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Scattering in Thin Foils Associated with Passive Proton and Ion Beam Focusing

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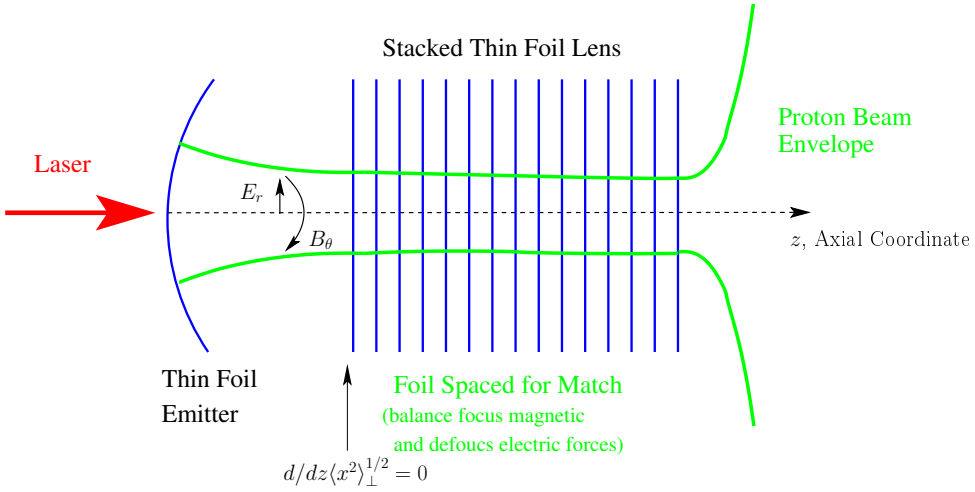
Outline

1. Principles of foil focusing of protons/ions
2. Scattering of Protons in Thin Foils
   - Overview of scattering results from Jackson
3. Simulation using SRIM/TRIM
   - Difference in model between Jackson's and SRIM/TRIM's
   - Comparision of results between SRIM simulations and Jackson's analytical formula
4. Modification of the Envelope Model
   - Envelope Equation
   - Approximation: "jump" of the emittance through each foil
5. Influence of emittance growth for the envelope solution
6. Comparisons against WARP simulations
7. Conclusion

Schematic

- Passive focusing of intense proton and ion beams using stacks of metallic foils.
- Foils attenuate the defocusing $\vec{E}$ field but allow the focusing $\vec{B}$ field to penetrate.
  - can result in net matching of the beam.
- Foils also stop co-moving electrons that can neutralize the beam.



- Using aluminium, hundreds of $\sim 0.5\ \mu m$ thick foils with tens μm gaps can be stacked.

Results without scattering

The group's PRSTAB paper[1] derives a long pulse. Results shown for:

- Initial beam $< x^2 >^{\frac{1}{2}} = 200\mu m$ and $\frac{d}{ds} < x^2 >^{\frac{1}{2}} = 0$
- $\varepsilon_{x,\text{rms}} = 0$ and $P_\theta = 0$
- $I = 400\ A$
- No applied focusing
- No neutralization
- No scattering
- L = foil spacing indicated

Simulation agrees with theory!

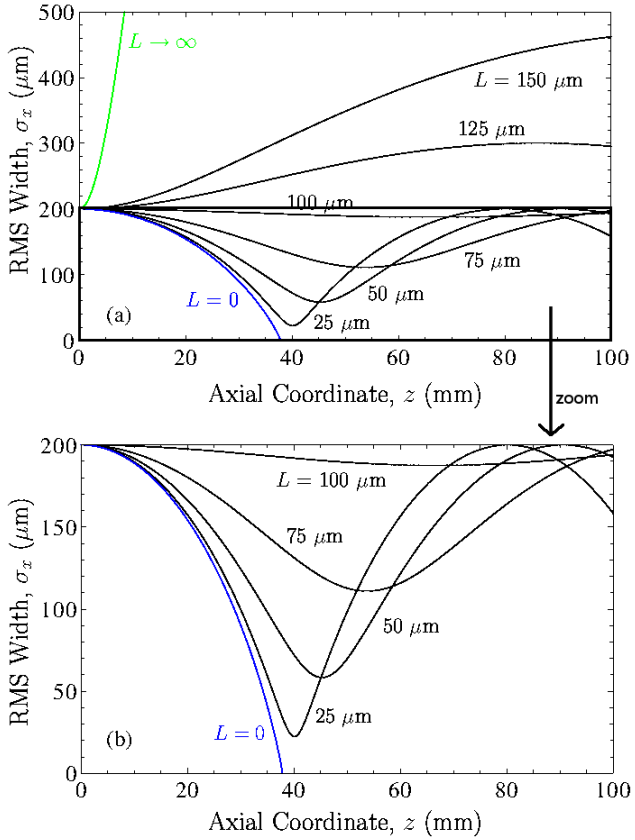


Figure: Envelope evolution for the transverse rms-beam

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[1] S.M. Lund, R.H. Cohen, P.A. Ni, Envelope model for passive magnetic focusing of an intense proton or ion beam propagating through thin foils, 2012, to be published

Possible issues

Many issues may alter the focusing effect from the idealized model.

- Scattering of ions in metallic foils can defocus beams.
- Can we neglect it?
  - hundreds foils may be traversed
  - Foils need to be thick enough to stop co-moving electrons and survive heating over the beam pulse.

Scattering effects will be investigated in this talk.
Addition issues not considered here:

- Electron (knock-on) neutralizing current.
- Foils having adequate thickness to stop co-moving electrons.
- Finite pulse duration and distribution structure.
- And a lot more!

Scattering of incident ions through matter

Following Jackson's treatment[2].

- Particle interactions:
  - *Atomic electrons of metal atoms* mostly decrease energy of incident particle without deflection
  - *Massive nuclei of metal atoms* mostly scatters incident particle without change of energy (elastic scattering) in case of an incoming proton beam
- Both effects largely classical
- However, nuclear charges can be effectively screened by atomic electrons at large impact factors, effectively reducing the Z of the nucleus.
- For small angle scattering, the Rutherford cross-section can be applied for a particle of charge ze scattering on a massive nucleus of charge Ze

[2] Jackson, Classical Electrodynamics, Wiley, 1988.

- Non-relativistic Rutherford cross-section formula for point charges[3]:

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{zZe^2}{4\pi\epsilon_0 E}\right)^2 \frac{1}{\theta^4} \tag{1}$$

  - z and Z atomic number of incident and foil atom.
  - θ = scattering angle in spherical coordinate
  - $E = \frac{1}{2}mv^2$ = kinetic energy of the incident particle of mass m and velocity $v = \beta c$ where c = speed of light
  - $d\Omega$ = solid angle
  - $\frac{d\sigma}{d\Omega}$ = cross-section = $\frac{Flux\ in\ solid\ angle\ d\Omega\ about\ \theta}{Incident\ flux\ per\ unit\ area}$ (in $[\frac{1}{L^2}]$)
- We need to introduce cut-off at small angle and large angle to take into account the finite size of the nucleus and the screening of its field by the electron cloud.
  Small-angle cut-off:

$$\theta_{min} \simeq \frac{Z^{\frac{1}{3}}}{192}\frac{1}{\beta} \tag{2}$$

[3] E. Segre, Nuclei and Particles, W. A. Benjamin, 1977

- Large-angle cut-off:

$$\theta < \theta_{max} \simeq \frac{274}{A^{\frac{1}{3}}} \frac{1}{\beta} \quad where \;\; A = mass\ number \;\simeq 2Z \tag{3}$$

- For particle traversing solid matter, the particle will undergo many small angle deflections and emerge at a small angle. That is the cumulative statistical superposition of many small angle deflections.
  - Large angle scatters are rare and should only happen once if at all in a thin foil.
  - Central limit theorem applies to the bulk core distribution and then implies that the net scattering angle should be Gaussian distributed.

