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Eliminating Parasitic Error in Dynamically Driven Flexure Systems

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INTRODUCTION

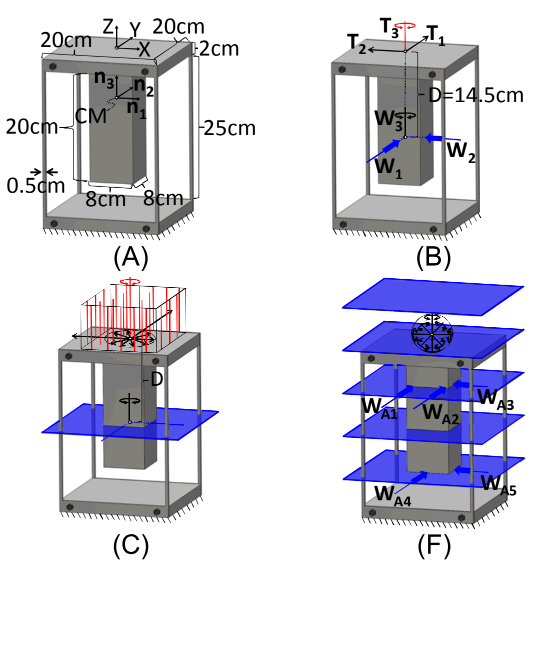
The aim of this paper is to guide designers in determining the optimal number and placement of actuators for driving general multi-axis flexure systems at any desired speed. Although the degrees of freedom (DOFs) of a flexure system are largely determined by the location and orientation of its flexure elements, the system's stage will tend to displace in unwanted directions (i.e., parasitic errors) while attempting to traverse its intended DOFs if it is not actuated correctly. The problem of correctly placing actuators is difficult because the optimal location changes depending on the speed with which the stage is driven. In this paper we introduce the mathematics necessary to generate geometric shapes, called dynamic actuation spaces, which enable designers to rapidly visualize all the ways actuators could be placed for driving a general flexure system with minimal parasitic error at any speed without having to move the actuators placed. The theory provided here impacts the design of precision motion stages in that it significantly simplifies their control and increases their bandwidth. Example systems that benefit from this research include flexure-based nano-positioners, high-speed assembly stages, and multi-axis micro-mirrors.

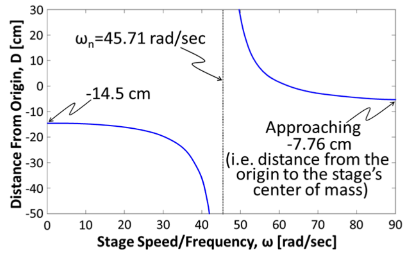
CONTRIBUTIONS

In this paper we *(i)* introduce the concept of dynamic actuation space, *(ii)* provide the complete library of these spaces, *(iii)* provide guidelines for placing the correct number of actuators within these spaces, and *(iv)* introduce the mathematics necessary to generate these spaces as well as calculate the selected actuators' output force magnitudes for driving a system with a desired DOF at a desired speed.

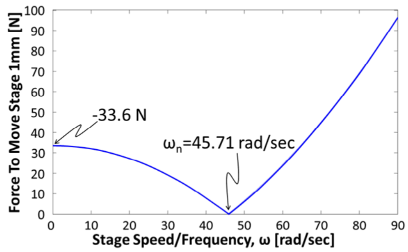
FUNDAMENTAL PRINCIPLES

Consider the steel flexure system in Fig. 1A (E=210GPa, G=79GPa, ρ=7700 kg/m^3). Its four wire flexures guide its T-shaped stage to move with three DOFs—two translations and one rotation. These DOFs may be modeled using twist vectors [1], $\mathbf{T_1}$, $\mathbf{T_2}$, and $\mathbf{T_3}$ (Fig. 1B). If we wish to quasi-statically actuate these DOFs, we could do so optimally (i.e., with minimum parasitic error) by pushing on the stage with two linear forces located in the middle of the flexures





