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Analytic Model for Richtmyer-Meshkov Turbulent Mixing Widths

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We discuss a model for the evolution of the turbulent mixing width $h(t)$ after a shock or a reshock passes through the interface between two fluids of densities ρ_A and ρ_B inducing a velocity jump Δv. In this model the initial growth rate is independent of the surface finish or initial mix width h_0, but its duration t^* is directly proportional to it: $h(t) = h_0 + 2\alpha A \Delta\text{v}t$ for $0 \le t \le t^*$, and $h(t) = h^*\left(1 + (\dot{h}^*/\theta h^*)(t - t^*)\right)^{\theta}$ for $t \ge t^*$. Here A is the Atwood number $(\rho_B - \rho_A)/(\rho_B + \rho_A)$, α and θ are dimensionless, A–dependent parameters measured in past Rayleigh-Taylor experiments, and β is a new dimensionless parameter we introduce via $t^* = (h_0/\Delta\text{v})\beta$. The mix width h and its derivative $\dot{h}$ remain continuous at $t = t^*$ since $h^* = h_0 + 2\alpha A \Delta\text{v}t^*$ and $\dot{h}^* = 2\alpha A \Delta\text{v}$. We evaluate $\beta \sim 6$ at $A \approx 0.7$ from air/SF_6 experiments and propose that the transition at $t = t^*$ signals isotropication of turbulence. The model makes several predictions for shock- as well as reshock-induced mix, such as a Kolmogorov $k^{-5/3}$ spectrum for the turbulent energy at late times $t >> t^*$. We show that it is consistent with past experiments on reshocks, and call for the construction of a "National Shock-Tube Facility" to further study turbulence generated by the Richtmyer-Meshkov instability.

Key words: Turbulent mix, shocks, reshocks, Rayleigh-Taylor, Richtmyer-Meshkov, National Shock-Tube Facility.

I. Introduction

Hydrodynamic instabilities at fluid interfaces have been known for a long time. For example, it is well-known that in a gravitational filed $\vec{g}$ one cannot support a heavy fluid above by a lighter fluid below – the fluids interpenetrate and mix. The linear regime of the instability where the amplitude $\eta(t)$ of a simusoidal perturbation remains much smaller than the wavelength λ was studied by Lord Rayleigh [1] and G. I. Taylor [2], hence the instability is known as Rayleigh-Taylor (RT). Somewhat later a shock-induced instability, Richtmyer-Meshkov or RM, was the focus of an analytic study by Richtmyer [3] and experimental study by Meshkov [4], who found that perturbations grow when a shock passes from a light fluid to a heavy fluid or vice versa. RT and RM instabilities have been extensively studied in recent years because they impact inertial confinement fusion capsules [5] as well as astrophysical phenomena such as supernova explosions [6]. The scales are vastly different, from millimeters to millions of kilometers, and therefore analytic studies or models expressing how the instabilities evolve are highly useful.

Single-scale perturbations grow fast in the linear regime and then slow down, but continue to grow, in the nonlinear regime where $\eta \geq \lambda$. Exact analytic expressions are practically nonexistent in this regime and a model must be used. A most useful nonlinear model, though not a panacea, is Layzer's model [7]. Originally proposed for RT, it was applied to RM more recently [8]. These were limited to a single fluid, hence $A=1$, where A is the Atwood number defined by $(\rho_B - \rho_A)/(\rho_B + \rho_A)$, ρ_B (ρ_A) being the density of the heavy (light) fluid. The model was extended to arbitrary A by Goncharov [9]. Although there are limitations to this model, we have applied it to cases where the acceleration is time-dependent [10].

When the initial perturbations are not single-scale, as in practically all real-life applications, turbulence is observed at the interface between the two fluids and a mix width, denoted by h, evolves and grows with time. Needless to say, there are no exact solutions for this case either. The turbulence was described in the pioneering work of Youngs [11] and Read [12] using advanced numerical simulation techniques [11] and experiments driven by rockets and hence known as "rocket-rig experiments" [12]. The mix width of the light fluid into the heavier fluid is denoted by h^b, with b standing for "bubble", while the mix width of the heavier fluid into the lighter fluid is denoted by h^s, with s for "spike", terminology derived from the single-scale nonlinear amplitudes η^b and η^s. Of course the total mix width is $h^{b+s} \equiv h^b + h^s$.