- Average scattering angle:

$$< \theta^2 > \equiv \frac{\int_{\theta_{min}}^{\theta_{max}} d\Omega \; \theta^2 \; \frac{d\sigma}{d\Omega}}{\int_{\theta_{min}}^{\theta_{max}} \frac{d\sigma}{d\Omega} d\Omega} \equiv 2\theta_{min}^2 \; ln(\frac{\theta_{max}}{\theta_{min}}) \tag{4}$$

- By using our previous definitions of θ_{max} and θ_{min}:

$$< \theta^2 > \simeq 4\theta_{min}^2 \; ln(\frac{204}{Z^{\frac{1}{3}}}) \tag{5}$$

- Projecting the polar scattering angle θ to the $x - z$ plane gives the RMS scattering angle for a single collision:

$$< x'^2 > \Big|_{one \; deflection} = \frac{1}{2} < \theta^2 > \simeq 2\theta_{min}^2 \; ln(\frac{204}{Z^{\frac{1}{3}}}) \tag{6}$$

- The RMS scattering angle for a single particle travelling through a material of thickness t is given by:
  - the number of collision it undergoes.
  - the RMS scattering angle due to a single collision.
- Number of collision n that a particle makes travelling a path length t in the metal

$$n \simeq N\sigma_{tot}\Delta_f \tag{7}$$

  - N = number of atoms per unit volume
  - Δ_f = thickness
  - σ_{tot} = total cross section = $\int_{4\pi} \frac{d\sigma}{d\Omega} sin\theta d\theta d\phi \simeq \pi(\frac{zZe^2}{4\pi\epsilon_0 E})^2 \frac{1}{\theta^2_{min}}$
- Final rms scattering angle exiting the material:

$$\begin{aligned} < x'^2 > \Big|_{exit\ \ foil} &\simeq n < x'^2 > \\ &\simeq 2\pi N(\frac{zZe^2}{4\pi\epsilon_0 E})^2\ ln(\frac{204}{Z^{\frac{1}{3}}})\Delta_f \end{aligned} \tag{8}$$

Description of SRIM/TRIM

SRIM/TRIM is a group of Monte Carlo computer programs free of charge which calculate the stopping and range of ions (up to 2 GeV/amu) into matter[4]. It includes:

- Quantum mechanical treatment of ion-atom collisions.
- Screened Coulomb collisions of the atoms and ions during collisions due to the overlapping electron shells.
- Electron excitations and plasmons.
- Effective charge $Z^* < Z$ due to the collective electron cloud.
- Large angle scattering.

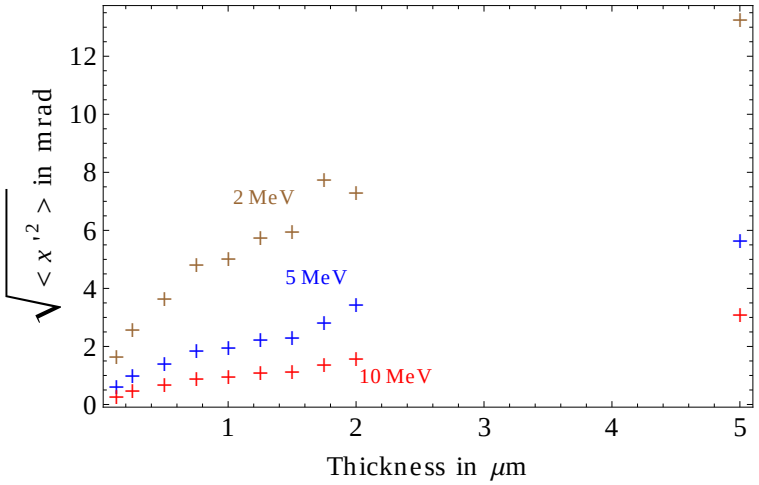
[4] J.F. Ziegler, http://www.srim.org

SRIM/TRIM Results on rms scattering angle and energy loss

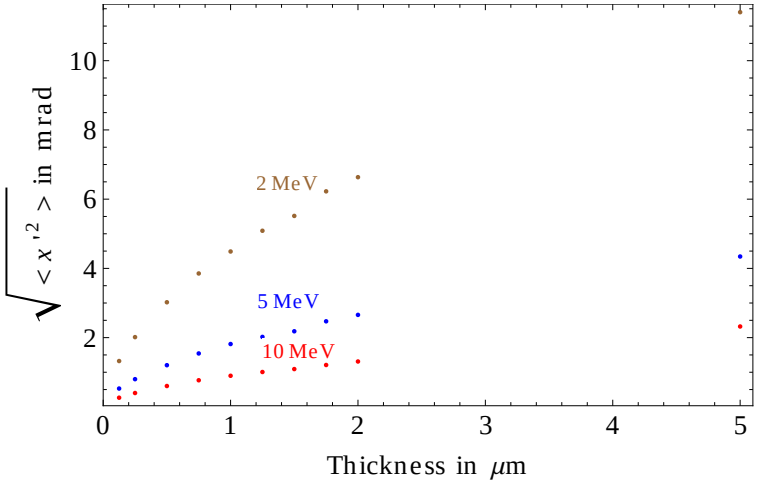
In each of our simulations using SRIM/TRIM, we shoot 3000 protons with normal incidence through a foil of Aluminum with a given thickness and compute the rms scattering angle. Only two parameters are modified.

- Energy of the incoming protons: 2 MeV, 5 MeV, 10 MeV.
- Thickness of the aluminum foil ranging from 0.125 μm to 5 μm

- SRIM results including both small and large angle scattering.



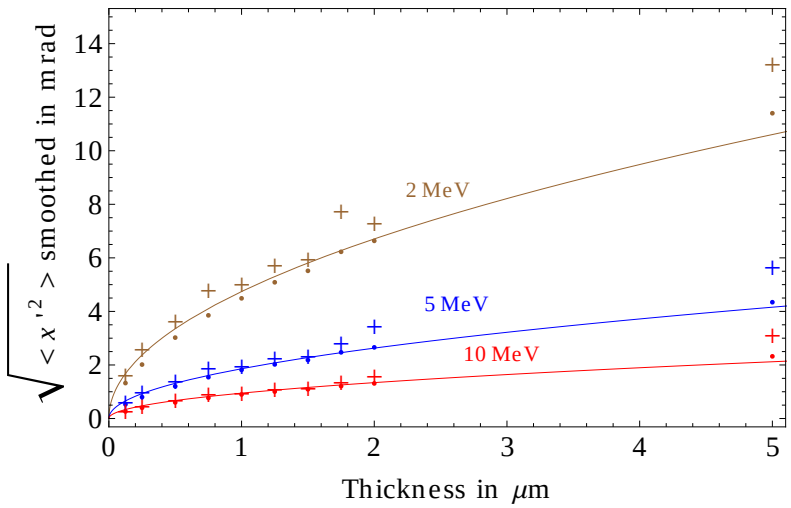
- Smoothed SRIM results with angles $|x'| > 5 < x'^2 >^{\frac{1}{2}}$ removed. Magnitude little changed but less noisy due to infrequent large angle events removed.



