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LLNL-JRNL-619752

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February 20, 2013

European Journal of Mechanics B/Fluids

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Comment on "The effect of viscosity, surface tension and non-linearity on Richtmyer-Meshkov instability" [European J. Mech. B Fluids 21 (2002) 511-526]

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The Richtmyer-Meshkov (RM) [1,2] instability refers to the evolution of perturbations $\eta\cos(kx)$ (η=amplitude, $k = 2\pi/\lambda$, λ=wavelength) at the interface between two fluids of densities ρ^l and ρ^h (l=light, h=heavy) after the passage of a shock inducing a velocity jump $\Delta\mathrm{v}$. The classical result in the absence of surface tension or viscosity is [1,2]

$$\eta(t)/\eta_0 = 1 + \Delta\mathrm{v}kAt \tag{1}$$

where $\eta_0 \equiv \eta(t=0)$ and $A \equiv (\rho^h - \rho^l)/(\rho^h + \rho^l)$. All formulas in this Comment apply to the linear regime, i.e. $\eta k << 1$.

When surface tension $T^{(s)}$ is present we found [3]

$$\eta(t)/\eta_0 = \cos(\omega t) + \frac{\Delta\mathrm{v}kA}{\omega}\sin(\omega t) \tag{2}$$

where $\omega \equiv \sqrt{\dfrac{k^3 T^{(s)}}{\rho^h + \rho^l}}$.

When viscosities μ^l and μ^h are present we found [4]

$$\eta(t)/\eta_0 = 1 + \frac{\Delta\mathrm{v}A}{2\nu k}(1 - e^{-2\nu k^2 t}) \tag{3}$$

where $\nu \equiv (\mu^h + \mu^l)/(\rho^h + \rho^l)$.

Subsequently, Carlès and Popinet (CP) presented [5]

$$\eta(t)/\eta_0 = 1 + \Delta\mathrm{v}kAt - \frac{\omega^2 t^2}{2}(1 + \Delta\mathrm{v}kAt/3) \tag{4}$$

when surface tension is present. As far as we know the derivation of this equation, and that of Eq. (5) below, has never been published citing that it requires "typically more than a hundred pages."[5].

We note here that Eq. (4) is merely a small-time expansion of Eq. (2), and is valid only for $t << 1/\omega$. It does not capture the oscillating nature of $\eta(t)$ and can be highly misleading if used for $t > 1/\omega$.

For viscosity, CP presented [5]

$$\eta(t)/\eta_0 = 1 + \Delta \mathrm{v} kAt(1 - Ct^{1/2}) \qquad (5)$$

where $C \equiv 16k\sqrt{\mu^l \mu^h \rho^l \rho^h} / [3\sqrt{\pi}(\sqrt{\mu^l \rho^l} + \sqrt{\mu^h \rho^h})(\rho^l + \rho^h)]$.

CP point out that C vanishes when only one of the fluids has viscosity, i.e. when $\mu^l = 0$ but $\mu^h \neq 0$ or *vice versa*, predicting no viscous damping in such a configuration, and they point out that this is not the case for the Rayleigh-Taylor instability. Here we point out that Eq. (5) should not be used in a configuration where one of the $\mu's$ is zero or much smaller than the other: There *are* viscous effects in the RM instability when only one of the fluids is viscous. In Eq. (3) viscous effects survive when one of the $\mu's$ is zero because ν in Eq. (3) depends on the *sum* $\mu^l + \mu^h$, while C appearing in Eq. (5) depends on the *product* $\mu^l \mu^h$.

Eq. (5), such as it is, has another serious limitation: Even for $\mu^l \neq 0$ *and* $\mu^h \neq 0$ it is limited to short times:

$$t < t^{CP} \qquad (6)$$

where $t^{CP} \equiv 4/9C^2$. Again, serious errors will be made if one uses Eq. (5) beyond t^{CP}. According to Eq. (5) $\dot{\eta} = 0$ at $t = t^{CP}$ and, for $t > t^{CP}$, it predicts that the amplitude will

decrease and eventually become negative (!) at late times. Nothing of this sort happens in actual calculations (experiments are sparse). In contrast, Eq. (3) predicts that $\dot{\eta} = 0$ only at $t = \infty$, or $t >> 1/2\nu k^2$.

