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A Comprehensive Evaluation of Robust Unit Commitment

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Executive Summary

For day-ahead scheduling of electric grid operations, system operators forecast the system condition for the following day, and determine which power generators will be used through a process called unit commitment (UC). New England's policymakers are seeking to increase the amount of renewable resources available to meet consumers' energy needs. These resources offer low-emission energy, but their variability poses a unique challenge for the reliable scheduling and operation of the region's power system. The existing deterministic UC model is not equipped with any effective uncertainty management strategy. As a result, the cost of the real-time system operation can be increased, and the reliability of the system can be jeopardized when the amount of stochastic elements (variability) associated with renewable resources is significant. For example, if the amount of wind energy is heavily over-forecasted, the uncommitted units with long lead time many not be able to start up quickly enough to make up for the difference between forecast and actual wind energy production. ISO New England (NE) enlisted Lawrence Livermore National Laboratory (LLNL)'s help in determining whether a new methodology, known as robust UC (Bertsimas et.al. (2013)), would improve system reliability while keeping the operation cost relatively low in the presence of renewable variability. The robust approach can produce commitment schedules that accommodate a forecasted generation range of variable resources as shown in Figure 1, rather than only the forecasted level as in the deterministic approach.

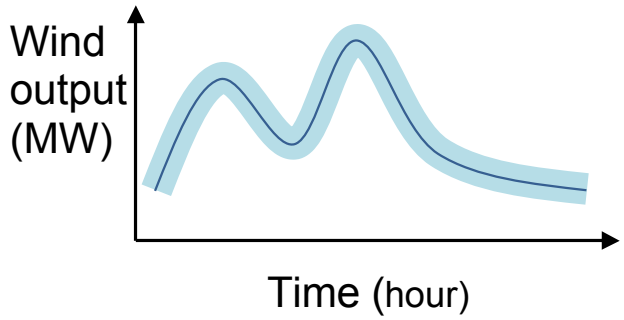

Figure 1: The black line shows expected wind generation level over a 24-hour period. The blue line represents the range of uncertain wind generation the UC model must accommodate between projected and actual wind output.

Over the course of one and half year, ISO NE used LLNL's high performance computing capabilities to run a large number of simulation to analyze and optimize robust UC solutions that would mitigate the impact of variability and uncertainty on the power system. During the first phase of the collaboration, because of the number of inputs, including wind penetration levels, weather conditions, and conservatism levels (or the forecast range), up to 2,500 robust UC and 1,250 deterministic UC configurations were analyzed – with each configuration taking an average of 30 minutes to compute. By parallelizing the software used to solve the problems, the team reduced

the computation time from 1800 hours to 90 minutes. During the second phase of the collaboration, the team ran simulations and identified the impact of various factors – such as the conservatism levels on the economic and operational benefits of robust UC over deterministic UC. The simulations used Monte Carlo sampling based on historical load and wind generation data combined with a Livermore-developed statistical model. One thousand dispatch problems, each taking 15 seconds, must be answered for every UC configuration. By efficiently parallelizing the dispatch simulation in batches, the total simulation time was reduced from 1.8 years to 4.8 hours.

Based on the simulation results, the optimal conservatism level that led to the best trade-off between the system cost and reliability under the robust approach was identified. The robust approach demonstrated the economic and operational advantages over the deterministic approach in terms of lowering the total costs and increasing the system reliability. From the generators' perspective, compared to the deterministic approach, the robust approach reduced the profit largely due to the depressed energy prices under the latter. The robust approach also exhibited stable performance when large forecast error occurred. The advantages of the robust approach are magnified when there are more renewable resources integrated into the system.

Processing thousands of simulations simultaneously increased the productivity, provided ISO NE with a more comprehensive evaluation of the robust commitment schedule, and allowed ISO NE to assess the effectiveness of the robust unit commitment approach under a broad range of energy use, wind and power generation scenarios.

Contents

Executive Summary iii
Abstract 1
Introduction 1
Methods 2
Simulation Results 6
Benefits to Company and LLNL 22
Conclusions 22
Acknowledgements 23
References 23
Appendix A Mathematical Formulations 25
Appendix B Features Used in Clustering Forecast Data 30
Appendix C 31
Model for the Observed Wind and Load and Parameter Estimation 31

Abstract

This report summarizes the procedure to compare the robust UC approach with the deterministic counterpart. The procedure involves solving the robust and deterministic UC problems under various scenario configurations, and then performing system dispatch simulations under each UC solution to generate economic and operational performance measures. The report describes the parallelization process for solving UC problems as well as for running system dispatch simulations. The report also summarizes the statistical model used for generating Monte Carlo simulation samples and forming forecast ranges used in the robust UC approach.

Some key findings are discussed as well. In particular, the economic and operational performance of the robust and deterministic approaches is compared under different 1) conservatism levels or the sizes of forecast range, 2) the wind penetration levels, and 3) the mixture of load and wind behaviors.

Introduction

To accommodate the operating characteristics of a large number of variable resources, the ISO will require an effective UC methodology that can provide the system operator with a wider operating range. ISO NE has been investigating robust optimization techniques, which take into account the uncertainty and variability of renewable resources as a forecast range, to develop a reliable UC process. A preliminary experiment conducted by the ISO's Business Architecture and Technology team in 2010 showed sizable savings on dispatch costs using robust UC techniques compared to the existing deterministic approach.

A concern with the robust approach is that it might be too conservative. When the forecast range is large, the cost associated with excessive commitment is high. On the other hand, a smaller range will lead to a lower cost due to less committed resources, but may possibly harm its operational performance. Therefore, some trade-offs need to be made when we choose the appropriate level of conservatism or the size of the forecast range for the robust UC.

Besides using the robust approach, another alternative to handle the forecast uncertainty in the UC process is to solve a deterministic UC with incremental reserve. A critical principle in the operation of a bulk power system is that a balance between generation and demand must generally be maintained. Power system operators carry operating reserves as a matter of standard practice to accommodate potential loss of generation. The introduction of significant variable resources causes the variability and uncertainty on the generation side of the balance to increase, requiring power system operators to plan for carrying incremental amounts of operating reserves, in this case necessary to accommodate potential changes in renewable generation. The term incremental reserve is used to denote the operating reserve that the system carries to respond to variability and uncertainty in renewable generation and demand. The deterministic approach with incremental reserve has computational advantage over the

robust approach, but the former cannot capture the uncertainty explicitly due to its modeling limitation. The mathematical models for the deterministic and robust approaches are presented in Appendix A.

Thus, the objective of this study is to answer the following two main questions:

- What is the optimal conservatism level of the robust UC that can balance cost and reliability?
- Would the robust UC be a more reliable and economic approach to scheduling resources to generate electricity for the grid than the deterministic UC approach?

To this end, a large number of simulations are required to evaluate the economic and operational performance of the robust and deterministic UC under various configurations. Two key components were necessary to perform the analysis. First, a statistical model that characterizes the load, wind and interchange uncertainty was developed. This model serves as a random events generator to simulate wind generation, load and interchange realizations in the system dispatch simulation. Moreover, it also provided stochastic information to construct forecast ranges for the robust UC.

Second, the large number of commitment and simulation problems demands parallelization. Solving thousands of UC problems and millions of dispatch simulation problem would take more than 3.5 years on one desktop computer. The ability to run a large number of simulations simultaneously at LLNL's supercomputers significantly improves productivity, enabling us to conduct comprehensive evaluations in a reasonable time span.

Thus, three essential goals of this report are to describe the analysis performed by ISO NE and LLNL to evaluate the robust UC, describe the statistical model developed to characterize the uncertainty, and present the key findings of the analysis.

Methods

We considered the following three sources of uncertainty in the New England power system: the consumption levels of eight load zones, the wind outputs of 13 (25) wind sites in the low (high) wind penetration case, and the import and export at 5 interconnections with neighboring areas. The high-level analysis procedure is illustrated in Figure 2.

More specifically, to determine the optimal conservatism level of the robust UC involved the following steps:

(A. 1) generating day-ahead forecast ranges for uncertain load, wind generation, and export/import interchange under various configurations, namely low and high wind penetration levels, and twelve load, wind and interchange behavior clusters, which are discussed in Appendix B.

(A. 2) varying the forecast ranges used in the robust unit commitment calculation to reflect different conservatism levels. This allowed us to determine what the optimal conservatism level is to achieve the balance between robustness and performance.

(A. 3) testing the system dispatch over the day for 1000 random realizations of load, wind generation and interchange under each robust UC solution, and

(A. 4) obtaining economic and operational performance measures.

The formulation for the dispatch problem in step (A.3) can be found in Appendix A.

We also ran the deterministic UC with incremental reserve and compared it with the robust counterpart. The procedure for the deterministic approach was similar to the one described above. In particular, it involved:

(B. 1) generating day-ahead forecast ranges for uncertainty under various configurations. This step is the same as the step (A.1).

(B. 2) adjusting the incremental reserve to calculate the corresponding deterministic unit commitment schedules to reflect different conservatism levels,

(B. 3) testing the system dispatch over the day for 1000 random realizations of load, wind generation and interchange under each deterministic UC solution, and

(B. 4) comparing the economic and operational performance measures between the robust and deterministic approaches.

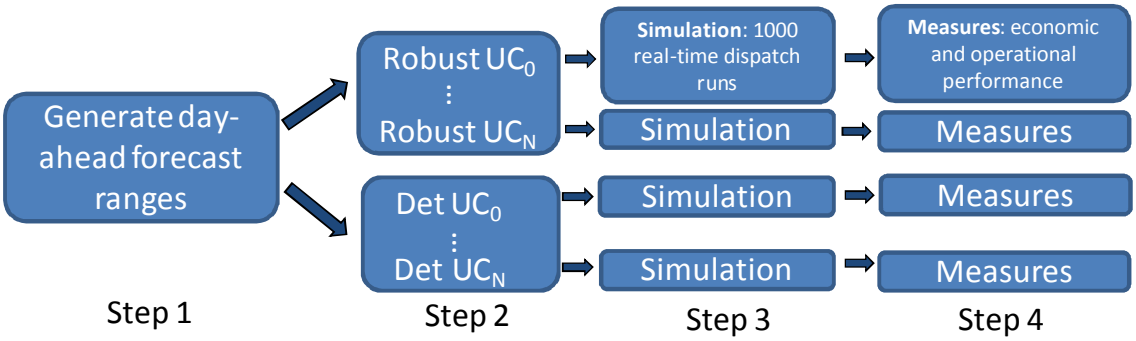

Figure 2: The procedure to calculate UC solutions and perform the simulation.

A Livermore-developed statistical model of load and wind behavior was used to generate the forecast range as well as to sample the possible realizations of load, wind and interchange in the simulation. We assumed that forecasts of either wind or load could be grouped by similar patterns and magnitudes, and that forecasts within any one group (cluster) shared the same statistical characteristics in terms of the *forecast error*. Thus, the knowledge of the day-ahead forecast and, more specifically, the cluster it belongs to, could inform the appropriate model of the forecast uncertainty. For example, load forecast errors for days with a high total load may come from a distribution with one set of parameters (such as the means and covariances), while those for days with a low total load may be described by possibly the same type of distribution, but a different set of parameters. The features used to categorize wind and load forecasts into clusters are discussed in Appendix B.