- Jackson's displayed a scaling of $< x'^2 >^{\frac{1}{2}} \propto \frac{\sqrt{foil\ \ thickness}}{particle\ \ energy}$
- Calculate scaling coefficient using least squares method and SRIM/TRIM data to obtain:

$$\sqrt{< x'^2 >} = G_{s,SRIM} \frac{\sqrt{t[\mu m]}}{E[MeV]}\ rad \qquad (9)$$

$$where\ G_{s,SRIM} = 0.0094 MeV.\mu m^{-\frac{1}{2}}$$



It fits the model quite well!

Comparison of the SRIM fit with Jackson coefficient for $\sqrt{< x'^2 >}$

- Using the formula from Jackson:

$$
\begin{aligned}
< x'^2 > \Big|_{exit\ \ foil} &\simeq 2\pi N(\frac{zZe^2}{E})^2\ ln(\frac{204}{Z^{\frac{1}{3}}})t \\
&\simeq \frac{G^2_{s,Jackson}}{E^2}t \qquad (10) \\
where\ G_{s,Jackson} &= \sqrt{2\pi N(zZe^2)^2 Log(204/Z^{\frac{1}{3}})}
\end{aligned}
$$

- For Al, find $G_{s,Jackson} = 0.024\ MeV.\mu m^{-\frac{1}{2}}$.
- $G_{s,Jackson}$ is 3.8$\times$ higher than $G_{s,SRIM}$.
  - Expect Jackson results to be higher due to large cut-off.
  - SRIM uses experimental tabular data.

Modification of Envelope Model due to Scattering

- From the PRSTAB paper, the derived envelope equation for foil focusing was:

$$\frac{d^2}{dz^2}\langle x^2\rangle_\perp^{1/2} + \kappa(z)\langle x^2\rangle_\perp^{1/2} + \frac{\gamma_b^2}{4}\left[(1-\mathcal{F}_J)\beta_b^2 - (1-\mathcal{F}_\rho)F\right]\frac{Q}{\langle x^2\rangle_\perp^{1/2}} - \frac{\varepsilon_{x,\mathrm{rms}}^2}{\langle x^2\rangle_\perp^{3/2}} = 0. \tag{11}$$

$$\varepsilon_{x,\mathrm{rms}} \equiv \left[\langle x^2\rangle_\perp\langle x'^2\rangle_\perp - \langle xx'\rangle_\perp^2\right]^{1/2} = const. \tag{12}$$

- The envelope equation in the PRSTAB paper must be modifed within the foils[5]. More details in the appendix.

$$\frac{d^2}{dz^2}\langle x^2\rangle_\perp^{1/2} + \kappa(z)\langle x^2\rangle_\perp^{1/2} + \frac{\gamma_b^2}{4}(1-\mathcal{F}_J)\beta_b^2\frac{Q}{\langle x^2\rangle_\perp^{1/2}} - \frac{\varepsilon_{x,\mathrm{rms}}^2}{\langle x^2\rangle_\perp^{3/2}} = 0. \tag{13}$$

$$\frac{d\varepsilon_{x,\mathrm{rms}}^2}{dz} = {C_s}^2\langle x^2\rangle_\perp \text{ where } C_s = \frac{G_{s,i}}{E} \text{ and } i = \text{SRIM or Jackson} \tag{14}$$

- Note: No electric defocus in foils: $F \to 0$
- Emittance evolves due to scattering.

[5] J. Barnard, USPAS Lecture, Beam Physics with Intense Space Charge, 2011 and private discussion

Approximation: "jump" of the emittance through each foil

- Foils are thin. Neglect change in $\langle x^2 \rangle_\perp$ through a single foil and take:

$$\langle x^2 \rangle_\perp \simeq \langle x^2 \rangle_\perp \Big|_{incident\ \ foil} = const. \tag{15}$$

- In the foils, neglect coupling to the envelope equation and take:

$$\frac{d\varepsilon_{x,\mathrm{rms}}^2}{dz} = {C_s}^2 \langle x^2 \rangle_\perp \Big|_{incident\ \ foil} \tag{16}$$

- For a foil thickness Δ_f:

$$\varepsilon_{x,\mathrm{rms}}^2 \Big|_{exit\ \ foil} \simeq \varepsilon_{x,\mathrm{rms}}^2 \Big|_{incident\ \ foil} + \Delta_f {C_s}^2 \langle x^2 \rangle_\perp \Big|_{incident\ \ foil} \tag{17}$$

Base Case without scattering

The group's PRSTAB paper derives a long pulse. Results shown for:

- Initial beam $< x^2 >^{\frac{1}{2}} = 200\mu m$ and $\frac{d}{ds} < x^2 >^{\frac{1}{2}} = 0$
- $\varepsilon_{x,\text{rms}} = 0$ and $P_\theta = 0$
- $I = 400\ A$
- No applied focusing
- No neutralization
- No scattering
- L = foil spacing indicated

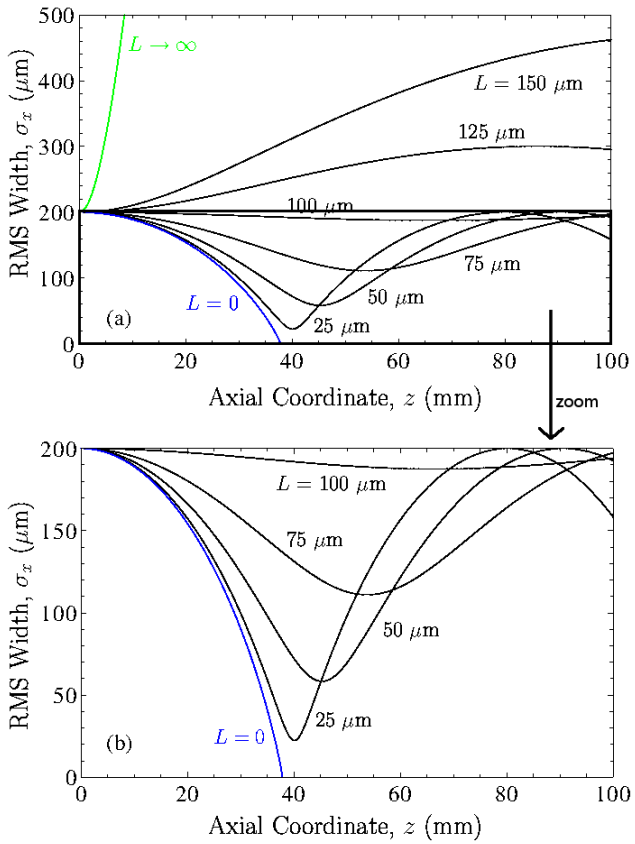


Figure: Envelope evolution for the transverse rms-beam

Rough estimate of the emittance growth (I)

Consider base case. Number of needed foil to reach the minimum spot size:

Foil Spacing L (μm)	Focal Spot Δz_{focus} (mm)	Number of Foils N
100	67	670
75	54	720
50	45	1100
25	40	1600

Rough estimate of the emittance growth (II)

- Suppose $\varepsilon_{x,\mathrm{rms}}\Big|_{initial} = 0$
- $< x^2 >_{\perp}^{1/2}\Big|_{though\ stack} \simeq 100\mu m = \mathrm{const.}$