It was found both numerically and experimentally [11,12] that h^b obeys a simple law when the acceleration $\vec{g}$ is constant:

$$h^b = \alpha^b A g t^2 \tag{1}$$

with $\alpha^b \approx 0.07$. Youngs also performed simulations, all in 2D, for shock-induced mix but did not propose an expression equivalent to Eq. (1) for the RM case. His numerical evaluation of α^b was close to the experimental value. Since then multiple calculations have been performed of which we mention only the "Alpha-Group Collaboration" with 3D codes [13].

On the experimental side several research groups have confirmed Eq. (1) with $\alpha^b \approx 0.03 - 0.07$: In chronological order Snider and Andrews [14], Dimonte and Schneider [15], Edwards *et al.* [16], Ramaprabhu and Andrews [17], and Olson and Jacobs [18]. No experimental correlation between initial conditions and late-time turbulent behavior has been reported, although such theories exist (e.g. Refs. [19, 20]). An experiment with a large variation of initial conditions found remarkably fast recovery of the buoyancy-driven RT mixing width indicating very weak, if any, correlation (Kraft and Andrews, Ref. [21]).

The first experimental report of the RT mixing width on the spike side, h^s, was made by Dimonte and Schneider [22] with a result similar to Eq. (1) with α^b replaced by $\alpha^s \geq \alpha^b$. At low Atwood numbers $\alpha^s \approx \alpha^b$ as expected [14, 17], and α^s increases slowly with A reaching $\approx 4\alpha^b$ at $A \approx 1$ [22].

Turning to RM or shock-driven mix, we proposed [23] a formula based on treating the shock as an instantaneous acceleration which induces a jump velocity $\Delta \mathrm{v}$ at the interface: Replace Eq. (1) by $d^2 h^b / dt^2 = 2\alpha^b A g$, replace $g \to \Delta \mathrm{v} \delta(t)$, where $\delta(t)$ is the Dirac delta function, and integrate twice assuming $h(0) \approx \dot{h}(0) \approx 0$ to obtain

$$h^b = 2\alpha^b A \Delta \mathrm{v} t \,. \tag{2}$$

Like RT mix, RM mix according to Eq. (2) is independent of initial conditions, although this will be modified in a rather strange way in Sec. III below. Comparisons with shock tube experiments will be presented in Sec. II. The extension of Eq. (2) and its implications will be presented in Sec. III. Conclusions and a call for a new type of experiments will be given in Sec. IV.

II. RM experiments and modeling

The first comparison of Eq. (2) with experiments was presented by Vetter and Sturtevant [24]. A large shock tube is needed to isolate the mix width from the boundary layer. Low/high density gases air/SF_6 are separated initially by a thin membrane supported by two wire plates. When the shock passes from the air into the SF_6 the mixing width depends on the relative orientation of the supporting plates and how the membrane is shredded. The shock bounces off the endwall of the shock tube and returns to reshock the interface. The growth after reshock was found to agree with Eq. (2) and was independent of how much mixing was generated by the first shock. Unlike RT experiments where h^b and h^s can be measured separately, RM experiments cannot distinguish them and measure only the sum $h^b + h^s$. Since h^s was not known at the time, Vetter and Sturtevant assumed $\alpha^s \approx \alpha^b \approx 0.07$ and, from Eq. (2),

$$h^b + h^s = 2(\alpha^b + \alpha^s) A \Delta \mathrm{v} t \approx 0.28 A \Delta \mathrm{v} t\,, \tag{3}$$

a result that agreed well with post-reshock mixing widths [24].