(D)



(E)

FIGURE 1. Flexure system (A), optimal actuator placement for static actuation (B), freedom and static actuation spaces (C), distance of static actuation space from origin verses stage speed (D), actuator force verses stage speed (E), and dynamic actuation space with five actuators (F)

as shown in Fig. 1B and torqueing the stage with one pure moment respectively. These actuation actions may be modeled using wrench vectors [1], $\mathbf{W_1}$, $\mathbf{W_2}$, and $\mathbf{W_3}$. Their locations correspond with the system's center of stiffness and are shown a distance, *D*, from the origin. As long as the system is not driven with any appreciable speed, these actions will minimize the axial loads in the flexures and will thus produce the corresponding DOFs with minimal parasitic error for small displacements.

Although the stage is capable of moving with three DOFs, it may also move with every combination of these DOFs. These motions are visually depicted by the system's freedom space [2]. Freedom space is a geometric shape that represents all the ways a system may move. The freedom space of this example consists of an infinitely large box of parallel rotation lines (shown red in Fig. 1C) and a disk of translation arrows that are perpendicular to these lines. If $\mathbf{T_1}$, $\mathbf{T_2}$, and $\mathbf{T_3}$ are linearly combined, this freedom space is generated. For the case of quasi-static actuation, the optimal locations and orientations of the actuators that successfully drive all the motions/twists within this freedom space also lie within a geometric shape called a static actuation space [2]. This space results from the linear combination of $\mathbf{W_1}$, $\mathbf{W_2}$, and $\mathbf{W_3}$ and consists of a plane of linear forces (shown blue in Fig. 1C) and an orthogonal moment.

The optimal location of this actuation space will remain at *D*=14.5cm only if the DOFs in the freedom space are actuated with quasi-static speeds. If, however, we wish to actuate the motions within the freedom space with an increasing sinusoidal frequency of ω, the optimal location of the actuation space will displace downwards until it is infinitely far away when ω reaches the system's first natural frequency (Fig. 1D). As ω increases to infinity above this natural frequency, the planar actuation space descends from positive infinity until it approaches the stage's center of mass. The effect of increasing ω on the absolute value of the actuation force necessary to displace the stage a sustained-amplitude of 1mm is shown in Fig. 1E.

Given these results, one may conclude that the only way to dynamically actuate a stage with minimal parasitic error is to move the actuators according to the desired actuation speed. This is fortunately not the case. If the wrench vectors within every planar actuation space for all values of ω are linearly combined, a new space emerges. This space is called dynamic actuation space. The dynamic actuation space of this example consists of an infinite number of stacked parallel planes that contain linear force wrenches (shown blue in Fig. 1F) and coupled moment/force wrenches as well as a sphere of pure moment wrenches. Unlike the planar static actuation space from Fig. 1C that contains three independent linear forces, the dynamic actuation space of Fig. 1F contains five such forces. Thus, while quasi-static actuation requires only three actuators from the plane of Fig. 1C to drive the system's three DOFs with minimal parasitic error, dynamic actuation requires at least five actuators from the parallel planes of Fig. 1F. As long as these five actuators' lines of action are all independent, like $\mathbf{W_1}$ through $\mathbf{W_5}$ shown in Fig. 1F, their output forces may be combined to actuate any motion within the system's freedom space with minimal parasitic error at any speed without the actuators needing to change locations at any time. A later section provides the mathematics necessary to *(i)* generate a general flexure system's dynamic actuation space, *(ii)* determine the fewest number of actuators that need to be selected from this space, and *(iii)* calculate the output force magnitudes of the selected actuators for achieving any desired DOF with minimal parasitic error at any speed, ω.

CONCLUSIONS

This paper introduces the concept of dynamic actuation space as a geometric shape that guides designers in placing static actuators for driving multi-axis flexure systems at various speeds with minimal parasitic error. A comprehensive library of these spaces as well as the mathematics necessary to generate them, have been provided. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. LLNL-CONF-XXXXX.

