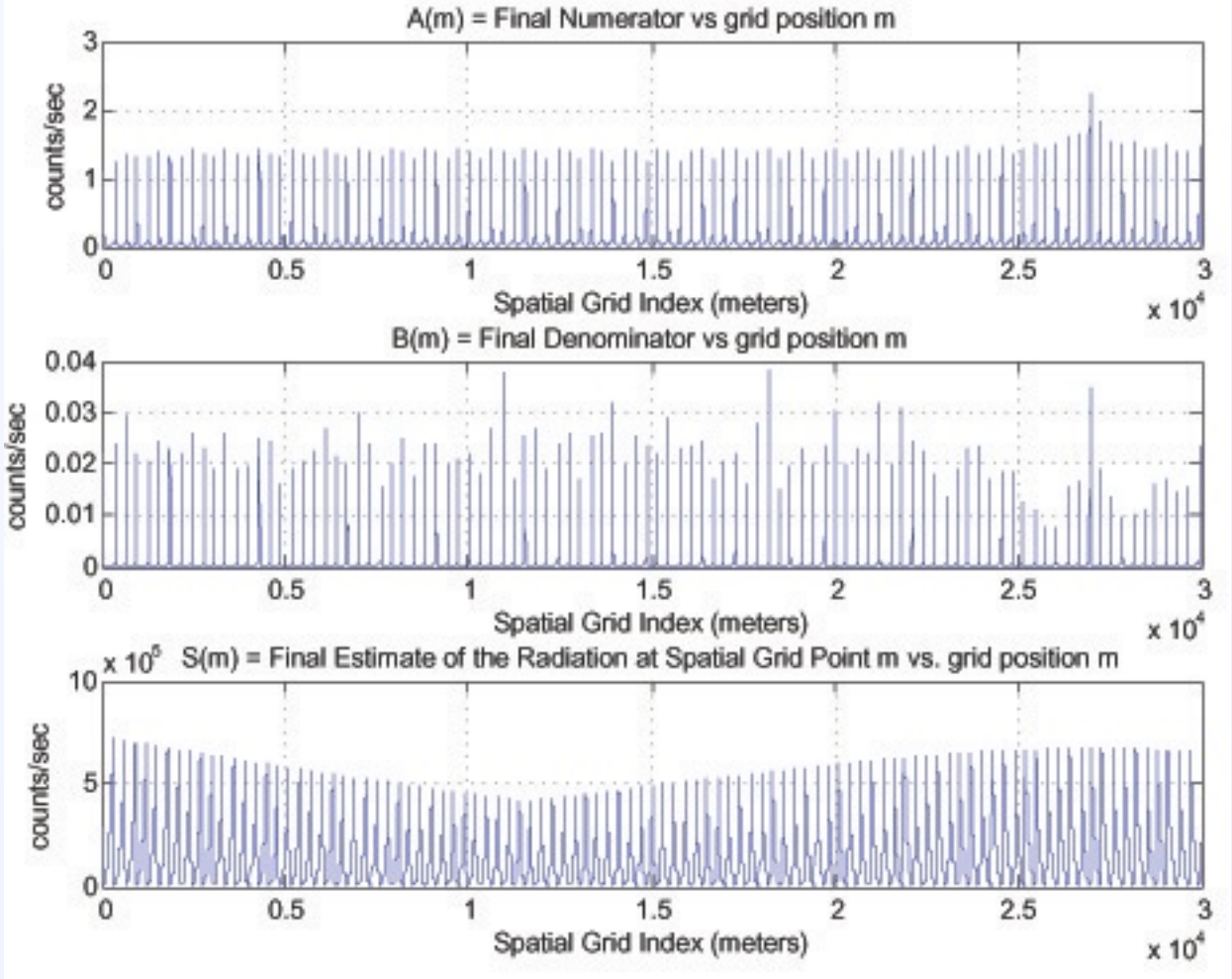
E2_Yinjected_w_SrcC.pdf

Histogram of the Sensor Measurement (w / Source)