We agree with CP that Eq. (3) can, in certain cases, underestimate the effects of viscosity. We want to clarify their claim that our viscous term can be "several orders of magnitude smaller," as shown in Fig.3(d) of CP [5]. $A = 0.6$ in that figure where the magnitude of the "viscous term" is plotted as a function of nondimensional $t' \equiv \Delta \mathrm{v} k t$. For example, for $\mathrm{Re} = 4000$ and at $t' = 0.1$ Eq. (3) predicts 2.4e-6, two orders of magnitude smaller than the 2.4e-4 predicted by Eq. (5). What this means is the following: $\eta / \eta_0 = 1 + At' - |viscous_term| = 1.0599976$ from Eq. (3) to be compared with 1.05976 from Eq. (5), a 0.02% difference. Similarly, at $t' = 1$, where our "viscous term" is some 32 times smaller than theirs (2.4e-4 vs. 7.6e-3), the difference in η / η_0 is less than 0.5%: 1.59976 vs. 1.5924.

Using the hydrocode CALE [6] we have run several direct numerical simulations to compare Eqs. (3) and (5) and to verify the above statements. In Fig. 1 we show the case $\eta_0 = 1\,\mathrm{mm}$, $\lambda = 13\,\mathrm{cm}$, $\rho^l = 0.166\,\mathrm{mg/cm^3}$, $\mu^l = \mu^h = 1\,\mathrm{poise}$, $\rho^h = 4\rho^l$, and $\Delta \mathrm{v} = 18.9\,\mathrm{cm/ms}$. This case was chosen to favor Eq. (5) [We do not show problems where one of the viscosities was very small and where Eq. (5) gave a completely wrong answer while Eq. (3) was in fair agreement with CALE]. From Fig. 1 we see that, compared to CALE, Eq. (3) does indeed underestimate viscous effects and hence overestimate $\eta(t)$, but by no more that 16%. There are compressional effects but they are kept at a minimum in the code. Eq. (5) comes closer to the simulation at early times, but then exhibits the unphysical behavior described above. For this case $t^{CP} \approx 2.1\,\mathrm{ms}$, and

indeed in Fig. 1 η^{CP} stops growing after t^{CP}, decreasing and becoming negative around 5ms. In contrast, as $t \to \infty$, the simulation approaches the asymptote

$$\eta_\infty / \eta_0 = 1 + \frac{\Delta \mathrm{v} A}{2vk} \tag{7}$$

given by Eq. (3). We will show elsewhere that this is indeed the correct asymptote.

ACKNOWLEDGMENT

This work was performed under the auspices of the U. S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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FIGURE CAPTION

Fig. 1. The evolution of the perturbation amplitude $\eta(t)$ for the problem described in the text, as calculated by Eq. (3), the hydrocode CALE, and Eq. (5).

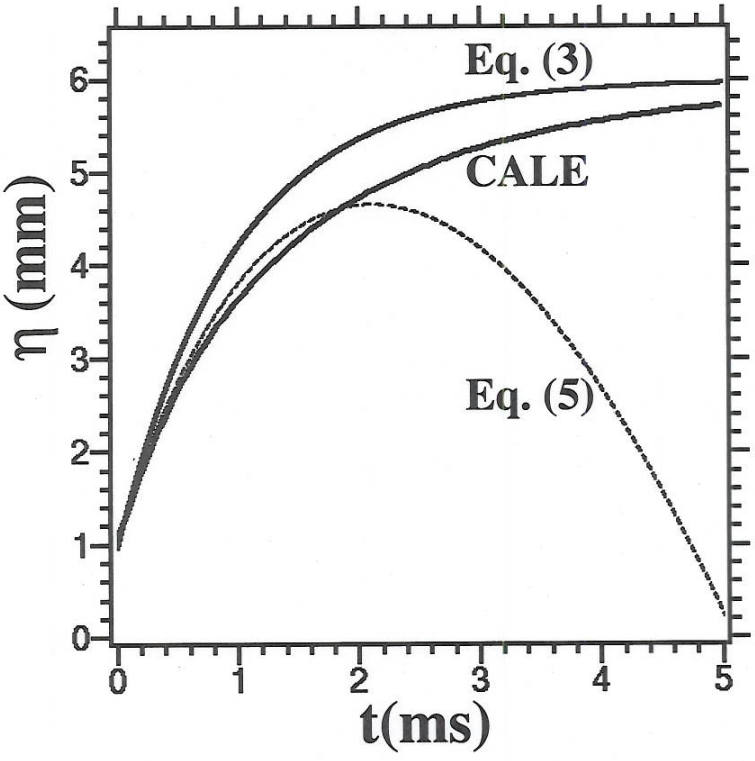

Fig.1