Subsequent experiments by Erez *et al*. [25], using strong or weak membranes, confirmed that post-reshock growth was independent of pre-reshock conditions. Those conditions depended on whether a weak or a strong membrane was used to initially separate the gases, but they did not affect the post-reshock growth rate [25].

These and other experiments, in particular those of Leinov *et al.* [26], are discussed in Ref. [27]. Leinov *et al*. found Eq. (3) to be in good agreement with three different types of reshock experiments once the more modern, increased value of α^s was used. For air/SF_6 one has $A \approx 0.72$ for which $\alpha^s \approx 1.8\alpha^b$ (see Ref. [22]). Combined with $\alpha^b \approx 0.06$, the coefficient in Eq. (3) reads 0.34, compared with ≈ 0.38 experimentally (see Fig. 19 in Leinov *et al.* [26]).

On the computational side, large direct numerical simulations of turbulence after reshock have been reported and they also support Eqs. (2) and (3) – See Refs. [28] and [29]. We should point out, however, that most of the experiments as well as the simulations have been on the air/SF_6 system and therefore we cannot ascertain the general validity of these equations. We mention one alternative model, that of Lombardini *et al.* [30], and it has received even less testing by other researchers.

III. Extension of the model

From the discussion in the previous section we see that practically all RM turbulent mix measurements have concentrated on the reshock. The reason, we believe, is that these are “cleaner”, highly reproducible experiments, free of membrane effects. In contrast, first-shock mixing widths depend on the membrane which is both obvious (there is no mix if the membrane survives the shock) and demonstrated experimentally [24, 25]. One can imagine, however, ideal experiments which have no membrane, of the type conducted by Jacobs and collaborators [31, 32] on single-scale perturbations, but now starting with multimode perturbations which lead to turbulent mix. The question then becomes: How does the first-shock-mixing-width evolve with time?

Our prediction [27] is that the mix width following *any* shock or reshock grows according to the *same* formula,

$$h = h_0 + 2\alpha A \Delta \mathrm{v} t\,, \tag{4}$$

where $h = h^b$ or h^s, $\alpha = \alpha^b$ or α^s, $h_0 = h_0^b$ or h_0^s is the initial ($t = 0$) mix width and t is the time after shock or reshock. For the first shock h_0 is very small and can be neglected, but of course h grows and, denoted by h_- just before reshock, must be included for the reshock. For example, let reshock occur at $t_{reshock}$ when $h = h_-$; after reshock h is given by $h_- + 2\alpha A \Delta \mathrm{v}(t - t_{reshock})$ in this model.

This is the same as the original model [23]. The extension given in [27] predicts that Eq. (4) remains valid only until a time $t*$, after which it is replaced by

$$h = h*\left(1 + \frac{\dot{h}*}{\theta h*}(t - t*)\right)^{\theta} \tag{5}$$

where $h* = h_0 + 2\alpha A\Delta\mathrm{v}t*$, $\dot{h}* = 2\alpha A\Delta\mathrm{v}$, and θ $(=\theta^b$ or $\theta^s)$ is a dimensionless parameter taken from RT experiments after the acceleration was turned off [22]. In words, the model takes $h \sim t$ for $t \le t*$ and $h \sim t^{\theta}$ for $t \ge t*$. During the transition from t to t^{θ} the mix width h and its derivative $\dot{h}$ remain continuous.

Fig. 1 taken from Ref. [27] shows $h(t)$ for 3 experiments of Leinov *et al.* [26]: $M_s = 1.33$ with $L = 80$ mm, $M_s = 1.20$ with $L = 80$ mm, and $M_s = 1.20$ with $L = 235$ mm. M_s is the Mach number of the incoming shock in air, and L is the length of the test section containing SF_6 which varied from a minimum of 80 mm to a maximum of 235 mm. Eqs. (4) and (5) have been used to draw the 3 curves in Fig.1 and they are in good agreement with the experiments (for details see [27]), which stopped when a third wave reached the interface.