We used a relative error model to describe the observed wind and load 24-hour trajectory (by each wind site or load zone) separately, as a function of the forecast trajectory (see Appendix C for more details of the model). To identify the features used to cluster the wind and load forecasts and to estimate the parameters in the

aforementioned model for each cluster, we used *forecast* and *observed* hourly wind production and load data for the New England power system for all days in the period from January 1st 2004 through December 31st 2006 (a total of 1096 24-hour trajectories of observed and forecast wind and load).

Once the clusters were identified (see Appendix B) and model parameters of each cluster were specified (see Appendix C), given a specific day-ahead wind forecast, one could then determine which wind cluster the forecast belongs to and obtain both the forecast range and the set of possible actual realizations needed to run the robust UC algorithm and economic dispatch simulation. The underlying assumption is that the wind generation realizations in the simulation follow the forecasted behavior pattern used in the commitment process. The same can be done with a given day-ahead forecast of the load.

We also had historical data for the observed 24-hour interchange trajectories. This dataset contained all days from January 1st, 2005 to December 31st, 2006, so we had a total of 730 such trajectories. Because interchange forecast errors have much more complex statistical characteristics than wind or load, we did not model these due to time constraints of the project. Instead, to obtain a set of actual realizations for a given day, we selected a set of trajectories randomly with replacement from the entire set of 730.

For each cluster, the wind forecast range for each location at each hour was chosen to equal a particular confidence level, i.e., the interval that covers the middle *p* percent of the distribution of possible wind output values. The forecast ranges for load and interchanges were determined in the same fashion. In the analysis, the value of *p* was selected to be 99 because it was assumed that a 99 confidence interval is the largest amount of uncertainty operators are willing to accommodate under the robust approach. Figure 3 illustrates the forecast ranges of the aggregated load, wind, interchange and net load, which is load less the sum of interchange and wind, over the 24-hour period in a representative cluster using the confidence level *p* set to 99. A concern with such a large confidence interval is that it might be too conservative. To address this, the operators' conservatism level is taken into account via the size of the forecast range around the expected value of uncertainty. We varied the size of the forecast range to explore the tradeoff between robustness and performance of the robust approach in the analysis. The details on varying the size of the forecast range or the conservatism level are discussed below. Under the deterministic approach, we used the difference between the upper bound of the net load forecast range and the net load mean as the incremental reserve. This corresponded to the most conservative deterministic UC solution. Similarly to the robust approach, we gradually reduced the amount of incremental reserve to control the conservatism of the deterministic UC solutions.

To obtain a comprehensive evaluation of the robust approach, we considered 2448 robust UC configurations of the combinations of the following elements:

(1) two wind penetration levels: low level (700 MW) and high level (2GW),

(2) twelve wind and load behavior patterns: 12 clusters produced by the clustering procedure (see Appendix B),

(3) fifty-one levels of conservatism: 0 to 1 with increment of 0.02, where 0 corresponds to a forecast point while 1 corresponds to the largest forecast range, i.e. the most conservative case, and

(4) two ways to vary the conservatism level: the uniform method and the non-uniform method. The uniform method reduces the conservatism level by proportionally reducing the variation range of each uncertainty source. The non-uniform forecast range, which is also known as budget uncertainty set (Bertsimas and Sim 2004), controls the conservatism level by limiting the number of uncertainty sources that can reach their maximum variation. Figure 4 illustrates these two methods assuming that there are two uncertainty sources. The mathematical description of the uniform and non-uniform forecast ranges is presented in Appendix A.

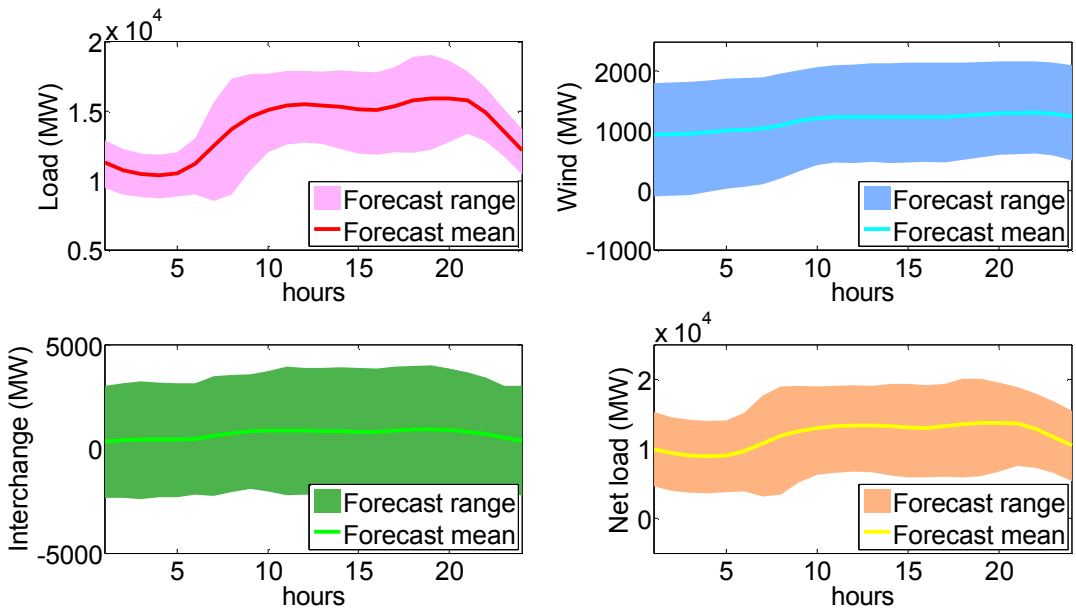

Figure 3: The forecast means (solid curves) and ranges of aggregated load, wind, interchange and net load for a 99 confidence interval.

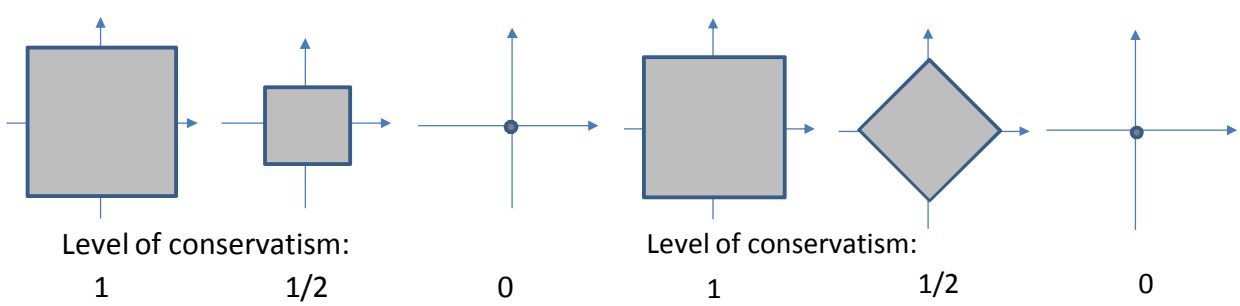

Figure 4(a): the uniform method

Figure 4(b): the non-uniform method

For the deterministic approach, we considered 1224 deterministic UC configurations of the combinations of elements (1) - (4) since there was only one way to vary the conservatism level of the deterministic solutions. In the deterministic approach,

the level of conservatism was adjusted by changing the incremental reserve, which implicitly handled uncertainty by committing additional capacity besides meeting forecast value in an ad-hoc fashion.

The configurations described above led to a total of 2448 robust UC and 1224 deterministic UC solutions that had to be computed and analyzed. The first half of the collaboration focused on the computation and analysis of the 2448 robust UC and 1224 deterministic UC solutions that resulted from the configurations described above. These solutions took an average of 30 minutes to compute. Solving this set of UC problems on one desktop computer would take about 1,836 hours. By parallelizing the software used to solve the problems, we reduced the computation time to just 90 minutes. The experiments were run on the Cab cluster at LLNL. It is a 1,200 node, 16 cores per node Linux cluster. Each node has 32G RAM. All 3,672 UC problems were run in parallel, using one core per one UC problem.

During the second half of the collaboration, we ran simulations. The simulations used Monte Carlo sampling based on the statistical model. One thousand dispatch problems, each one taking 15 seconds, must be solved for every UC configuration, for a total dispatch-solving time of 15,300 hours, or approximately 1.8 years, on a desktop.

Two joint efforts were taken to efficiently parallelize the dispatch simulation. First, ISO NE and LLNL teamed up with GAMS, a software company whose product is used to code UC and dispatch models. GAMS used the GAMS scenario solver, known as Gather-Update-Solve-Scatter (GUSS) facility, to solve 1,000 dispatch problems sequentially in an efficient fashion. GUSS reduced the computation time of solving 1,000 dispatch problems from 15,000 seconds to 450 seconds. Next, LLNL parallelized the 7.5 million dispatch problems in batches on the Cab cluster. In particular, one single core was used to solve every 1,000 dispatch problems with GUSS facility. Because running 1,000 dispatch problems took large memory (about 10GB), only three cores were used on each node. The total 3.7 million dispatch problems were run in batches using around 150 nodes, and 1800 cores, many of which were idle. At the end, the calculation time for simulations was reduced to 4.8 hours.

Many files were written out by GAMS (the largest almost 2GB for combining a UC problem and 1,000 dispatch problems). To handle this, LLNL sent all outputs to a parallel file system connected to the Cab cluster. More specifically, the outputs were sent to lscratchd, which has a 2PB capacity, and bandwidth of 20 GB/s from the Cab cluster. In addition, LLNL used MOAB as the job scheduler to schedule the jobs on various nodes and cores. The Moab Workload Manger is the batch scheduling system at LLNL. Under the covers, MOAB uses SLURM for reserving nodes, allocating jobs to nodes or cores. It allows for fine-grained control, including setting the number of cores per nodes, which was needed for both UC and dispatch runs.

Simulation Results

We compare the performance of the two robust approaches, namely the robust approach with non-uniformly reduced forecast range (R-NU) and the robust approach with uniformly reduced forecast range (R-U), to the deterministic approach with

incremental reserve (Det) in the following six aspects: 1) commitment, dispatch and total costs, 2) energy and reserve prices, 3) profit of generators, 4) load shedding events, 5) wind spillage and 6) the sensitivity to the accuracy of forecast. These factors are selected to assess the impact of the UC decisions on the economic efficiency and operational reliability. The production cost is an indication of the economic efficiency. The prices and the profit of generators reveal the economic impact on generators. The frequency of load shedding events measures the reliability of the real-time dispatch operation under different UC decisions. Because the ISO wants to use the low cost wind as much as possible, the wind spillage implies how efficiently wind generation is utilized. The definition of the wind spillage is given at the Appendix A. The sensitivity to the forecast accuracy shows the robustness of UC decisions against forecast errors.

The main findings are the following:

(1) An optimal conservatism level can be identified for each of the R-NU, R-U and Det approaches.

(2) At the optimal conservatism level, the robust UC solutions have a lower total cost than the deterministic solutions, indicating better economic efficiency of the robust approaches. The robust UC solutions also result in lower cost volatility than the deterministic solutions, implying a more stable performance of the former.

(3) The robust approaches depress energy prices, as well as lower generators' profit, relative to the deterministic approach. However, they lower the volatility of the profit, thus reducing risk, relative to the deterministic approach.

(4) The robust approaches are more resilient to forecast errors than the deterministic approach.