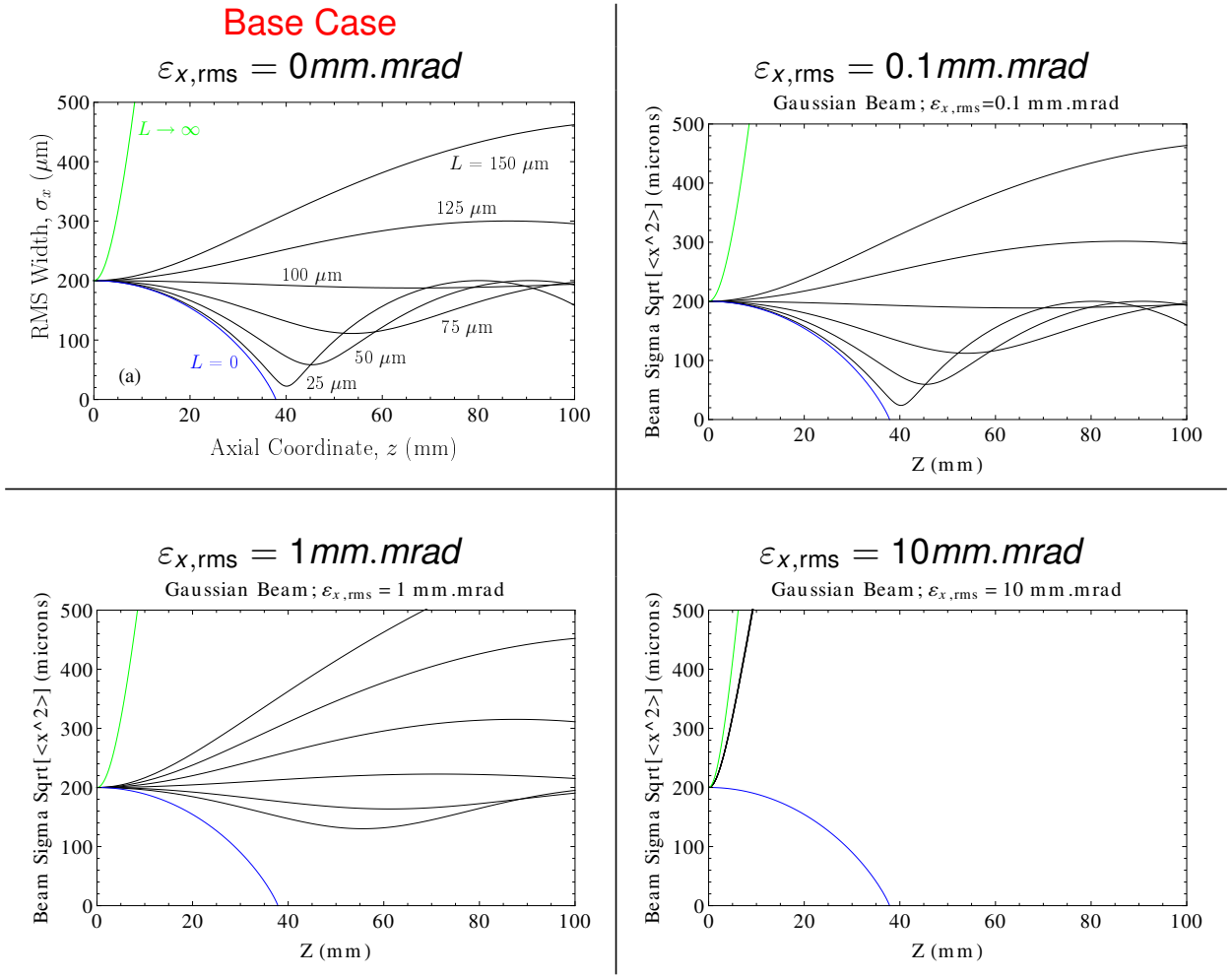
$\rightarrow \varepsilon_{x,\mathrm{rms}}^2\Big|_{exit\ stack} = N\Delta_f C_s^2 < x^2 >\Big|_{through\ stack}$

Foil Thickness Δ_f (μm)	Foil Spacing L (μm)	Focal Spot Δz_{focus} (mm)	Number of Foils N	Final Emittance $\varepsilon_{x,\mathrm{rms}}$ (mm.mrad)
$\Delta_f = 1\mu m$	100	67	670	6.2
	75	54	720	6.4
	50	45	1100	8.0
	25	40	1600	9.6
$\Delta_f = 0.1\mu m$	100	67	670	2.0
	75	54	720	2.0
	50	45	1100	2.5
	25	40	1600	3.0

The $\varepsilon_{x,\mathrm{rms}}$ is not negligible!

The following plots show the same curves and notation as this previous slide.

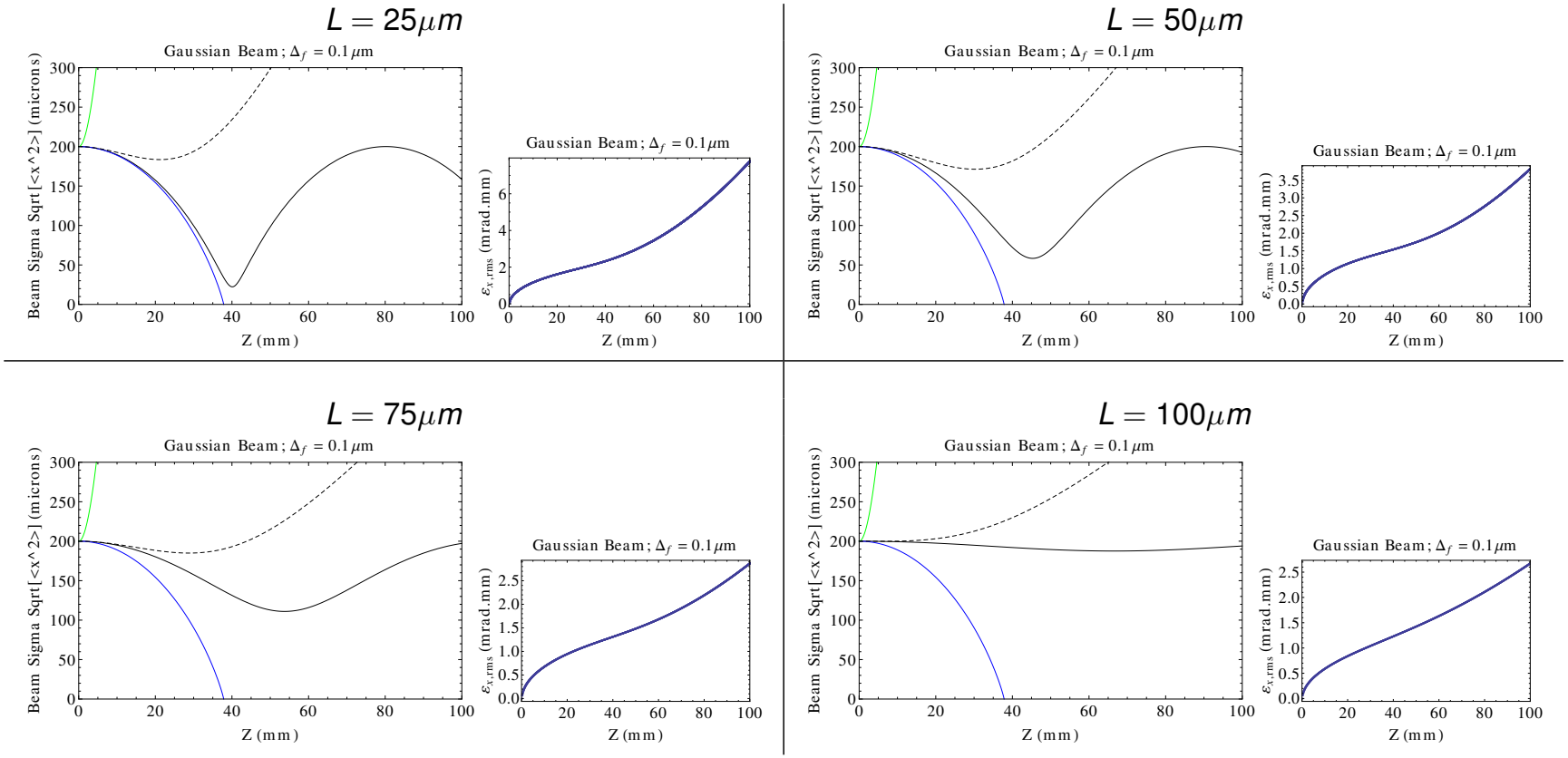
Comparison between Base Case and Cases with fixed non-zero emittance



Emittance growth $\gtrsim 0.1 mm.mrad$ is a problem.