Perhaps the most surprising element of the model is the evaluation of $t*$, the transition time, and its connection to h_0. In Ref. [27] we argued that

$$t* = \frac{h_0}{\Delta\mathrm{v}}\beta \tag{6}$$

where β is a new dimensionless "constant" although it can, *a priori*, depend on A, M_s, ratio of sound speeds, or any other dimensionless parameter in the problem. For air/SF_6 and $M_s = 1.20$ we evaluated $\beta \approx 6$ and suggested that it may be truly a "constant" like

α or θ. In that case Eq. (6) predicts that $t^*_{shock} << t^*_{reshock}$, i.e. the mix width begins to decay earlier after a shock than after a reshock. This follows because the jump velocities $\Delta\mathrm{v}$ after shock or reshock are similar within factors ~2 but h_0, the mix width before the shock, is much smaller than h_-, the mix width before reshock, and therefore

$$t^*_{shock} = \frac{h_0}{\Delta\mathrm{v}_{\mathrm{shock}}}\beta \sim \frac{h_0}{\Delta\mathrm{v}_{reshock}}\beta << \frac{h_-}{\Delta\mathrm{v}_{reshock}}\beta = t^*_{reshock}\,. \tag{7}$$

This effect is seen in Fig.1 where the transition from t to t^θ behavior occurs ~0.1 ms after the shock but much later, ~0.4 ms after reshock in the long ($L = 235$ mm) test section.

In Ref. [27] we proposed that the transition from t to t^θ was the result of the mixing width forgetting the direction of the shock, i.e., turbulence becoming isotropic around $t = t*$. Eq. (6) simply says that interfaces with large initial perturbations retain memory of the direction of the shock longer than interfaces with small initial perturbations. Since reshocks see larger h_- than shocks, they decay later, which is the essence of Eq. (7).

Combining Eq. (6) with the definition $h^* \equiv h_0 + 2\alpha A \Delta\mathrm{v}t*$ we obtain

$$\frac{h*}{h_0} = 1 + 2\alpha\beta A \tag{8}$$

which is independent of $\Delta\mathrm{v}$ and is a function of A only. For air/SF_6 $A \approx 0.72$, $\alpha^s \approx 1.8\alpha^b$, hence $h*/h_0 \approx 1 + 2\times 2.8\alpha^b\beta A \approx 2.5$ for $\alpha^b = 0.06$ and $\beta = 6$, meaning that the total mix width begins to decay after it has grown to about 2.5 times its initial value, i.e., when $h/h_0 \approx 2.5$ for the shock and $h/h_- \approx 2.5$ for the reshock as well. In Fig. 1

$h_- \approx 6.2\,\text{mm}$ in the long shock tube just before reshock, so the decay begins when h, growing linearly with time, reaches ~16 mm after which it switches to a t^{θ} growth.

From Eq. (8) h^*/h_0 is an increasing function of A, assuming β is fixed at ~6, and therefore $(h^*/h_0)_{\max} = (h^*/h_0)_{A=1} = 1+12\alpha^b(1+\alpha^s/\alpha^b)$, where α^s is the value of spike parameter for $A=1$. An air/water experiment with $A \approx 1$ (Ref. [33]) reported $\alpha^s/\alpha^b \approx 2.5$, leading to $(h^*/h_0)_{A=1} \approx 3.5$. From Ref. [22], however, $\alpha^b \approx 0.05$ and $\alpha^s/\alpha^b \approx 4$ near $A=1$, giving $(h^*/h_0)_{A=1} \approx 4$, meaning that in all cases the transition, which appears like a "decay", must occur before h reaches at most 4 times its preshock or pre-reshock value. Lower Atwood numbers lead to earlier transitions.