E2_A_B_SC.pdf

A(m), B(m) and S(m), where S = A./B

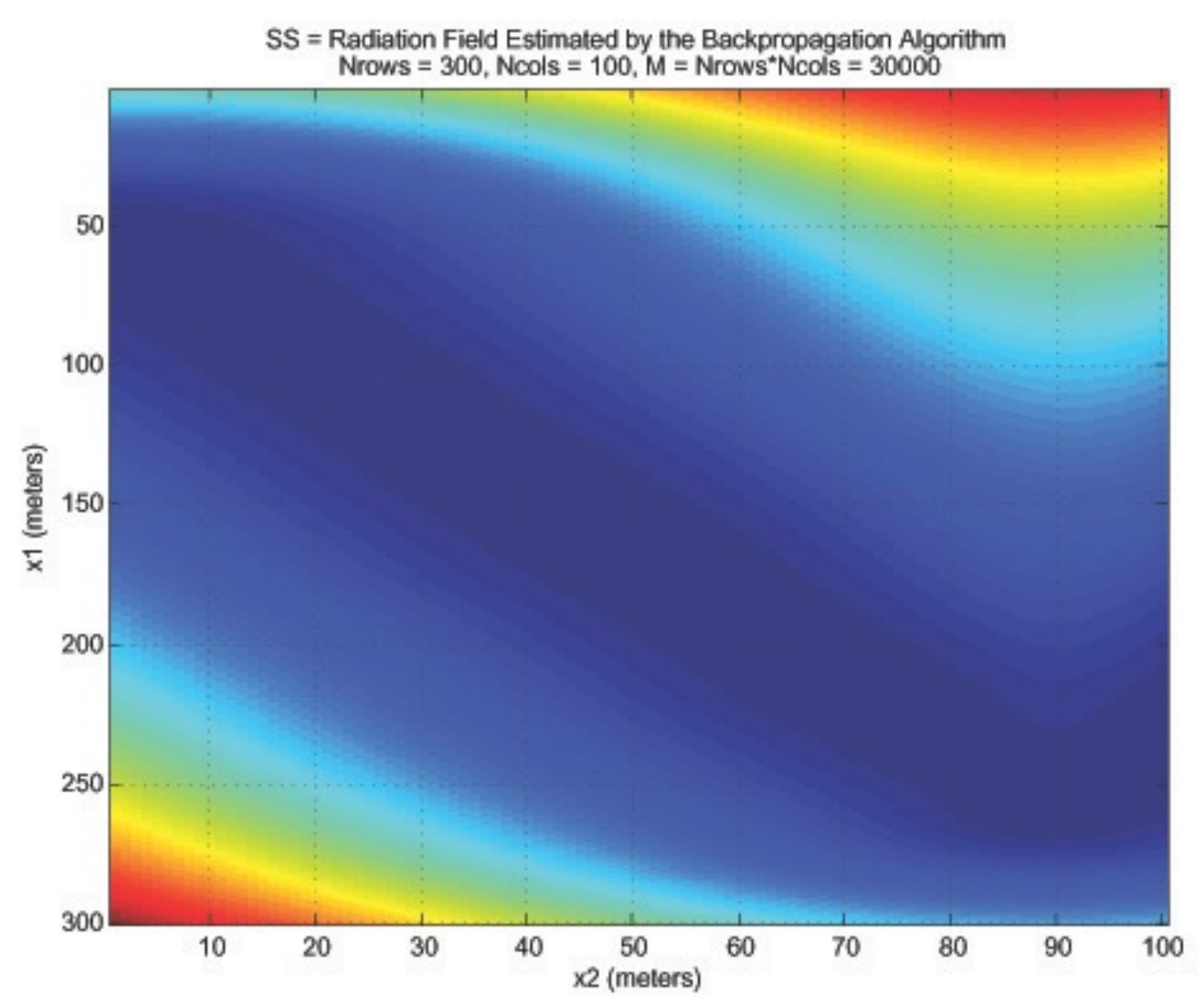


S(m) denotes the Estimated Radiation at Grid Position m

E2_BP_MapC.pdf

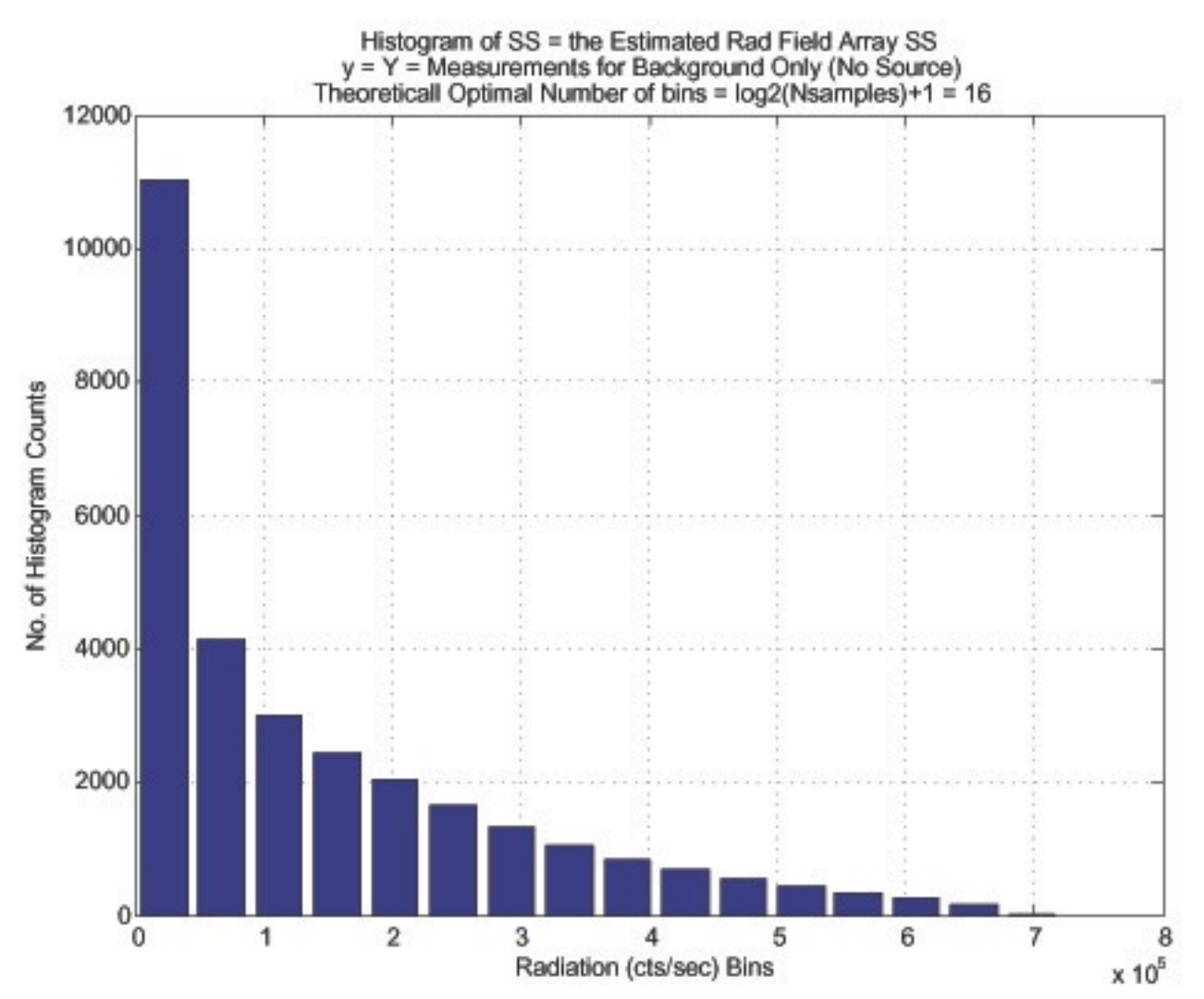
Estimated Radiation Map Using Back Propagation

- Red = High
- Blue = Low



E2_SS_HistoC.pdf

Histogram of the Estimated Radiation Map



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