(5) The advantages of the robust approaches are amplified when the wind penetration level is high.

(6) It is more costly to operate the system under the high wind penetration than the low wind penetration for all three approaches.

(7) The performances of the R-NU and R-U approaches are similar, but the latter shows slight advantages over the former in terms of improving system reliability, saving total cost, and reducing the volatility of dispatch cost and energy prices.

Therefore, we recommend the R-U approach as the most appropriate tool to manage the uncertainty in the UC process.

The detailed discussion is presented below. We focus on the high wind penetration case in subsections A, B and C. Finally, we analyze the impact of the wind penetration level in subsection D.

A. Cost efficiency and the choice of conservatism level

By properly adjusting the conservatism level, the robust approaches achieve lower total cost and lead to a more reliable system than the deterministic approach. The R-NU and R-U approaches are comparable. When the R-NU and R-U approaches are

tuned to be at their optimal conservatism levels, the latter has a slightly better performance than the former in terms of increasing cost efficiency and operational reliability. In particular, the R-U approach always results in lower total cost volatility and a more stable performance than the Det approach. As for the R-NU approach, its total cost can be the most volatile of the three approaches in some uncertainty behavior clusters, which is not desirable.

Figure 5 plots the average commitment cost of all the clusters in the high wind penetration case. The penalty cost for loss of load is not considered in the commitment cost. As the level of conservatism increases from 0 to 1 on the x-axis, the size of the forecast range increases from the point forecast to the most conservative case, corresponding to the 99 confidence interval range, under the robust approaches. The incremental reserve increases from 0 to the difference between the largest net load of the 99 confidence interval and expected net load under the Det approach. It can be seen that the commitment cost under both robust approaches increases as the level of conservatism increases. The commitment cost of the R-U approach is constantly lower than that of the R-NU approach. The commitment cost does not increase monotonically under the Det approach as it does under the robust approaches. In particularly, the commitment cost under the Det approach reaches its peak at the conservatism level 0.55, and decreases afterward. This is because the system does not have enough reserve capacity to meet the incremental reserve requirement when the conservatism level is greater than 0.55. As a result of the reserve shortage, suboptimal UC solutions are obtained, leading to a decrease in the commitment cost.

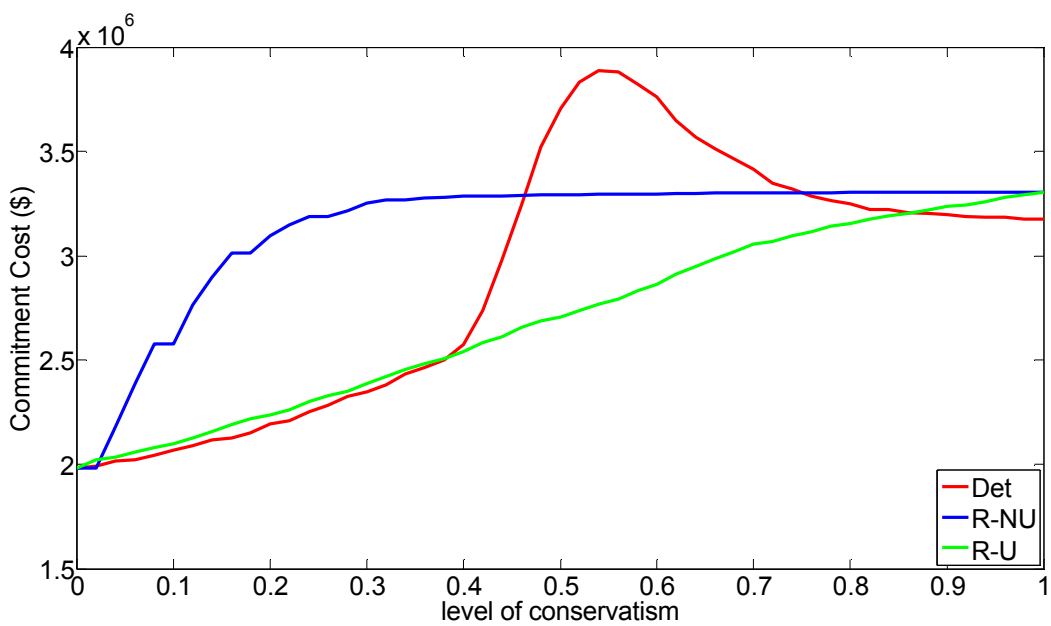

Figure 5: The commitment cost for the high wind penetration case

Figure 6 shows the load shedding event frequency under three alternative approaches for each cluster. The twelve clusters are partitioned into three columns, each corresponding to low, mid and high net load, respectively, and each row corresponds to different wind clusters in Figure 6. Load shedding frequency is used as

an indicator of system reliability. The smaller the frequency, the more reliable the system is. As illustrated in Figure 6, the load shedding events are reduced to almost none as the conservatism level increases under all three approaches when the net load is either low or medium. This means that the system can operate reliably under all approaches, as long as the forecast range of the robust approaches and the incremental reserve of the deterministic approach are chosen to be large enough. However, when the net load is high, the system is stressed to a sufficiently high degree that even the most conservative UC solution leads to a high load shedding frequency. The deterministic approach results in much more load shedding frequency than the robust counterparts. This means that more expensive emergency actions, such as dispatching fast-start units or load-shedding, have to be taken to maintain system reliability under the Det approach than the other approaches. In addition, the suboptimal deterministic UC solutions associated with the conservatism level greater than 0.55 leads to an increase in the corresponding load shedding frequency under the Det approach when the net load is high.

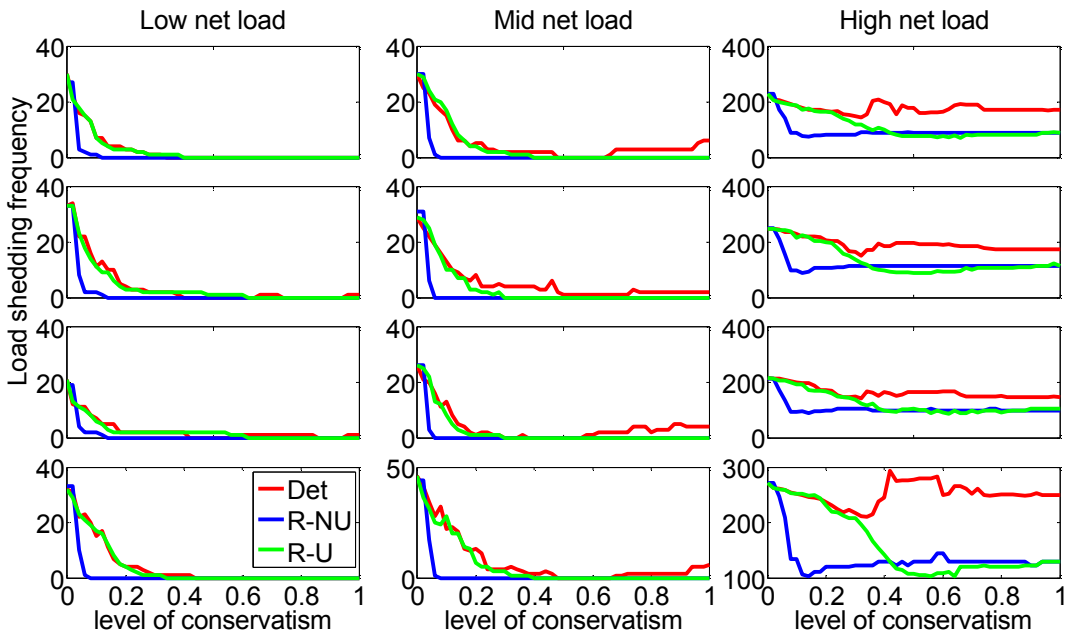

Figure 6: The load shedding frequency for the high wind penetration case.

In Figure 7, the average real-time dispatch costs are plotted as solid curves. The load shedding penalty cost is not included in the dispatch cost. The legend of Figure 7 is the same as Figure 6. All the average dispatch cost curves display U-shape, and the squares on the solid curves specify the minimal dispatch cost points. The dashed vertical lines in Figure 7 represent the conservatism level corresponding to the minimal load shedding frequency or the most reliable UC solutions[1]. It can be seen that the UC solution associated with the least dispatch cost does not always coincide with the most reliable performance.

[1] The red dashed vertical line overlaps with the green dashed vertical line in row 3 and column 2 of Figure 7.

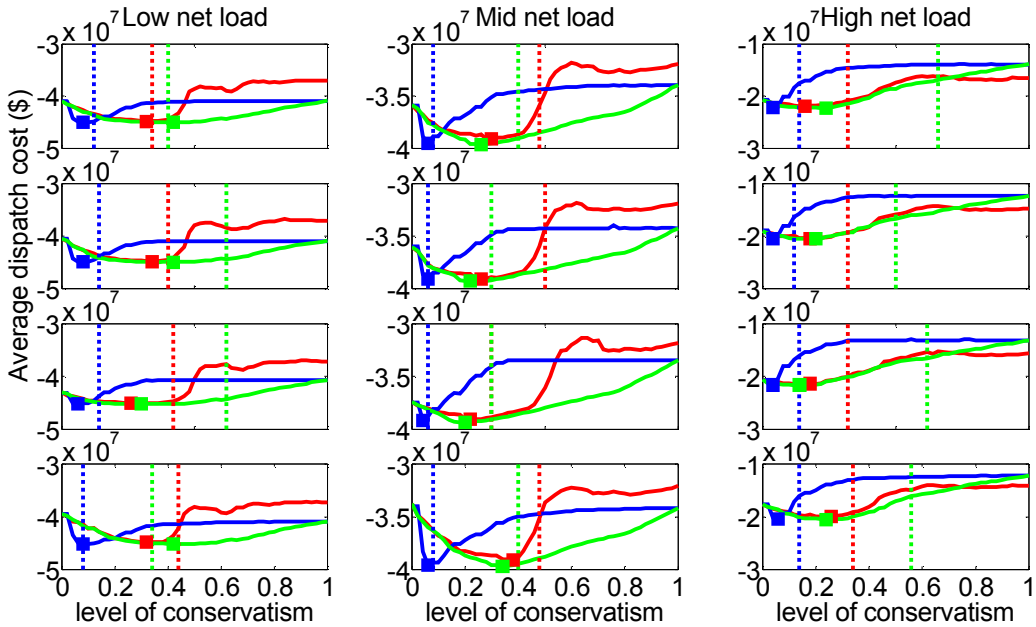

Figure 7: The average dispatch cost for the high wind penetration case.

Next, we want to find the optimal conservatism level that balances between the average total cost, i.e. the sum of commitment and dispatch costs, and the operational reliability. We multiply the load shedding frequency by 10^5 to scale it up to the same order of magnitude as the average total cost[2]. Then, we use the sum of the scaled load shedding frequency and the average total cost to measure the overall cost and reliability performance. The conservatism level that returns the smallest sum is considered to be the optimal conservatism level. Table 1 reports the optimal conservatism level for each cluster. We observe that the R-NU attains its optimal performance when the conservatism level is relatively small, e.g. around 0.08 when the net load is low. This optimal conservatism level is consistent with the findings in [Bertsimas et al. 2013]. The optimal conservatism levels of the Det and R-U approaches are close, and their optimal conservatism levels are usually above 0.2.