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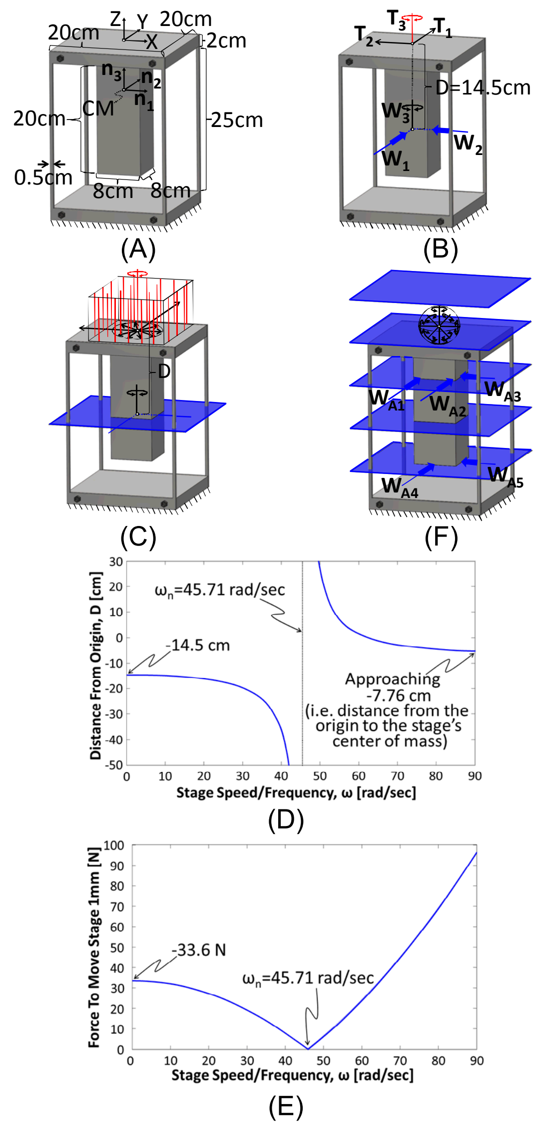

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There are a finite number static and dynamic actuation spaces. These spaces are provided in Fig. 2. They are identical to the constraint spaces described in Hopkins [3]. They consist of linear forces depicted as blue lines, pure moments depicted as black lines with circular arrows about their axes, and coupled moment/force wrenches depicted as orange lines. It is not important to the purpose of this paper that the reader understand the details of these spaces. What is important to recognize is that each space is numbered and belongs to one of seven columns. Each column is labeled with the number of independent wrenches (i.e., minimum number of necessary actuators) contained within its actuation spaces. Note that the actuation space of Fig. 1C is the first space in the "3 Actuators" column of Fig. 2 and that the actuation space of Fig. 1F is the third space in the "5 Actuators" column.

Thus, when a system's dynamic actuation space has been determined, Fig. 2 may be used to identify the fewest number of actuators necessary to actuate the systems DOFs with minimal parasitic error. The only actuation spaces that contain enough linear force actuators to achieve all of the system's DOFs are those shown in the pyramid outlined in black

(Fig. 2). If the system's actuation space lies outside of this pyramid, it will require at least one pure moment or coupled moment/force actuator to drive all of the system's DOFs.