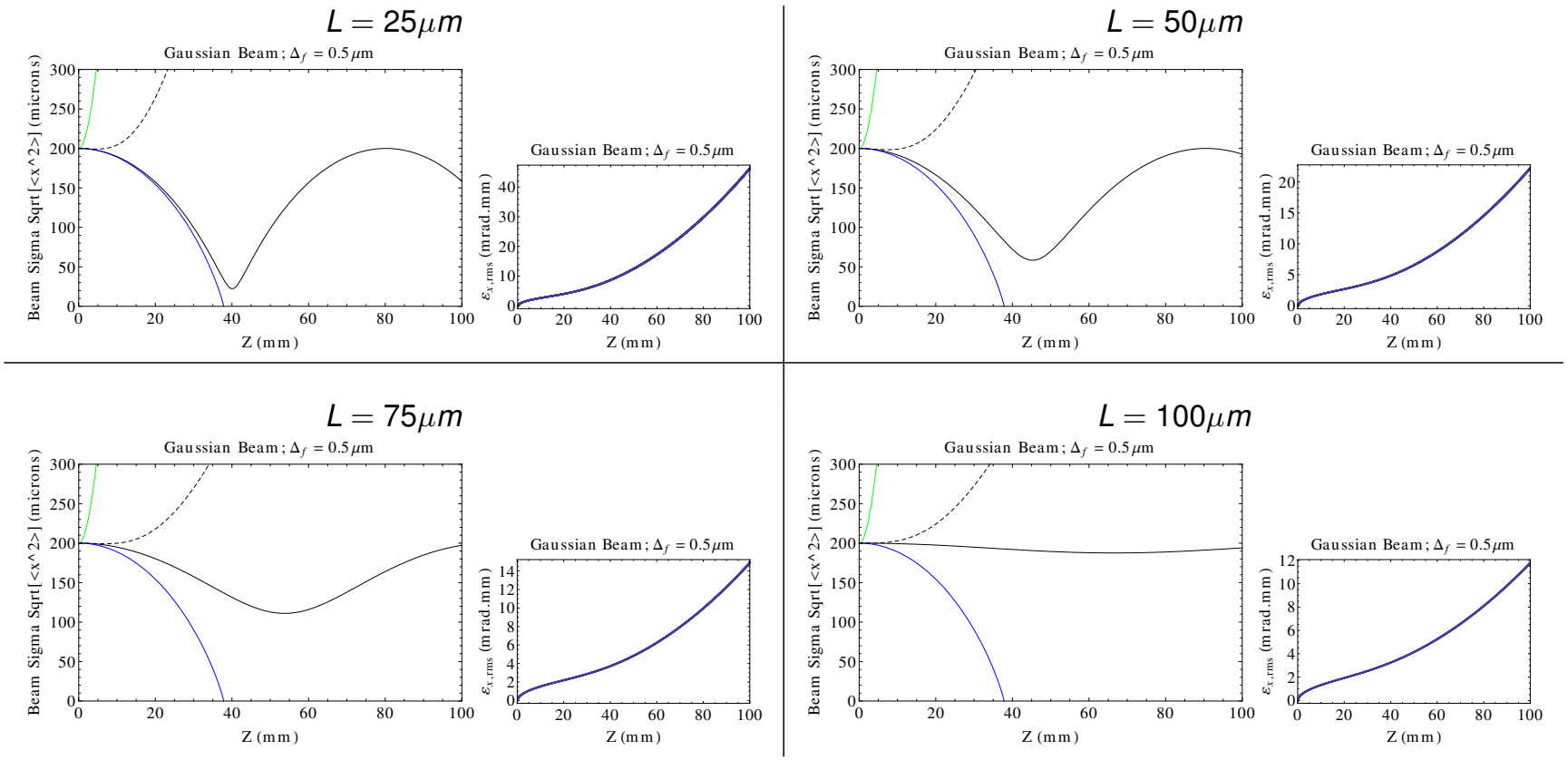
Comparison between Base Case and Cases with the actual formula for emittance growth (I)

The dashed lines represent the cases that include the emittance growth.
If $\Delta_f = 0.1\mu m$:



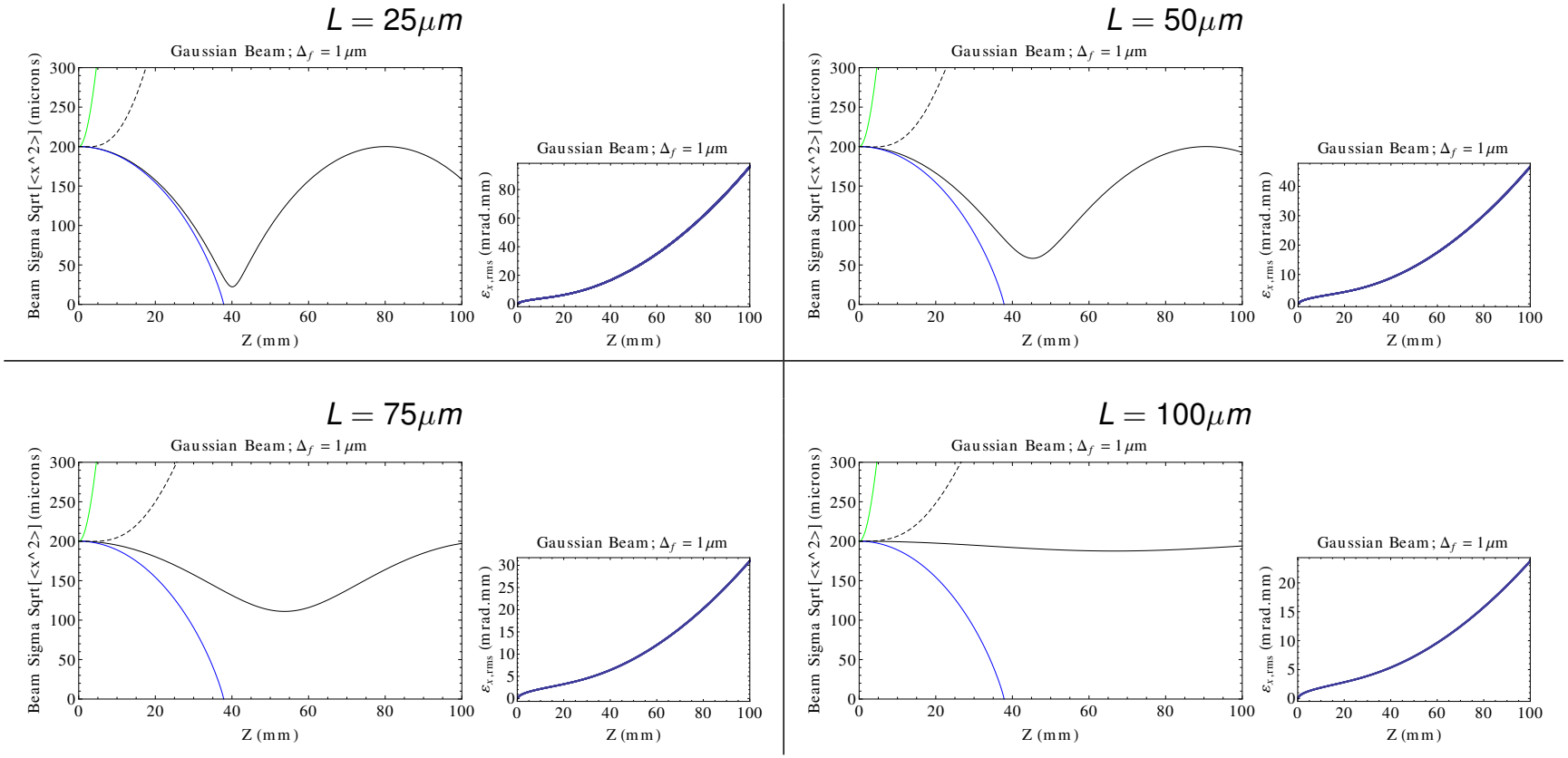
Comparison between Base Case and Cases with the actual formula for emittance growth (II)