This extended model makes many explicit predictions, but little of it can be said to be verified experimentally so far. As we mentioned, Leinov *et al.* [26] concentrated on the reshock. They did, however, present h_- values for 6 different L's, and these values agree with the values predicted by Eq. (5). Another prediction of the model is that after transition the turbulence becomes isotropic and hence is expected to have a $k^{-5/3}$ spectrum, which is verified by the recent RM experiments of Weber *et al.* [34]. Before the transition, the turbulent energy is predicted have a k^{-1} spectrum [23]. These two regimes are shown schematically in Fig. 2. Needless to say, the transition from the early $E \sim k^{-1}$ behavior to the late $E \sim k^{-5/3}$ does not occur suddenly at $t = t^*$ but only gradually as the direction of the shock is forgotten by the turbulence, and indeed the $k^{-5/3}$ spectrum was observed in the experiments only at late times [34]. This is one of the few cases where the turbulent spectrum is decidedly time-dependent and the model, we believe, provides a reasonable explanation as loss of memory of the direction singled out by the shock, and subsequent return to isotropy.

IV. Concluding remarks and future work

In our model [23, 27] the general behavior after the first shock is the same as after the second (or third) shock: Linear growth followed by decay. As h grows from one shock, the next one starts with a larger initial mix width h_{-}, and a larger h_{-} means only a longer period t^* of linear growth. Since the very first shock starts with a small h_0, h decays quickly. The next shock sees a larger mix width and therefore decays later. If there is a third or fourth shock they are predicted to decay even later. For a given system the Atwood number A changes little from shock to shock and therefore, for $A \approx 0.7-1$, the decay begins after the mix width has reached ~2.5-4 times the value it started with.

What is somewhat strange in this model is that the early growth rate is independent of h_0 or h_{-} which control only the *duration* of this early growth. We show in Fig. 3 two cases having the same basic flow: $M_s = 1.20$ with $L = 235$ mm. The first starts with $h_0 = 0.64$ mm, the second starts with a 5 times larger h_0. As discussed above, the early growth is the same in both cases, but it lasts 5 times longer in the second case, and hence reaches a larger h_{-} by $t_{reshock}$ (2.2 ms in Fig. 3). But now it is only 3 times larger. Upon reshock, both mixing widths again grow at the same rate, $2\alpha A \Delta \mathrm{v}_{\mathrm{reshock}}$, but the second case lasts 3 times longer than the first which begins to decay at ~2.6 ms, some 0.4 ms after the reshock. By 3.2 ms, when a third wave arrives, the first case has grown to $h \sim 24$ mm while the second case is still growing linearly with time (It would begin to decay at 3.4 ms, i.e. $2.2+3\times0.4$ ms, but is “interrupted” by the third wave). Note that at 3.2 ms the second case is only 1.8 times larger than the first. Clearly, initially large ($5\times$) differences in h_0 result in smaller ($1.8\times$) differences after a shock and a reshock.

We have ignored any membrane effect which affects the first shock, and assumed that $2\alpha A\Delta \mathrm{v}_{\mathrm{reshock}}$, the growth rate induced by the reshock is much larger than the preshock $\dot{h}_-$. Larger shock tubes are preferable not only to bypass any boundary-layer effect but also because at very late times h is affected by the cross-sectional area $H\times H$ of the tube if $h\sim H$ ($H\times H$ was about 26cm$\times$26cm in [24] and 8cm$\times$8cm in [25, 26]). Another consideration is the length L of the shock tube; clearly, in a longer L one will see the decay not only after shock but after reshock also.

Such considerations have led us to propose what may be called a “National Shock-Tube Facility” sketched in Fig. 4. Two drivers, labeled 1 and 2, facing each other and firing independently, at a chosen delay, shocks one toward the other passing through fluids of densities ρ_A and ρ_B. We will not enumerate the many configurations that can be obtained by varying fluids A and B, the order and strength of firing each driver, and the delay between them. The possibilities are many and quite obvious, and we hope an organization in the fluid dynamics community will undertake such a project.

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Figure Captions

Fig. 1. Evolution of the total mixing width $h^b + h^s$, calculated using Eqs. (4) and (5), for three air/SF_6 experiments conducted by Leinov *et al.* [26]. The experiments are listed by M_s, the Mach number of the shock generated in air, and L, the length of the test section containing SF_6. The sharp breaks in the curves indicate reshocks. We have taken $h_0 = 0$, $t^* = 0.1\text{ms}$ for the shock and 0.4 ms for the reshock in the long-test-section experiment. The calculations, like the experiments, stop when a third wave arrives at the interface. For more details see Refs. [26, 27].