To quantify the comparison, we use the average total cost of the Det approach at its optimal conservatism level as the benchmark, and define the relative total cost reduction of the robust approaches at their optimal conservatism levels as (Det average total cost - robust average total cost) / Det average total cost. In other words, we compare the measurement of the best-tuned Det UC solution with the best-tuned robust UC solutions. The larger the total cost reduction, the more cost-effective the corresponding robust approach. Table 1 shows that the optimal R-U solution always leads to a greater total cost reduction and lower load shedding frequency than the optimal Det solution. The advantages of the R-U approach are especially great under the high net load clusters, where the R-U approach can save up to 2.73%, or $467 K, total cost relative to the Det approach while shedding less load. The performance of the

[2] The scaling factor of the load shedding frequency affects the optimal conservatism level. The larger the scaling factor, the more weight decision makers put on operational reliability relative to total cost. We want to weigh total cost and reliability equally by choosing 10^5 as the scaling factor in the analysis. If decision makers consider reliability more important than cost, a greater scaling factor can be used.

R-NU approach is similar to that of the R-U approach at the optimal conservatism level, but the former is slightly worse than the latter in terms of the total cost reduction and load shedding frequency, as shown in Table 1.

Cluster		The optimal conservatism level			The average total cost under the Det approach and the relative total cost reduction under the R-NU and R-U approaches at the optimal conservatism level[3]			The load shedding frequency at the optimal conservatism level			The standard deviation of the total cost under the Det approach and the relative standard deviation reduction under the R-NU and R-U approaches at the optimal conservatism level		
		Det	R-NU	R-U	Det ($\times 10^7$ \$)	R-NU (%)	R-U (%)	Det	R-NU	R-U	Det ($\times 10^6$ \$)	R-NU (%)	R-U (%)
Low net load	L1	0.33	0.08	0.35	-4.32	0.12	0.25	1	2	2	3.07	-1.30	11.40
	L2	0.35	0.08	0.35	-4.31	0.12	0.22	2	2	2	2.73	-6.96	8.06
	L3	0.27	0.08	0.25	-4.35	0.00	0.19	2	2	2	2.62	20.61	3.05
	L4	0.33	0.10	0.41	-4.32	0.46	0.53	1	0	0	3.39	26.25	20.65
Mid net load	M1	0.35	0.08	0.35	-3.68	1.68	1.70	4	1	1	4.44	3.38	14.19
	M2	0.22	0.06	0.20	-3.69	0.35	0.67	1	3	1	4.80	-1.25	7.92
	M3	0.29	0.08	0.27	-3.68	1.03	1.39	2	1	2	4.24	17.92	9.67
	M4	0.25	0.08	0.24	-3.69	-0.28	0.57	4	0	2	4.77	24.74	6.08
High net load	H1	0.27	0.08	0.29	-1.71	2.22	2.73	219	209	208	1.24	2.42	3.23
	H2	0.20	0.06	0.22	-1.77	0.02	0.29	218	211	201	1.23	-2.44	2.44
	H3	0.18	0.06	0.25	-1.92	1.44	1.56	172	171	150	1.22	0.00	7.38
	H4	0.20	0.06	0.16	-1.86	0.64	0.81	173	172	172	1.20	-0.83	0.00

Table 1: The optimal conservatism level, average total cost, load shedding frequency and standard deviation of the total cost for the Det, R-NU and R-U approaches for the high wind penetration case.

We also compare three approaches in terms of the standard deviation of the total cost. Everything else being equal, an approach with a lower standard deviation is preferable since a high standard deviation implies a greater uncertainty in the cost. We still use the optimal deterministic UC solution as the benchmark, and obtain the relative standard deviation reduction of the robust solutions at the optimal conservatism levels, which is defined as (Det standard deviation of total cost – robust standard deviation of total cost)/ Det standard deviation of total cost. As shown in Table 1, the R-U approach

[3] Because demand bids are considered, it results in negative total cost.

can drastically lower the total cost standard deviation, up to 20.65%, relative to the Det approach. The relative standard deviation reduction of the R-U approach is greater for low and medium net load values than for high values. On the other hand, the R-NU approach does not always lead to a smaller standard deviation than the Det approach, which is reflected by several negative relative standard deviation reduction values in Table 1.

We also compare three approaches with the load shedding scaling factor equal to 10^4 and 10^6, respectively, in addition to 10^5. The comparison result using the scaling factor 10^4 is similar to the one using 10^5 scaling factor. The R-U approach still outperforms the other two approaches in terms of reducing the average and standard deviation of total cost as well as decreasing the load shedding frequency. This suggests that if the importance of load shedding is valued less, the R-U approach can remain its economic efficiency, beneficial effect on dispatch cost volatility, as well as keeping system reliable. On the other hand, when the scaling factor is 10^6, the advantage of the robust approaches over the Det approach is weakened in the sense that the Det approach can lead to lower average and standard deviation of dispatch cost in some clusters. However, the robust solutions are much more resilient than the Det approach in that load shedding frequency under the robust approaches is much lower than that under the Det approach. This is because when more weight is put on the load shedding, the optimal robust UC solutions are more robust, but less economic than the deterministic solutions.

B. Impact on prices and generators' revenue

In this subsection, we show that the robust approaches depress energy prices in most of the clusters. However, the robust approaches do not necessarily result in a lower reserve price than the deterministic approach. Because the generators' profit mainly comes from the energy revenue, the robust approaches lead to a lower average generators' profit, but reduce the volatility of the profit, or the risk, compared to the Det approach.

Figure 8 shows the trend of the energy, average spinning and non-spinning reserve prices under various levels of conservatism. The spinning reserve is procured from the committed generators while the non-spinning reserve is procured from the uncommitted generators. Instead of displaying all twelve clusters, we select one representative cluster from each net load level in Figure 8. Both energy and spinning reserve price curves show a decreasing pattern, as illustrated in the first two rows of Figure 8. This is because as more generators are committed with the increasing conservatism level, the increasing amount of available capacity and spinning reserve depress the energy and spinning reserve prices. Similarly, because more units are committed under the robust approaches than the Det approach in most of the cases, it leads to lower spinning reserve prices under the former approaches than the latter. As a result, there is less offline units providing non-spinning reserve under the robust approaches than the Det approaches. Due to the supply and demand relationship, the non-spinning reserve prices are higher under the robust approaches than their deterministic counterpart. In addition, as the net load increases from low to high, energy and reserve prices tend to

increase, as expected. In particular, when the net load is high, energy and reserve prices spike more often due to more frequent scarcity conditions, causing the average prices to go up.

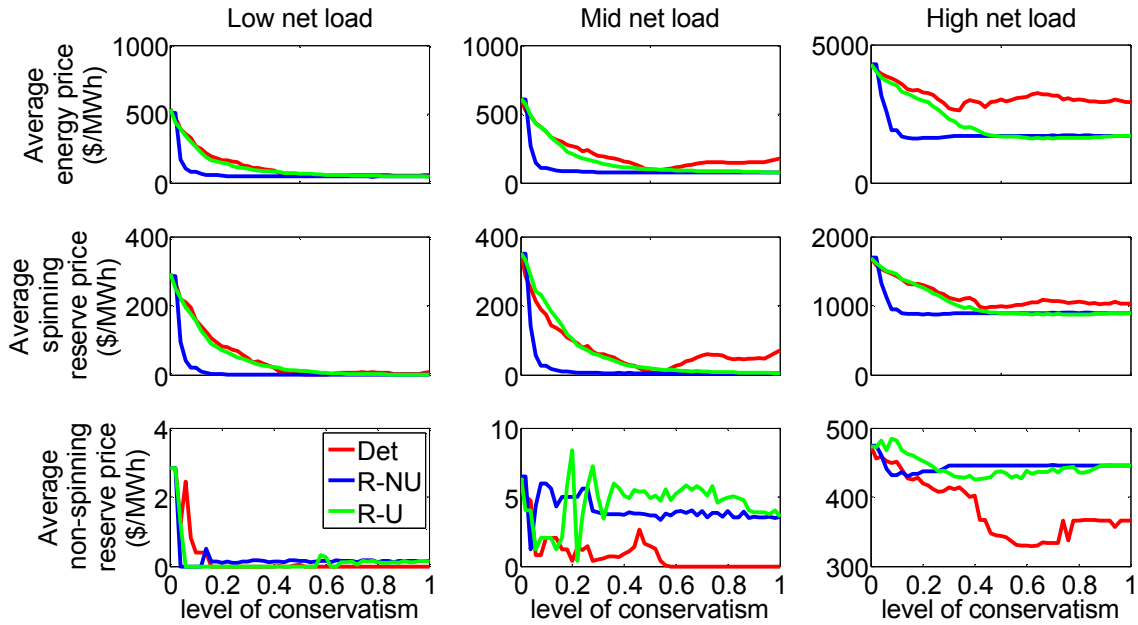

Figure 8: The average of spinning and non-spinning reserve prices for the high wind penetration case.

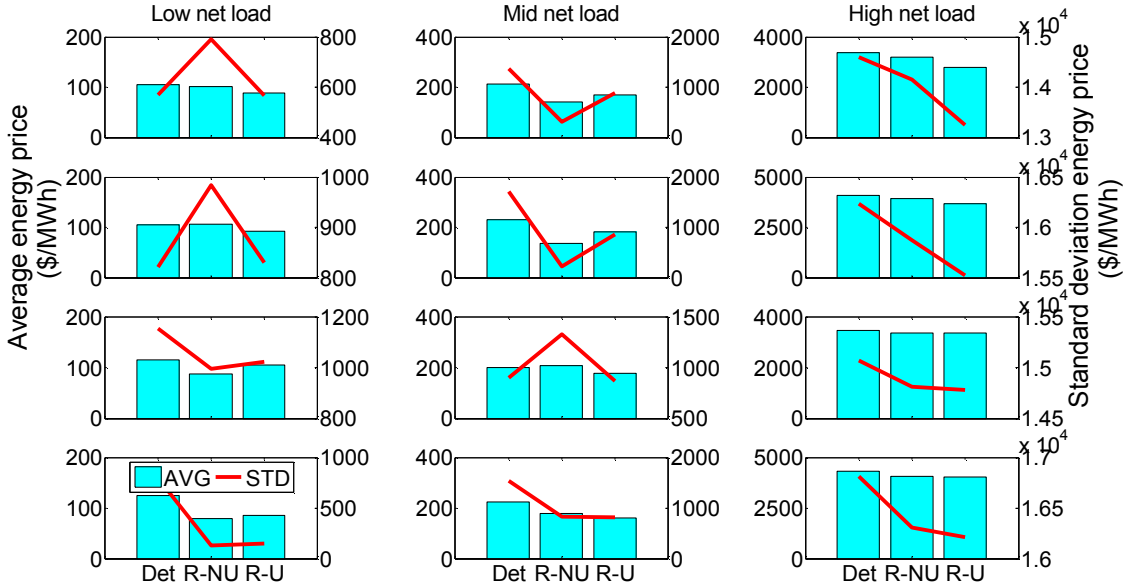

Figure 9: The average and standard deviation of energy prices at the optimal conservatism level for the high wind penetration case.