The dashed lines represent the cases that include the emittance growth.
If $\Delta_f = 0.5\mu m$:



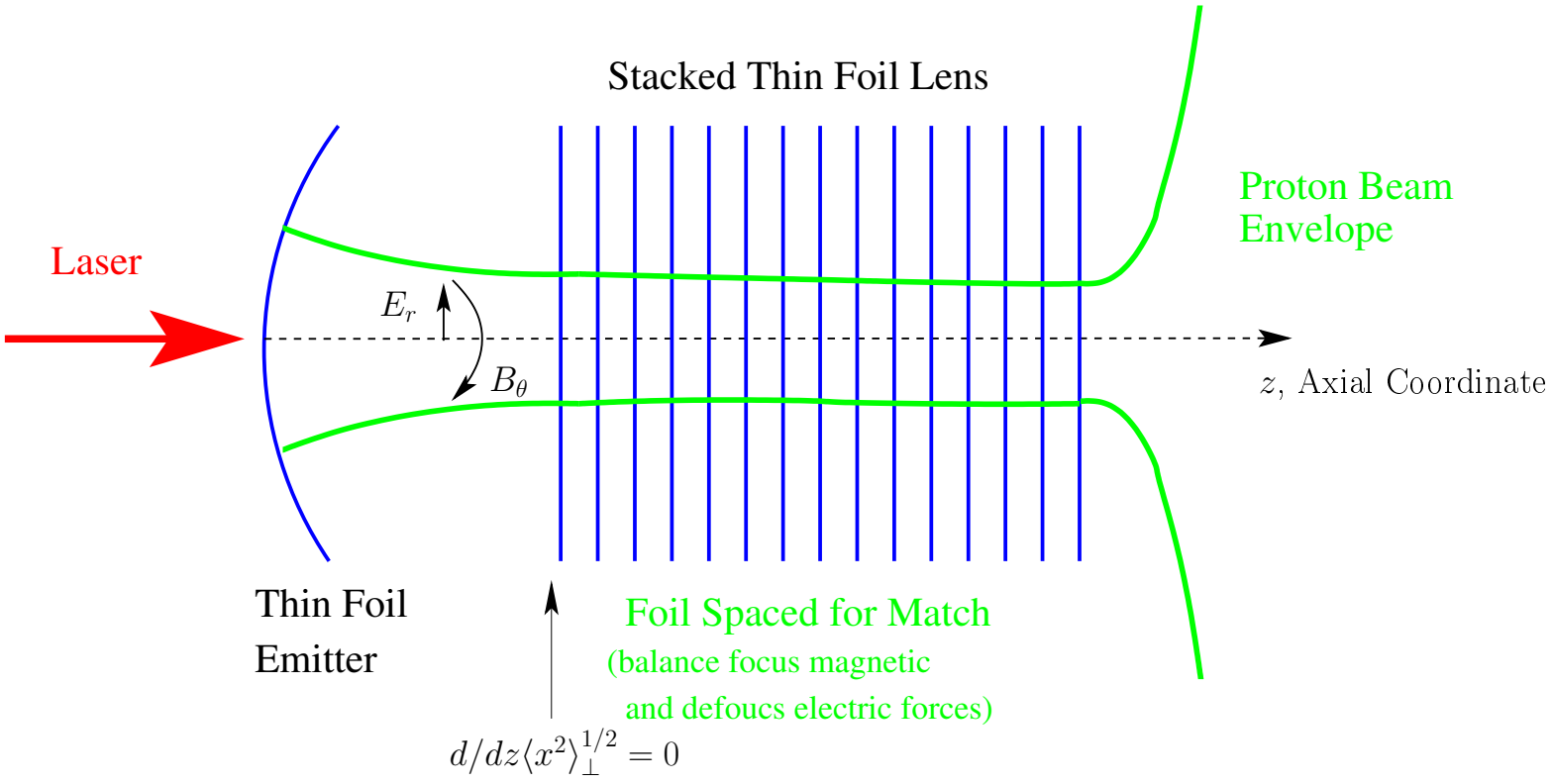
Comparison between Base Case and Cases with the actual formula for emittance growth (III)

The dashed lines represent the cases that include the emittance growth.
If $\Delta_f = 1\mu m$:



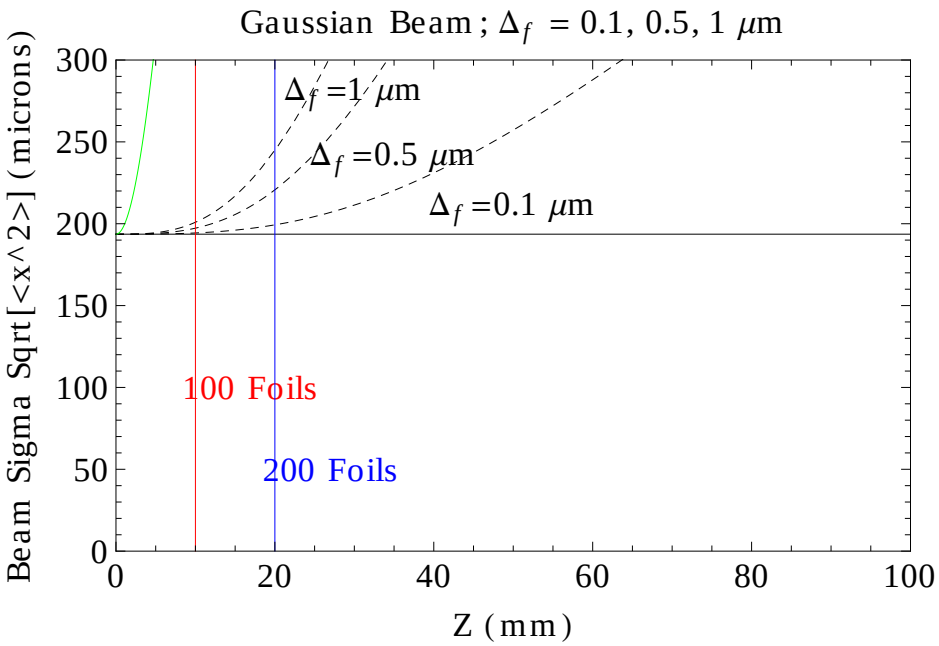
Schematic for Proof of Principle Experiment