Fig. 2. Schematic illustration of the turbulent energy spectrum at early ($t < t^*$) and late ($t > t^*$) times. The spectrum changes at about $t \sim t^*$ because the turbulence becomes isotropic forgetting the direction singled out by a shock or reshock.

Fig. 3. The effect of the initial surface finish h_0 on the total mixing width for the long-test-section experiment of Ref. [26]. The upper curve has a $5\times$ larger surface finish than the lower curve. That difference is reduced to a factor of 3 by reshock time, and further reduced to a factor of 1.8 at 3.2 ms.

Fig. 4. A proposed "National Shock Tube Facility" for the study of RM mix: Two drivers firing shocks in opposite directions at a set delay-time. The shocks pass through fluids A and B. The initial interface may have a single-scale perturbation (shown) or a multimode (more natural) perturbation.

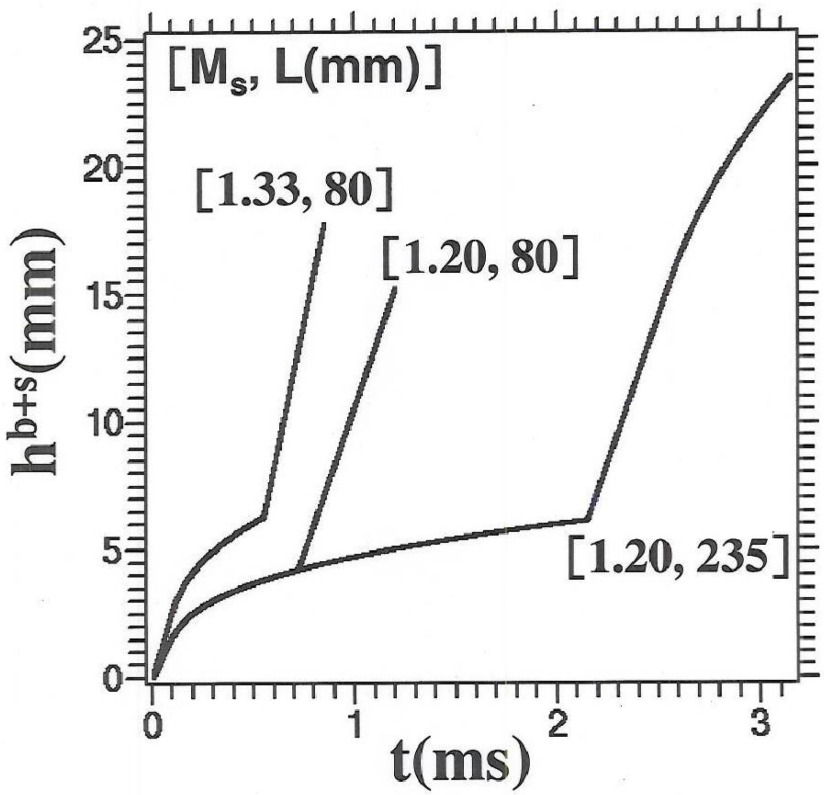
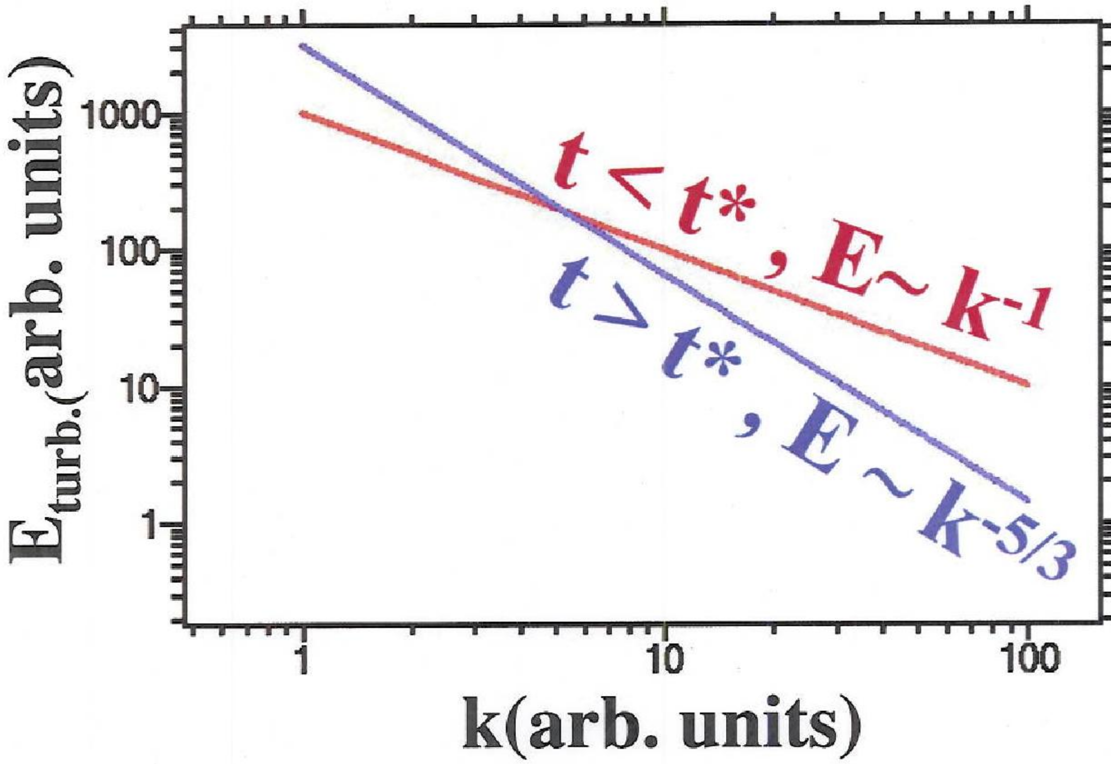