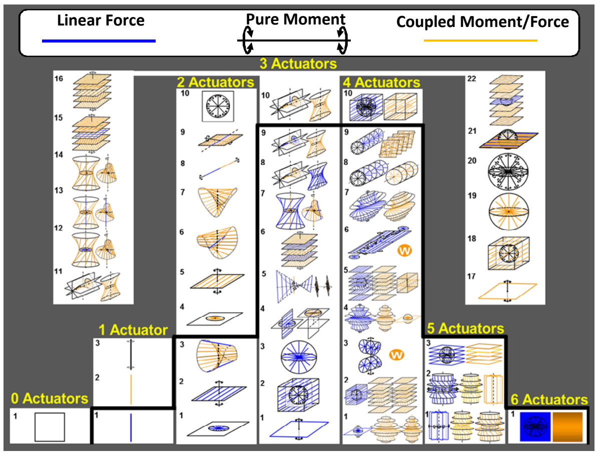

FIGURE 2. Complete library of actuation spaces

MATHEMATICAL TOOLS

This section provides the mathematical tools necessary to *(i)* generate a general parallel flexure system's dynamic actuation space, *(ii)* select appropriate actuators within this space, and *(iii)* calculate the output force magnitudes of the selected actuators for achieving any desired DOF with minimal parasitic error at any speed.

If we wish a general system's stage to move with a twist, $\mathbf{T_i}$, at a speed of ω, the wrench, $\mathbf{W_i}$, required to drive the stage may be calculated using

$$\mathbf{W}_\mathrm{i} = \left(-\omega^2[M_{TW}]+[K_{TW}]\right)\mathbf{T}_\mathrm{i}, \tag{1}$$

where $[M_{TW}]$ is the system's twist-wrench mass matrix [1] defined by

$$[M_{TW}]=[N][\Delta][in][N]^{-1}, \tag{2}$$

where $[N]$ is defined by

$$[N]=\begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}\times\mathbf{n}_1 & \mathbf{L}\times\mathbf{n}_2 & \mathbf{L}\times\mathbf{n}_3 & \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 \end{bmatrix}. \tag{3}$$

The vector $\mathbf{L}$ is a 3x1 location vector that points from the system's origin to the stage's center of mass (labeled CM in Fig. 1A), $\mathbf{n_1}$, $\mathbf{n_2}$, and $\mathbf{n_3}$ are 3x1 orthogonal unit vectors, $\mathbf{0}$ is a 3x1 zero vector, and $[\Delta]$ from Eq. (2) is

$$[\Delta]=\begin{bmatrix} [0] & [I] \\ [I] & [0] \end{bmatrix}. \tag{4}$$

Matrix $[0]$ is a 3x3 zero matrix, $[I]$ is a 3x3 identity matrix, and $[in]$ from Eq. (2) is a 6x6 inertia matrix defined by

$$[in]=diag\begin{bmatrix} I_1 & I_2 & I_3 & m & m & m \end{bmatrix}, \tag{5}$$

where I_1, I_2, and I_3 are the stage's mass moments of inertia (centered about the stage's center of mass) that correspond with the directions of $\mathbf{n_1}$, $\mathbf{n_2}$, and $\mathbf{n_3}$ respectively, and m is the stage's mass. For the plots of the example of Fig. 1, 1/3 of the four wire flexures' mass protruding from the system's stage is considered in the values of Eq. (5). This is done to take the inertia of the moving section of the flexures into account as well as the rigid stage. The twist-wrench stiffness matrix $[K_{TW}]$ in Eq. (1) may be calculated using the theory provided in Hopkins [2]. This stiffness matrix is constructed using information pertaining to the system's flexure element topology as well as geometry and material properties.

The static actuation space of a general flexure system is generated by linearly combining the wrench vectors, $\mathbf{W_i}$, that result from substituting all of the system's DOF twists, $\mathbf{T_i}$, into Eq. (1) for ω equal to zero. The dynamic actuation space of a general flexure system is generated by linearly combining the wrench vectors, $\mathbf{W_i}$, that result from substituting all of the system's DOF twists, $\mathbf{T_i}$, into Eq. (1) for every value of ω. In practice, however, only a few values of ω are necessary to substitute into the equation before a sufficient number of independent wrench vectors are identified that are capable of generating the entire dynamic actuation space.