Next, we compare three approaches at their optimal conservatism levels identified in Table 1. Figure 9 plots the average and standard deviation of energy prices

under the optimal robust and deterministic UC solutions for all twelve clusters. The columns correspond to three load clusters while the rows correspond to four wind clusters. Compared to the Det approach, the robust approaches depress the average energy price in most of the clusters, as shown in the bar plots in Figure 9. One of the contributing factors for the low average energy price is that there are fewer energy price spikes under the robust approaches than the Det approach. This is because of the fact that energy price spikes occur when load is shed. The low load shedding frequency under the robust approaches demonstrated in Figure 6 implies less energy spikes. In terms of the energy price standard deviation, which is plotted in red curves in Figure 9, none of the approaches consistently outperforms the others, but the R-U approach is never the one with the largest standard deviation. Overall, the energy price is relatively low and stable under the R-U approach.

Generators' energy revenue, which is the product of energy price and quantity, is highly correlated with energy prices. As a result, the impact of the three alternative approaches on energy revenue at their optimal conservatism levels is similar to the impact on the energy prices, as shown in Figure 9. Generators receive 13.83% and 12.55% less energy revenue under the R-U and R-NU approaches, respectively, than the Det approach, and face less energy revenue volatility under the R-U approach, compared to the Det approach.

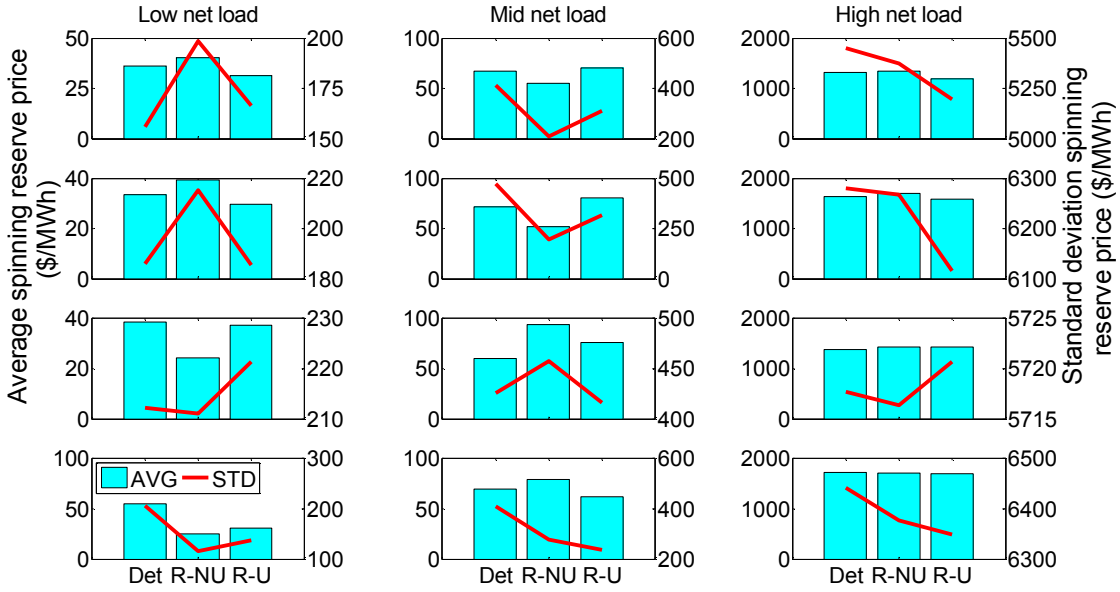

Figure 10: The average and standard deviation of spinning reserve prices at the optimal conservatism level for the high wind penetration case.

Figure 10 plots the average and standard deviation of spinning reserve prices under the optimal robust and deterministic UC solutions. Similar to Figure 9, the columns correspond to three load clusters while the rows correspond to four wind clusters. In contrast to energy prices, the spinning reserve prices do not behave consistently: for some clusters, they are higher for the robust approaches than the Det approach. The same is true for the reserve revenues. In addition, none of the approaches

demonstrates a consistent advantage over the rest in terms of the standard deviation of the spinning reserve price.

Because the energy revenue contributes to the majority of the generators' profit, the generators' total profit is 16.82% and 15.42% lower under the R-U and R-NU approaches, respectively, than the Det approach. The R-U approach results in a lower standard deviation than the Det approach.

C. Robustness against forecast errors

The forecast ranges used in the robust approaches are generated based on the prediction of the behavior pattern of wind, load and interchange, which are categorized into 12 clusters in this study, as described in Appendix B. In practice, the predicted behavior pattern can be very different from the actual realization. For example, it might be forecasted that the wind is high and trending up in the following day, but the actual wind generation turns out to be very low and trending down. It is therefore important for a UC solution to have stable economic and operational performance even in the presence of large forecast errors. The simulation results show that the robust approaches exhibit this desirable property. In comparison, the performance of the Det approach is more affected by forecast errors.

We consider two scenarios, namely wind over-forecast and load under-forecast. We use the UC solutions derived from a cluster with a high wind and low load pattern, as illustrated at the first row of Figure 11. Then, we run system dispatch simulations using wind samples generated from the same forecasted wind pattern, but with the load samples from a high load cluster under the load over-forecast scenario, as shown in the lower left plot of Figure 11. Under the wind over-forecast scenario, we use load samples generated from the same forecasted load pattern, but with the wind samples from a low wind cluster, as shown in the lower right plot of Figure 11.

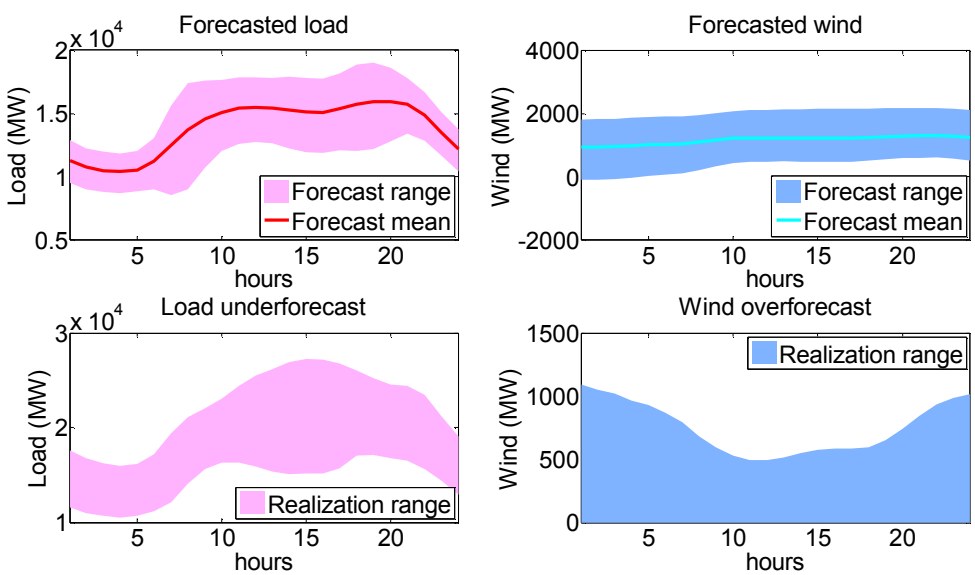

Figure 11: The forecasted load and wind patterns. The realized load and wind patterns for the load under-forecast and wind over-forecast scenarios.

Figure 12 plots the difference of the average and the standard deviation of dispatch cost between a poor and an accurate forecast of the uncertainty behavior under the three approaches. A poor forecast means either the wind over-forecast or the load under-forecast scenario. An accurate forecast means that the forecasted and realized wind and load are from the same cluster, or that the prediction and realizations are consistent in terms of load and wind behavior pattern. The difference of the average dispatch cost is defined as the dispatch cost of the poor forecast less that of the accurate forecast. Similarly, the difference of the standard deviation dispatch cost is equal to the standard deviation dispatch cost of the poor forecast less that of the accurate forecast. A smaller magnitude of either of these differences implies a more robust corresponding UC solution.

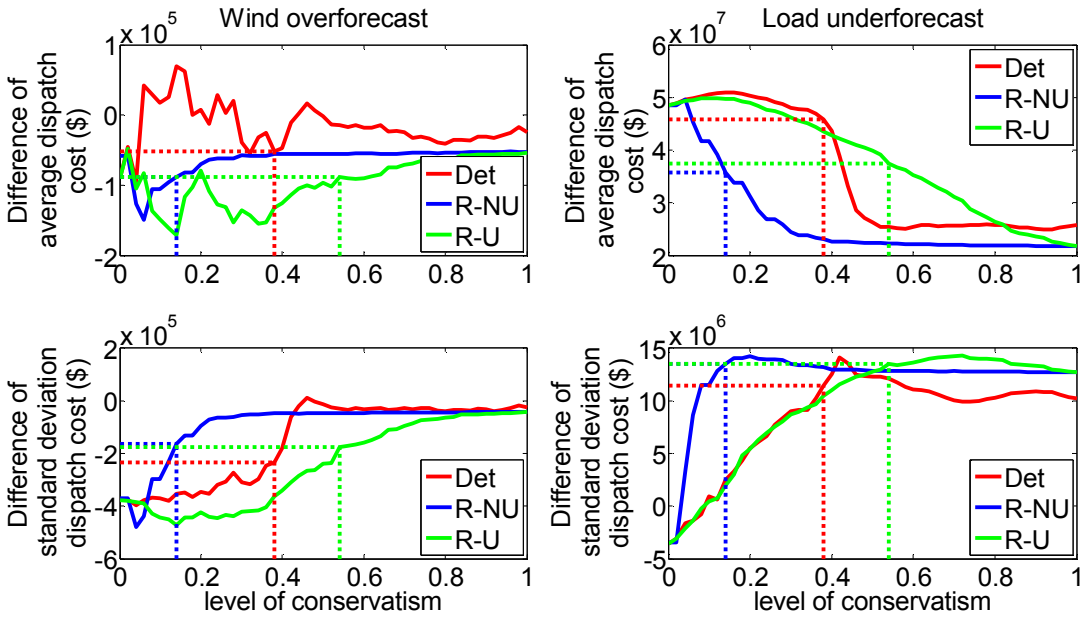

Figure 12: Impact of forecast errors on average and standard deviation dispatch cost for the high wind penetration case.

The left column in Figure 12 shows the results for the wind over-forecast scenario while the right column shows the results under the load under-forecast scenario. The dashed lines indicate the results at the optimal UC solutions. Under the wind over-forecast scenario, the differences of the average and standard deviation dispatch cost are negative at the optimal UC solutions. In other words, the dispatch cost and its volatility are reduced when less low-cost wind power is produced than forecasted. This is counterintuitive. The reason for the decrease in volatility is that when wind is over-forecasted, there is less wind power volatility in the dispatch simulation. As a result, it leads to a smaller standard deviation in dispatch cost. In addition, since less wind power is produced than forecasted, the system has sufficient flexibility provided by the over committed units to resolve congestion problems. Consequently, the fewer congestion problems result in a lower average dispatch cost. In the load under-forecast scenario, we observe that the average and standard deviation of dispatch cost both increase at the optimal UC solutions compared to the accurate forecast results. This is expected because the UC solutions under low load forecast do not commit enough

generators. In turn, it causes tight system operating conditions when the actual load realizations are high. Comparing three approaches, we observe that the behaviors of two robust approaches are similar. The impact on the average dispatch cost is smaller under the robust approaches than the deterministic counterpart. However, the Det approach leads to lower volatility in the dispatch cost than the robust approaches.