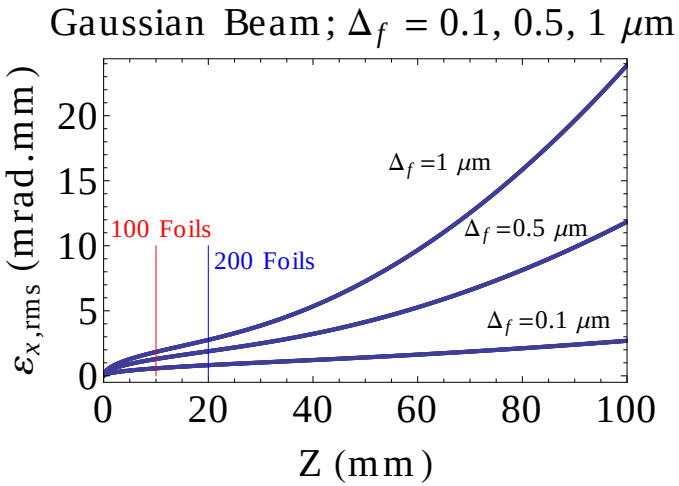
Tuned for zero envelope divergance at the first focusing foil using a laser produced beam from a curved foil emitter and matched balance between focusing magnetic and defocusing electric forces.



Proof-of-principle experiment at PHELIX (GSI) - currently conducted (I)

- Start with a 10 MeV "matched" proton beam through stack of Aluminum foils: $< x^2 >^{1/2} / L = 1.94$ for initial balance of focusing magnetic and defocusing electric forces.
- Case where $L = 100 \mu m$, or equivalently $\sqrt{< x'^2 >} = 194 \mu m$

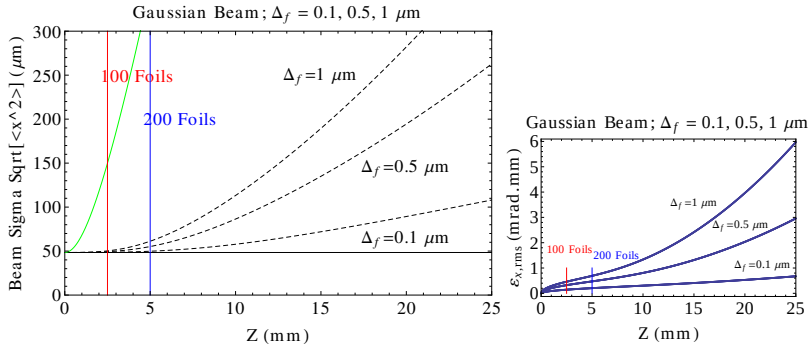




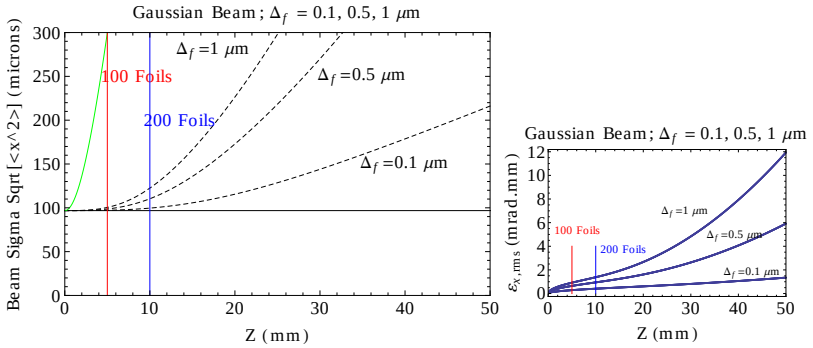
Proof-of-principle experiment at PHELIX (GSI) - currently conducted (II)

Cases with different gap spacing L to correspond to smaller initial "matched" beam.

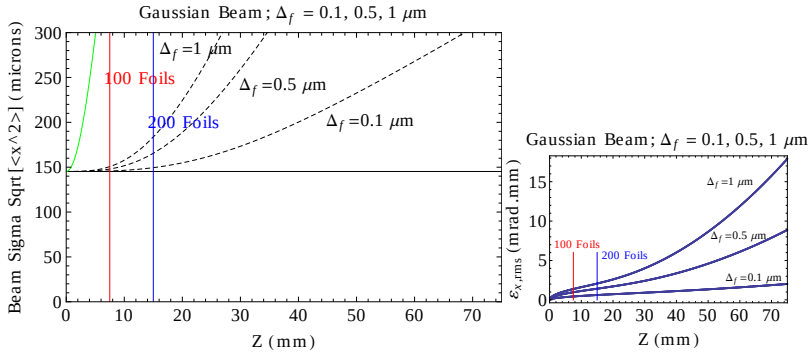
$L = 25\mu m \Leftrightarrow \sqrt{< x'^2 >} = 48\mu m$



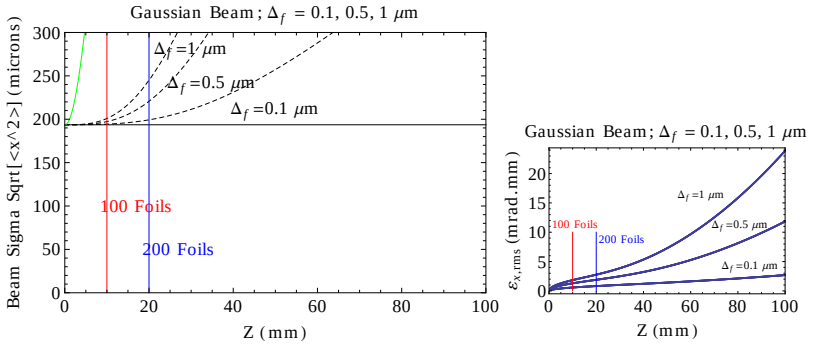
$L = 50\mu m \Leftrightarrow \sqrt{< x'^2 >} = 97\mu m$