Once a system's dynamic actuation space has been identified, designers may use the library of Fig. 2 to identify the minimum number of necessary actuators that must be selected from within the space depending on the column under which the space is categorized. This number is never less than the number of system DOFs, nor is it ever more than six. If it equals the number of system DOFs, the system is well designed and requires the fewest number of actuators to eliminate parasitic error for both static and dynamic scenarios. Such systems are generally symmetric and/or possess a center of stiffness that is coincident with their stage's center of mass. Their static and dynamic actuation spaces are identical. An example of such a flexure system that achieves the same three DOFs as the system from Fig. 1 with minimal parasitic error but with three actuators instead of five is shown in Fig. 3. The system's three actuators are sufficient to drive the stage's DOFs at any speed because the system's static and dynamic actuation spaces are synonymous (i.e., a plane of linear forces and a perpendicular moment).

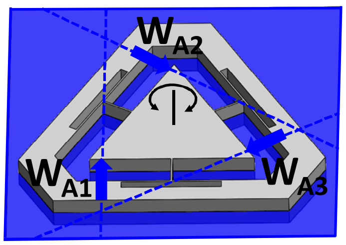

FIGURE 3. Improved flexure system

Once designers know how many actuators to select from the system's dynamic actuation space, they must select them such that their lines of action are independent and correspond with their actuator's type (e.g., blue lines correspond with linear actuators, black lines correspond with pure moment actuators, and orange lines correspond with coupled moment/force actuators). To ensure that the actuators selected are independent, designers may use the concept of sub-constraint spaces described in Hopkins [2-3], or they may apply Gaussian elimination to their actuators' wrench vectors. Actuators should be arranged with as much symmetry as possible and designers should ensure that their lines of action are orthogonal to the stage where they act. Stages should be designed with as few protrusions as possible to minimize inertia and thus increase the system's natural frequencies.

After the actuators have been placed, their 6x1 wrench vectors, $\mathbf{W_{Aj}}$, should be arranged in a matrix $[W_A]$ according to

$$[W_A] = \begin{bmatrix} \mathbf{W_{A1}} & \mathbf{W_{A2}} & \cdots & \mathbf{W_{Aj}} \end{bmatrix}, \qquad (6)$$

where the force magnitude of each vector is set to unity and j is the number of actuators placed. If any of these vectors are pure moment vectors, the magnitude of their moment is also set to unity. Note that j is five for the system of Fig. 1F. The actuators' output force magnitudes, F_{Aj}, that are necessary to drive a stage with a desired twist, $\mathbf{T_i}$, at a speed of ω, can be calculated according to

$$\begin{bmatrix} F_{A1} & F_{A2} & \cdots & F_{Aj} \end{bmatrix}^T = \left([W_A]^T \cdot [W_A]\right)^{-1} \cdot [W_A]^T \cdot \mathbf{W_i}, \qquad (7)$$

where $\mathbf{W_i}$ is calculated using Eq. (1).

The six mode shapes of a general parallel flexure system are the Eigen vectors of its [*Mode*] matrix defined by

$$[Mode] = [M_{TW}]^{-1}[\mathrm{K}_{\mathrm{TW}}]. \qquad (8)$$

The natural frequency, ω_n, associated with each mode shape is the square root of the mode shape's Eigen value. The first i mode shapes of a parallel flexure system correspond with the i independent DOF twists that lie within the system's freedom space.

Finally, note that the theory provided in this paper is specific to parallel flexure systems that are not subject to damping or internal losses. Extending this theory such that it applies to more general flexure systems is ongoing research.

CONCLUSIONS

This paper introduces the concept of dynamic actuation space as a geometric shape that guides designers in placing static actuators for driving multi-axis parallel flexure systems at various speeds with minimal parasitic error. A comprehensive library of these spaces as well as the mathematics necessary to generate them, have been provided. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. LLNL-CONF-XXXXX.

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[3] Hopkins, J.B., Culpepper, M.L., 2011, "Synthesis of Precision Serial Flexure Systems Using Freedom and Constraint Topologies (FACT)," Prec. Eng., 35(4): pp. 638-649