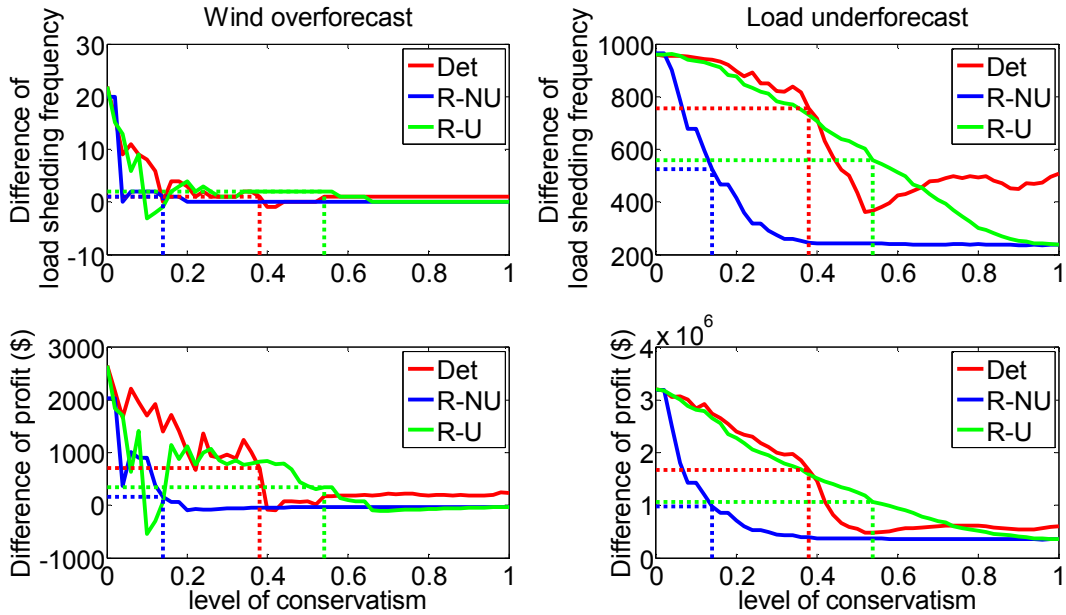

Figure 13: Impact of forecast errors on load shedding frequency and generators' profit for the high wind penetration case.

Figure 13 plots the impact of forecast errors on the load shedding frequency and the generators' average profit. The differences of load shedding frequency and profit are defined as the load shedding frequency and generators' profit of the poor forecast minus the counterparts of the accurate forecast, respectively. They are plotted in solid curves. The smaller this difference, the better the approach, everything else being equal. As in Figure 12, the corresponding differences at the optimal UC solutions are illustrated in dashed lines. It can be seen that the performance of two robust approaches are comparable. The system reliability under the optimal robust approaches is more immune to the forecast errors than the Det approach. This benefit is even greater when the load is under-forecasted, reflected by the fact that the load shedding frequency is approximately 25% lower under the former than the latter. The economic impact on the generators, which is measured by the profit difference, is also lower under the optimal robust approaches relative to the deterministic counterpart. Since the amount of load in the system is one magnitude larger than the wind capacity, the effect of wind over-forecast is much smaller than the effect of load under-forecast as shown in Figures 12 and 13.

D. Impact of wind penetration

In this section, we study the impact of wind penetration on the performance of three approaches. We find that it costs more to operate the system in the high wind penetration case than the low wind penetration case we investigated. This is because there is more uncertainty associated with high wind case, which requires the system to commit additional resources to hedge against the larger magnitude of wind variability. In the low wind penetration case, the robust approaches still outperform the Det approach in improving system reliability, saving operating cost, and resilience to forecast errors. However, the beneficial effect of the robust approaches in the low wind penetration case is not as considerable as it is in the high wind penetration case.

Table 2 shows the same information as Table 1, but for the low, rather than high, wind penetration case. It shows that the optimal conservatism levels of each approach are smaller than their counterparts in the high wind penetration case in Table 1. Compared to the deterministic approach, the advantages of the robust approaches in reducing the average and standard deviation of the total cost in the low wind penetration case are not as significant as it is under the high penetration case. This is because there is less uncertainty associated with low wind installment, which in turn diminishes the benefit of the robust approaches. However, the R-U approach still appears to have the best overall performance among the three approaches.

The comparison of the total cost standard deviation under the Det approach in Tables 1 and 2 reveals that the standard deviation is smaller for the low wind than for the high wind penetration case. The same is true for the standard deviations of the robust approaches. This is expected because the volatility of wind output is lower for the low wind penetration case, which in turn leads to a less volatile total cost.

The average total cost decreases as the wind penetration changes from high to low. In particular, the average total cost decreases by 1.71%, 1.25% and 1.27% for the deterministic, R-NU and R-U approaches, respectively. This means it costs more to operate the system for the high wind than for the low wind penetration case. This is because the system needs to commit additional units to hedge against additional uncertainty associated with wind generation in the high wind penetration scenario. Furthermore, the change of wind penetration form high to low also leads to a decrease in the average dispatch cost by 1.61%, 1.13%, and 1.17% for the deterministic, R-NU and R-U approaches, respectively. Because more units are committed to handle possible loss of large amount of wind in the high wind penetration case, some expensive units may be idle at their minimal generation level during the real-time dispatch. Especially, when the actual wind output turns out to be very large, the system has to curtail wind generation in order to balance the minimum generation levels of the committed units. Due to the excessive high cost units being committed, the average dispatch cost is higher when there is more wind in the system. This implies that additional cheap wind power can be detrimental unless the variability in the wind generation can be controlled.

Cluster		The optimal conservatism level			The average total cost under the Det approach and the relative total cost reduction under the R-NU and R-U approaches at the optimal conservatism level			The load shedding frequency at the optimal conservatism level			The standard deviation of the total cost under the Det approach and the relative standard deviation reduction under the R-NU and R-U approaches at the optimal conservatism level		
		Det	R-NU	R-U	Det ($\times 10^7$ \$)	R-NU (%)	R-U (%)	Det	R-NU	R-U	Det ($\times 10^6$ \$)	R-NU (%)	R-U (%)
Low net load	L1	0.29	0.06	0.29	-4.37	-0.23	0.00	2	5	1	2.67	-25.1	3.00
	L2	0.27	0.06	0.25	-4.36	0.00	0.23	0	0	0	2.31	-24.7	0.00
	L3	0.29	0.06	0.24	-4.36	0.00	0.23	1	2	1	2.33	-30.0	-14.2
	L4	0.27	0.06	0.27	-4.32	-0.23	0.23	1	1	1	2.86	-20.6	4.55
Mid net load	M1	0.20	0.06	0.20	-3.74	0.27	0.27	0	0	0	4.14	6.76	6.52
	M2	0.22	0.06	0.20	-3.78	0.53	0.53	4	4	4	4.12	2.43	2.18
	M3	0.20	0.06	0.20	-3.72	0.27	0.27	4	3	4	4.13	4.12	4.36
	M4	0.20	0.06	0.18	-3.76	0.27	0.27	2	2	2	4.25	7.29	2.35
High net load	H1	0.18	0.06	0.16	-1.96	0.51	1.02	190	181	189	1.16	0.86	-0.86
	H2	0.12	0.06	0.14	-1.91	0.00	0.52	183	171	177	1.16	3.45	1.72
	H3	0.14	0.06	0.12	-1.85	0.00	1.08	198	186	196	1.25	2.40	8.80
	H4	0.14	0.06	0.14	-1.82	0.00	1.10	197	194	195	1.22	1.64	-0.82

Table 2: Optimal conservatism level, average total cost, load shedding frequency and standard deviation of the total cost for the Det, R-NU and R-U approaches in the low wind penetration case.

Figure 14 shows the wind spillage as a function of the level of conservatism for both the high and the low wind penetration cases. It shows that the amount of wind spillage is similar in both cases when the net load is high. When the net load is medium, there is less wind spillage in the low wind penetration case than in the high wind penetration case. However, when the net load is low, the opposite is true. Additionally, it can be observed from Figure 14 that wind spillage increases with the amount of conservatism. This is because the additional units committed in the higher conservatism levels have to run at their minimum generation levels once they are online. As a result, the system is more likely to incur the supply surplus condition where cheap wind power has to be spilled to maintain system balance.

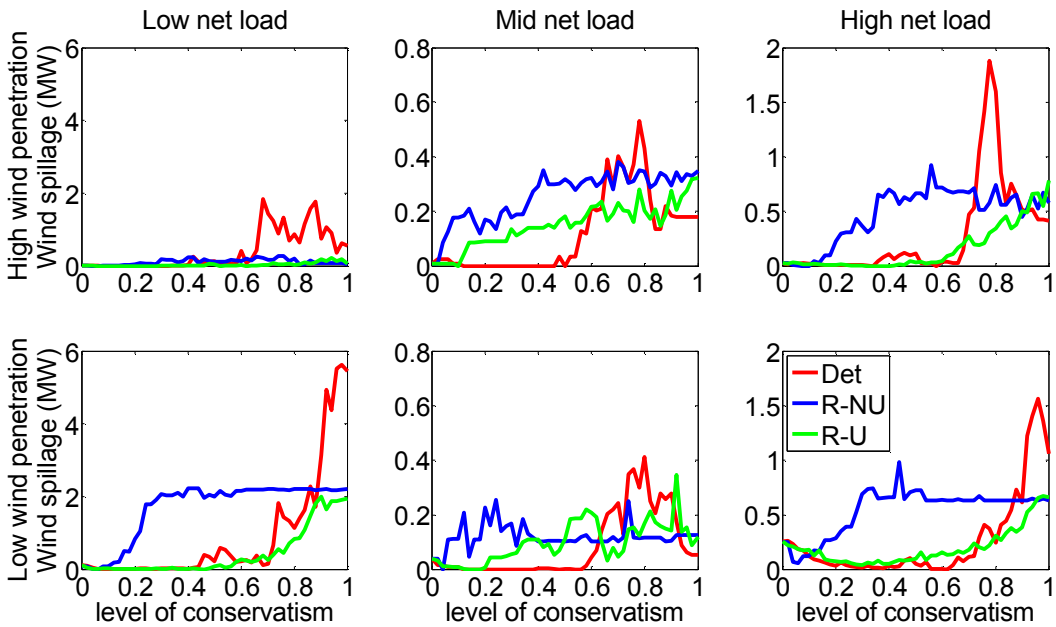

Figure 14: Wind spillage for the high and low wind penetration cases.