$L = 75\mu m \Leftrightarrow \sqrt{< x'^2 >} = 146\mu m$



$L = 100\mu m \Leftrightarrow \sqrt{< x'^2 >} = 194\mu m$

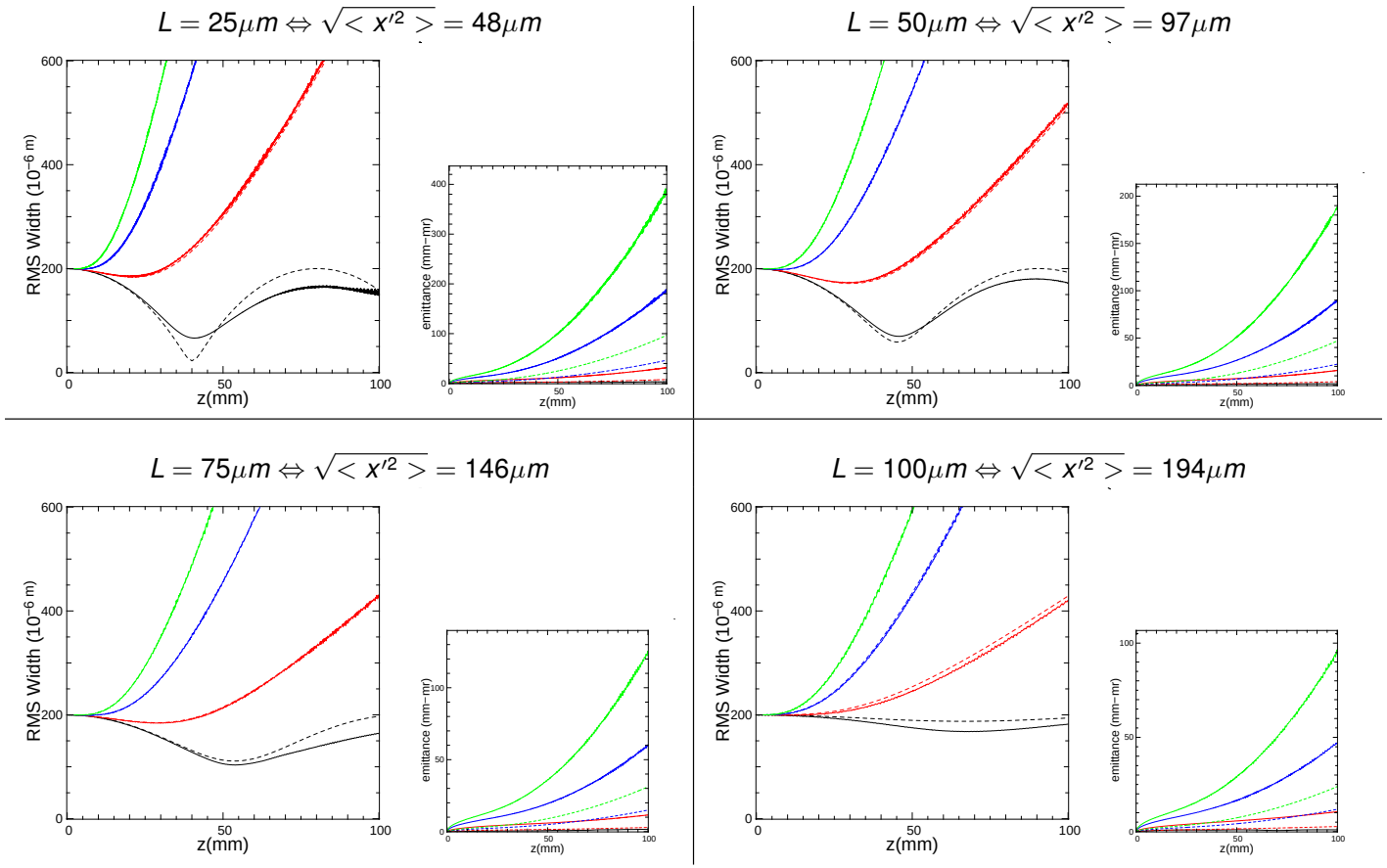


Significant envelope divergance should result from scattering beyond few hundred foils.

Comparisons against WARP simulations[6]

Theory: dashed lines. WARP simulations: full lines.

Colors: $\Delta_f = 0$, 0.1, 0.5, 1 μm



While the theory and simulations match for the RMS Width σ_X, it does not for the emittance.

[5] See simulation details: PP8.00145 : Theory and Simulation of Passive Focusing of an Ion Beam Propagating Through Thin Foils

Conclusion

- The emittance growth due to scattering cannot be neglected in likely applications.
- What can be done in near term proton proof-of-principle experiments at GSI/LLNL:

$$\varepsilon^2_{x,\mathrm{rms}}\Big|_{exit\ \ at\ \ Nth\ \ foil} \simeq \varepsilon^2_{x,\mathrm{rms}}\Big|_{initial} + \sum_{i=1}^{N} \Delta_f \frac{{G_s}^2}{E^2} \langle x^2 \rangle_\perp \Big|_{exit\ \ of\ \ ith\ \ foil} \tag{18}$$

  - Thinner foils Δ_f: ${\Delta_f}^{1/2}$ reduction on emittance.
  - Fewer foils N: $N^{1/2}$ reduction on emittance.
  - Higher energy E: E reduction on emittance.
- Future work
  - Pursue in WARP simulation and benchmark
  - Explore minimum foil thickness for electron stopping.
  - Quantify the number of knock-on electrons due to the passage of ion beam through foils.
  - To support experiments, use more realistic distributions in simulations instead of idealized beam distributions.

Appendix: Modification of the envelope equation

Contribution due to scattering: $< x'^2 > (z) \simeq 2\pi N(\frac{zZe^2}{4\pi\epsilon_0 E})^2 \, ln(\frac{204}{Z^{\frac{1}{3}}})z = C_s z$

Therefore: $\frac{d<x'^2>(z)}{dz} = C_s$

Using the standard equation of motion:

$$x'' + \kappa x - \frac{Q}{2 < x^2 >^{(1/2)}} x = 0$$

to derive 2nd order moments and show that:

$$\frac{d\varepsilon^2_{x,\text{rms}}}{dz} = 0 \;\; with \;\; \varepsilon^2_{x,\text{rms}} =< x^2 >< x'^2 > - < xx' >^2$$

and taking into account the scattering, the standard moment equations are modified:

$$\begin{cases} \frac{d<x^2>}{dz} = 2 < xx' > \\ \frac{d<xx'>}{dz} =< x'^2 > -\kappa < x^2 > + \frac{Q}{4} \\ \frac{d<x'^2>}{dz} = -2\kappa < xx' > - \frac{Q<xx'>}{2<x^2>} + C_s \end{cases}$$

These can now be used to derive coupled envelope and emittance equations:

$$\frac{d < x^2 >^{(1/2)}}{dz} = \frac{< xx' >^{(1/2)}}{x^2}$$

$$\frac{d^2 < x^2 >^{(1/2)}}{dz^2} = \frac{\frac{d < xx' >}{dz}}{< x^2 >^{(1/2)}} - \frac{1}{2}\frac{< xx' >}{< x^2 >^{(3/2)}}\frac{d < x^2 >}{dz}$$

$$= \frac{< x'^2 >}{< x^2 >^{(1/2)}} - \kappa < x^2 >^{(1/2)} + \frac{Q}{4 < x^2 >^{(1/2)}} - \frac{< xx' >^2}{< x^2 >^{(3/2)}}$$

$$= -\kappa < x^2 >^{(1/2)} + \frac{Q}{4 < x^2 >^{(1/2)}} + \frac{\varepsilon^2_{x,\mathrm{rms}}}{< x^2 >^{(3/2)}} = 0$$

Next examine $\varepsilon^2_{x,\mathrm{rms}}$:

$$\frac{d\varepsilon^2_{x,\mathrm{rms}}}{dz} = < x'^2 > \frac{d < x^2 >}{dz} + < x^2 > \frac{d < x'^2 >}{dz} - 2 < xx' > \frac{d < xx' >}{dz}$$

$$= \cancel{2 < x'^2 >< xx' >} - \cancel{2\kappa < x^2 >< xx' >} - \cancel{Q\frac{< x^2 >< xx' >}{2 < x^2 >}}$$

$$+ < x^2 > C_s - \cancel{2 < xx' >< x'^2 >} + \cancel{2\kappa < xx' >< x^2 >} - \cancel{2\frac{Q}{4} < xx' >}$$

$$= < x^2 > C_s$$

Hence the coupled envelope and emittance equations:

$$\boxed{\begin{aligned} \frac{d^2 < x^2 >^{(1/2)}}{dz^2} + \kappa < x^2 >^{(1/2)} - \frac{Q}{4 < x^2 >^{(1/2)}} - \frac{\varepsilon^2_{x,\mathrm{rms}}}{< x^2 >^{(3/2)}} = 0 \\ \frac{d\varepsilon^2_{x,\mathrm{rms}}}{dz} = < x^2 > C_s \end{aligned}}$$