Fig. 1



$E_{turb.}$(arb. units)
1000
100
10
1
1
10
100
k(arb. units)
$t < t^*$, $E \sim k^{-1}$
$t > t^*$, $E \sim k^{-5/3}$

Fig. 2

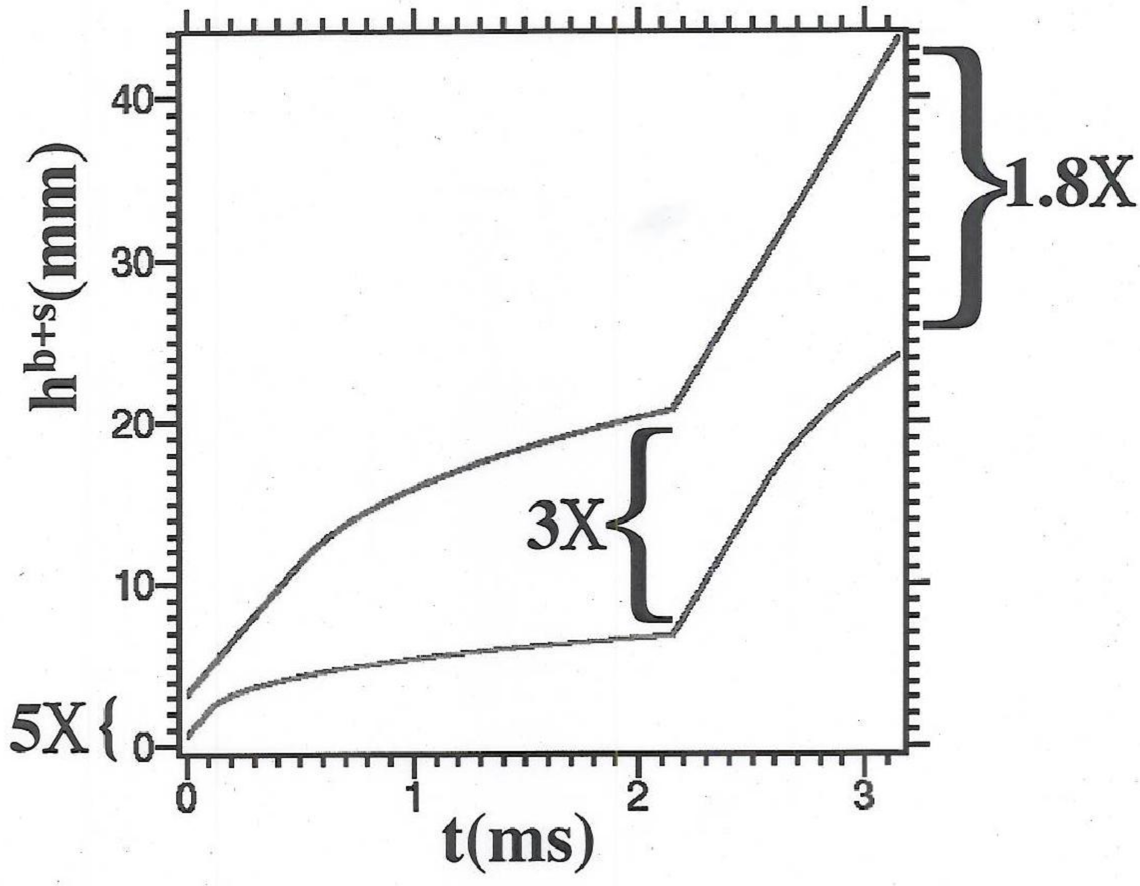


Fig. 3

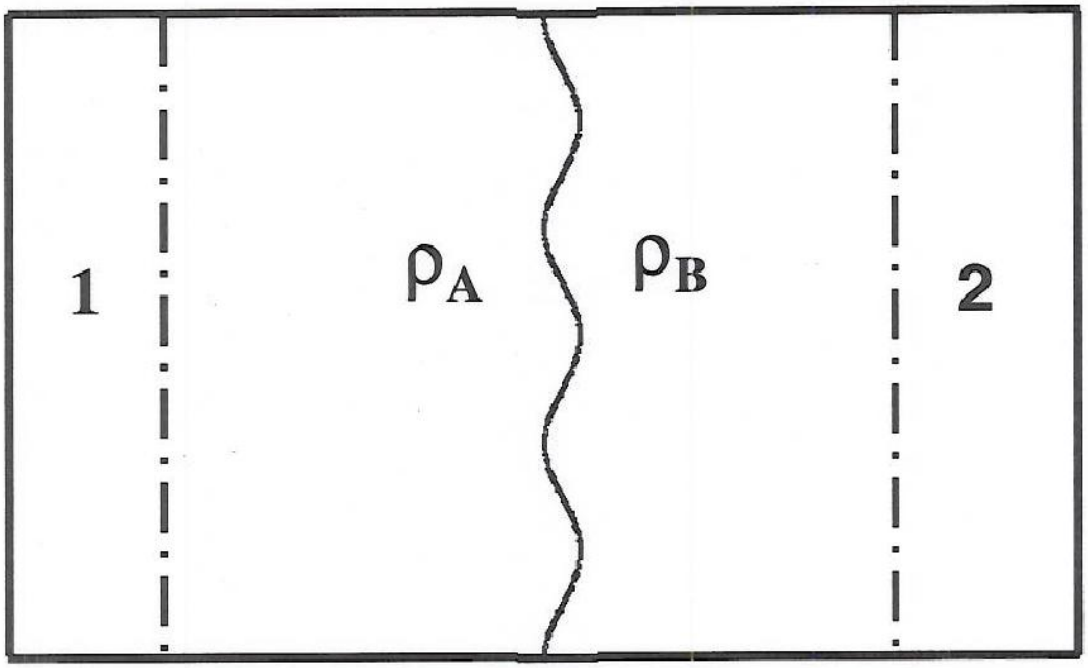


Fig. 4