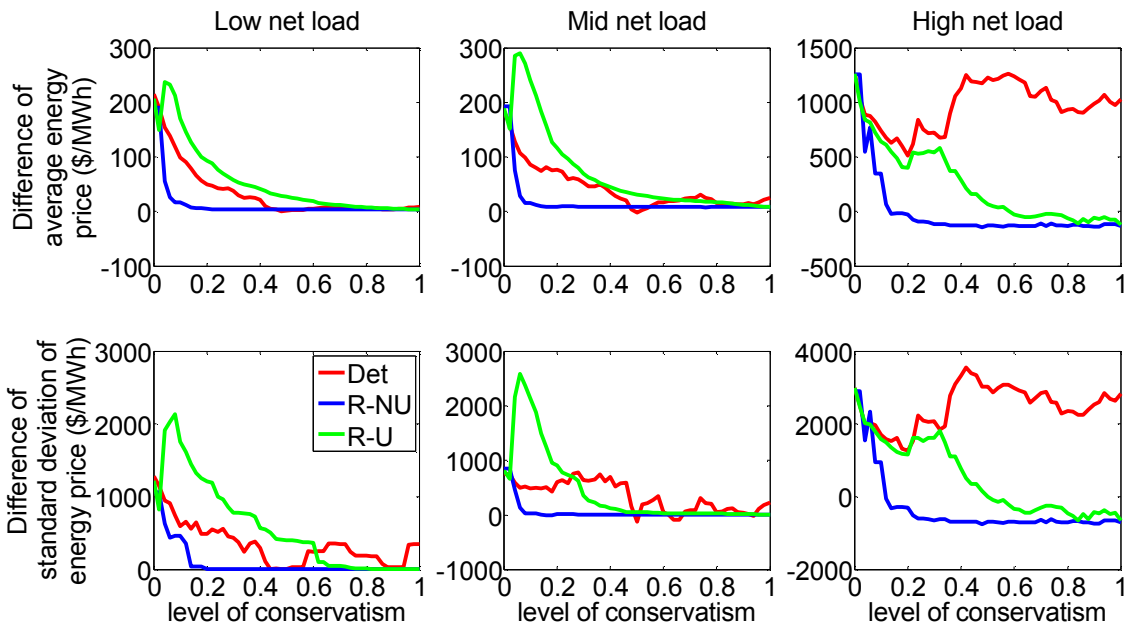

Figure 15: Difference in average and standard deviation of energy price between the high and low wind penetration cases.

Figure 15 shows the impact of wind penetration level on the average and the standard deviation of the energy price. The difference in the average (standard deviation) of energy price is defined as the average (standard deviation) of energy price under high wind penetration – the average (standard deviation) of energy price under low wind penetration. The variation of the difference with respect to the conservatism level is plotted as the solid curve in Figure 15. It shows that the R-NU approach leads to the lowest difference in average and standard deviation of energy price. This implies

that the energy price is least sensitive to the level of wind penetration under the R-NU approach among three approaches. When the net load is low or medium, all the curves are above zero. It suggests that the average and standard deviation of energy price are higher in the high wind penetration case than the low wind penetration case. On the other hand, when the net load is high, the curves can be below zero. This means that the energy price decreases due to more wind installment. The reason is that wind is needed to relieve the tight system condition under the high net load condition. In turn, it lowers the energy price.

The impact of forecast errors on dispatch cost and the impact on load shedding frequency and generators' profits in the low wind penetration case are shown in Figures 16 and 17, respectively. The dashed lines indicate the results at the optimal conservatism level. We can see that all the measurements are affected less under the optimal robust solutions than the optimal deterministic solutions. In addition, the robust approaches exhibit an advantage over the Det approach in terms of reducing the volatility of dispatch cost in the low wind penetration case. However, the resilience against forecast errors of the robust approaches is lower in the low wind penetration case than in the high wind penetration case shown in Figures 12 and 13.

In Figure 16, under the wind over-forecast scenario, the differences of the average and standard deviation dispatch cost are positive at the optimal UC solutions. This means that the dispatch cost and its volatility go up when actual wind power is less than the forecasted in the low wind penetration case, in contrast to the decrease in cost and volatility as shown in Figure 12. The reason is that the congestion is less an issue in the low wind case than it is in the high wind case. In turn, when there is less wind power is actually generated than the forecast value, more expensive units have to be used to meet the load, increasing the dispatch cost.

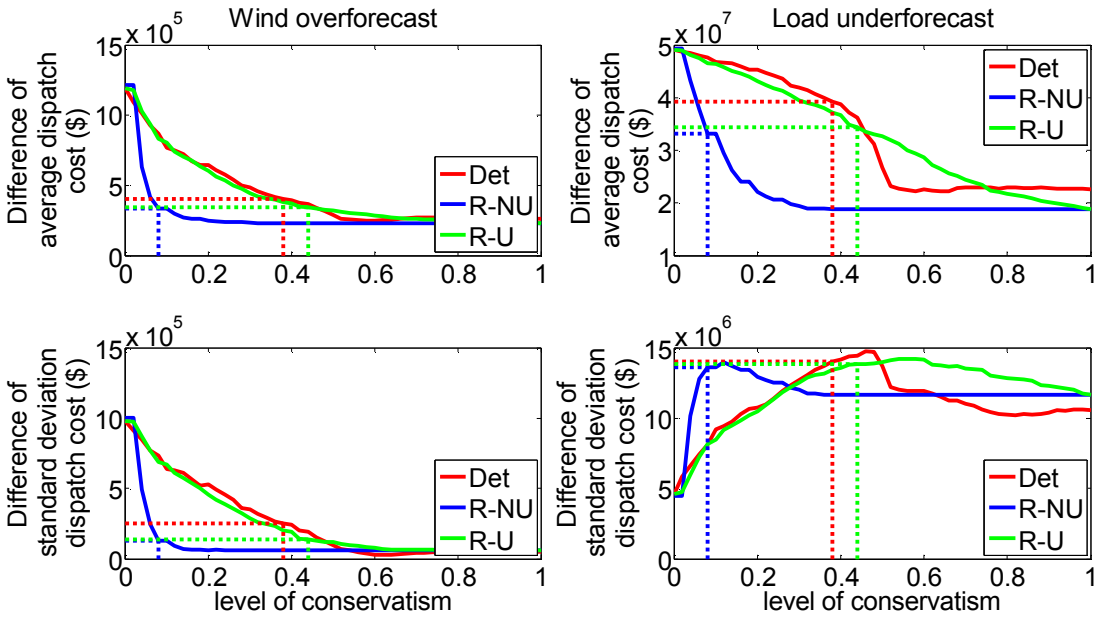

Figure 16: Impact of forecast errors on average and standard deviation dispatch cost for the low wind penetration case.

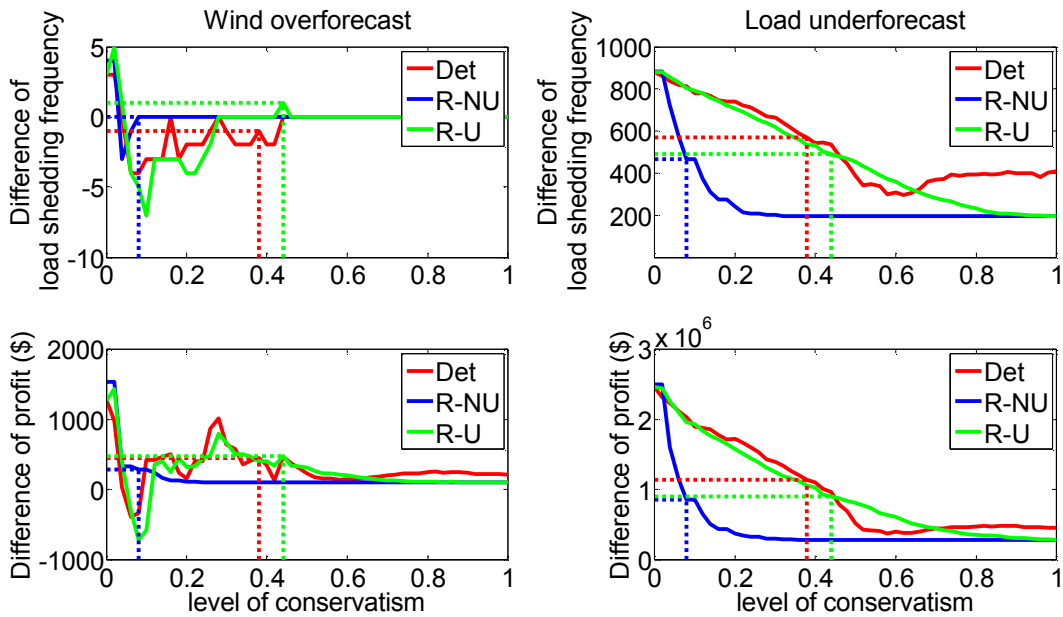

Figure 17: Impact of forecast errors on load shedding frequency and generators' profit for the low wind penetration case.

Benefits to Company and LLNL

The robust model is a powerful tool to handle uncertainties, but the immense simulation effort required limits ISO NE's ability to assess the benefit of applying robust solutions. The parallel computing capability of HPC significantly expedited the research efforts on evaluating the robust approach. This study is the first comprehensive study of robust UC using a large-scale power system. The findings can greatly facilitate the UC improvement process, and help the ISO to identify an appropriate risk management tool that can mitigate the impact of variability and uncertainty on system reliability and economics.

In turn, LLNL participants were exposed to the important problems faced by the energy industry, as well as new algorithms and methods required to solve them This project, as a part of the hpc4energy incubator (Smith 2012), joins a growing number of DOE-sponsored partnerships that are helping to connect energy businesses with HPC resources, while aiding the nation in meeting its carbon emission and energy security goals.

Conclusions

In this study, a comprehensive evaluation of robust unit commitment was conducted. The objectives of this study were to identify the optimal conservatism level that balances the economic efficiency and operational reliability of robust UC solutions,

as well as compare the robust and deterministic approaches. A statistical model characterizing the wind, load and interchange uncertainty was created to construct forecast ranges for robust UC and to generate random samples for the simulation. Parallelization techniques were employed to solve UC problems and perform dispatch simulation efficiently on supercomputers.

By simulating the performance of robust solutions under various levels of conservatism, we obtained a set of well-tuned robust UC solutions that result in low cost and load shedding. We also found that the robust approaches exhibit sizable reductions in total cost, significantly reduce the volatility of dispatch cost and improve the reliability of the power system operation relative to the deterministic approach. The robust UC solutions also showed resilient performance in the presence of large forecast errors. The advantages of robust approaches were more significant when more wind resources were integrated into the system. From the perspective of generators, the profit was lower for the robust approaches than the deterministic approach due to the fact that the former depresses energy prices. However, generators faced less risk under the robust approaches than the deterministic one.

Besides the robust approach, stochastic optimization is another popular method to manage uncertainty for the UC problem. The stochastic approach explicitly incorporates probability distributions of possible wind generation, interchange and load values, instead of assuming a fixed uncertainty range as is done in the robust approach, to compute UC solutions that guarantee system reliability. It would be interesting to compare the performance of robust and stochastic UC solutions in the future.

Acknowledgements

This study includes extensive collaboration between ISO NE and LLNL. Many thanks go to the team members below:

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References

Bertsimas, D., E. Litvinov, X. Sun, J. Zhao and T. Zheng (2013) “Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem”, IEEE Transactions on Power Systems, vol. 28, No. 1, pp. 52- 63, 2013.

Bertsimas, D. and M. Sim (2004) “The Price of Robustness”, Operations Research, vol. 52, No. 1, pp. 35 – 53, 2004.

Hartigan, J.A. and M.A. Wong (1979) “Algorithm AS 136: A K-Means Clustering Algorithm". Journal of the Royal Statistical Society, Series C 28 (1): 100–108.

Smith, C (2012) “Incubator Busy Growing Energy Technologies”, LLNL Science & Technology Review, June 2012, pp. 24–25. Online: https://str.llnl.gov/June12/smith.html

Appendix A
Mathematical Formulations

Nomenclature

Parameters:

N_g, N_d, N_w, N_a, T : The number of generators, loads, wind generators, interchanges, and time periods (in hours)

S_i^t, G_i^t, F_i^t : Start-up, shut-down, and no-load costs of generator i at time t.

$C_i^t(\square)$: Production cost of generator i at time t.

$p_i^{\max}$, $p_i^{\min}$: Maximum and minimum production levels of generator i.

RU_i^t, RD_i^t : Ramp-up and ramp-down rates of generator i at time t.

$MinUp_i^t$, $MinDw_i^t$: Minimum-up and minimum-down times of generator i.

$f_l^{\max}$: Flow limit on transmission line l in base case.

$f_{l,i}^{\max}$: Flow limit on transmission line l in contingency i (i.e., line l is tripped.)

$\mathbf{B}_p$, $\mathbf{B}_d$, : Network incidence matrices for generators and load.

$\mathbf{a}_l$: Network shift factor vector for line l for the base case.

$\mathbf{a}_{l,i}$: Network shift factor vector for line l in contingency i

$\bar{d}_j^t$: Expected demand at node j at time t.

$\bar{w}_i^t$: Expected output of wind generator i at time t.

$\bar{a}_i^t$: Expected interchange i at time t.

$\bar{q}_i^t$: Reserve capacity of generator i at time t.

$\bar{q}^t$: System reserve requirement at time t.

M : Penalty factor for load shedding

Variables:

$x_i^t \in \{0,1\}$: If generator i is on at time t, $x_i^t = 1$; otherwise $x_i^t = 0$.

$u_i^t \in \{0,1\}$: If generator i is turned on at time t, $u_i^t = 1$; otherwise $u_i^t = 0$.

$v_i^t \in \{0,1\}$: If generator i is turned down at time t, $v_i^t = 1$; otherwise $v_i^t = 0$.

$p_i^t \in [0,\infty)$: Production of generator i at time t.

$q_i^t \in [0,\infty)$: Reserve of generator i at time t.

$d_j^t \in [0,\infty)$: Uncertain demand at node j at time t.

$w_i^t \in [0,\infty)$: Uncertain output of wind generator i at time t.

a_i^t : Uncertain interchange i at time t.

s_+^t , $s_-^t \in [0,\infty)$: Slack variables at the balance constraint at time t.

LS^t : The amount of load shedding at time t.

WS : The amount of wind spillage at time t.

First, we present the deterministic UC model below, which also serves as the base model to its robust counterpart.

$$\min_{\substack{\mathbf{x},\mathbf{u},\mathbf{v},\\ \mathbf{p},\mathbf{q},\mathrm{LS},\mathrm{WS}}} \sum_{t=1}^{T}\sum_{i=1}^{N_g} x_i^t F_i^t + u_i^t S_i^t + v_i^t G_i^t + C_i^t(p_i^t) + M \times (s_+^t + s_-^t) \tag{1.1}$$

s.t.

$$x_i^{t-1} - x_i^t + u_i^t \geq 0 \text{ , } \forall i \text{ , } t \tag{1.2}$$

$$x_i^t - x_i^{t-1} + v_i^t \geq 0 \text{ , } \forall i \text{ , } t \tag{1.3}$$

$$x_i^t - x_i^{t-1} \leq x_i^\tau \text{ , } \forall i \text{ , } \forall \tau \in [t+1, \min\{t + MinUp_i - 1, T\}] \text{ , } t \in [2,T] \tag{1.4}$$

$$x_i^{t-1} - x_i^t \leq 1 - x_i^\tau \text{ , } \quad \forall i \text{ , } \quad \forall \tau \in [t+1, \min\{t + MinDw_i - 1, T\}] \text{ , } t \in [2,T] \tag{1.5}$$

$$\sum_{i=1}^{N_g} p_i^t + \sum_{i=1}^{N_w} \overline{w}_i^t + \sum_{i=1}^{N_a} \overline{a}_i^t + s_+^t - s_-^t = \sum_{j=1}^{N_d} \overline{d}_j^t \text{ , } \forall t \tag{1.6}$$

$$-RD_i^t \leq p_i^t - p_i^{t-1} \leq RU_i^t \text{ , } \forall i \text{ , } t \tag{1.7}$$

$$-f_l^{\max} \leq \mathbf{a}_l^{'}(\mathbf{B}_p\mathbf{p}^t + \mathbf{B}_w\overline{\mathbf{w}}^t + \mathbf{B}_a\overline{\mathbf{a}}^t - \mathbf{B}_d\overline{\mathbf{d}}^t) \leq f_l^{\max} \text{ , } \forall t \text{ , } l \tag{1.8}$$

$$-f_{l,i}^{\max} \leq \mathbf{a}_{l,i}^{'}(\mathbf{B}_p\mathbf{p}^t + \mathbf{B}_w\overline{\mathbf{w}}^t + \mathbf{B}_a\overline{\mathbf{a}}^t - \mathbf{B}_d\overline{\mathbf{d}}^t) \leq f_{l,i}^{\max} \text{ , } \forall t \text{ , } \forall l \in CT \text{ ,} \tag{1.9}$$

$$p_i^{\min} \leq p_i^t + q_i^t \leq p_i^{\max} \text{ , } \forall i \text{ , } t \tag{1.10}$$

$$p_i^{\min} x_i^t \leq p_i^t \leq p_i^{\max} x_i^t \text{ , } \forall i \text{ , } t \tag{1.11}$$

$$q_i^t \leq \overline{q}_i^t \text{ , } \forall i \text{ , } t \tag{1.12}$$

$$\sum_{i=1}^{N_g} q_i^t \leq \overline{q}^t \text{ , } \forall t \tag{1.23}$$

The objective function (1.1) of the above deterministic UC model is to minimize the total cost consisting of the start-up cost, shut-down cost, no-load cost, production cost and penalty cost. Constraints (1.2) and (1.3) are the logic constraints between on and off status and the turn-on and turn-off actions. Equations (1.4) and (1.5) are the constraints of minimum up and minimum down times for each generator. Constraints (1.6) and (1.7) are energy balance and ramping constraints, respectively. The slack variables s_+^t, s_-^t in the balance constraint (1.6) ensure a feasible solution to the optimization problem (1) when the system cannot be balanced physically. Equation (1.8) is the transmission flow constraint for the base case, where all transmission lines are functioning. Equation (1.9) is the transmission constraint for the ith contingency where transmission line i is tripped. Equation (1.11) is the resource capacity constraint. Equation (1.12) is the generator reserve capacity constraint. Equation (1.13) is the system reserve constraint.

Next, we present the robust UC model. The first step to build a robust model is to construct an uncertainty set. We consider the following two uncertainty sets, which correspond to the R-NU and R-U approaches, respectively. We denote $\mathbf{Y} \equiv (\mathbf{d}, \mathbf{w}, \mathrm{a})$ as the triple of uncertain energy level of load, wind and interchange.

Under the R-NU approach, the non-uniform uncertainty set, or the non-uniform forecast range, of wind generation, load, and inter change at each time period t is defined as follows:

$$\mathcal{U}^{NU}(\mathbf{Y}) \equiv \left\{ \begin{array}{l} \mathbf{Y} \in \mathbb{R}^{N_d} \times \mathbb{R}^{N_w} \times \mathbb{R}^{N_a} : \sum_{j=1}^{N_d} \frac{|d_j^t - \bar{d}_j^t|}{\hat{d}_j^t} + \sum_{i=1}^{N_w} \frac{|w_i^t - \bar{w}_i^t|}{\hat{w}_i^t} + \sum_{i=1}^{N_a} \frac{|a_i^t - \bar{a}_i^t|}{\hat{a}_i^t} \le \Gamma^t, \\ \qquad d_j^t \in [\bar{d}_j^t - \hat{d}_j^t, \bar{d}_j^t + \hat{d}_j^t], w_i^t \in [\bar{w}_i^t - \hat{w}_i^t, \bar{w}_i^t + \hat{w}_i^t], \\ \qquad a_i^t \in [\bar{a}_i^t - \hat{a}_i^t, \bar{a}_i^t + \hat{a}_i^t] \end{array} \right\}$$

The parameter Γ^t is the "budget of uncertainty" (Bertsimas and Sim 2004), taking values between 0 and $N_d + N_w + N_a$. When $\Gamma^t = 0$, $\mathcal{U}^{NU}$ is reduced to a singleton $\mathcal{U}^{NU} = \{\bar{\mathbf{d}}, \bar{\mathbf{w}}, \bar{\mathbf{a}}\}$, the expected load, wind and interchange levels. As Γ^t increases, the size of the uncertainty set $\mathcal{U}^{NU}$ enlarges. This means that larger total deviation from the expected values is considered, so that the resulting robust UC solutions are more conservative and the system is protected against a higher degree of uncertainty. When $\Gamma^t = N_d + N_w + N_a$, $\mathcal{U}^{NU}$ equals to the hypercube defined by the intervals for $\mathbf{d}$, $\mathbf{w}$, and $\mathbf{a}$.

Under the R-U approach, the uniform uncertainty set at each time period t is defined as followed:

$$\mathcal{U}^{U}(\mathbf{Y}) \equiv \left\{ \begin{array}{l} \mathbf{Y} \in \mathbb{R}^{N_d} \times \mathbb{R}^{N_w} \times \mathbb{R}^{N_a} : d_j^t \in [\bar{d}_j^t - \alpha^t \hat{d}_j^t, \bar{d}_j^t + \alpha^t \hat{d}_j^t], w_i^t \in [\bar{w}_i^t - \alpha^t \hat{w}_i^t, \bar{w}_i^t + \alpha^t \hat{w}_i^t], \\ \qquad a_i^t \in [\bar{a}_i^t - \alpha^t \hat{a}_i^t, \bar{a}_i^t + \alpha^t \hat{a}_i^t] \end{array} \right\}$$

The parameter α^t varies between 0 and 1. Similar to $\mathcal{U}^{NU}$, when $\alpha^t = 0$ and $\alpha^t = 1$, the uncertainty set $\mathcal{U}^U$ becomes a singleton and hypercube, respectively. As α^t increases, the size of the uncertainty set $\mathcal{U}^U$ also enlarges. However, how the shape of $\mathcal{U}^U$ changes is different from the way $\mathcal{U}^{NU}$ does. Figure 4 illustrates the difference for a simple case with only two uncertain sources.

The robust counterpart of the deterministic UC model (1) is the following:

$$\min_{\substack{\mathbf{x},\mathbf{u},\mathbf{v},\\ \mathbf{p}(\square),\mathbf{q}(\square),\mathrm{LS}(\square),\mathrm{WS}(\square)}} \left(\sum_{t=1}^{T}\sum_{i=1}^{N_g} x_i^t F_i^t + u_i^t S_i^t + v_i^t G_i^t + \max_{\mathbf{Y}\in\mathcal{U}} C_i^t(p_i^t(\mathbf{Y})) + M \times (s_+^t(\mathbf{Y}) + s_-^t(\mathbf{Y})) \right) \tag{2.1}$$

s.t. $(\mathbf{x},\mathbf{u},\mathbf{v})$ satisfies (1.2) – (1.5)

$$\sum_{i=1}^{N_g} p_i^t(\mathbf{Y}) + \sum_{i=1}^{N_w} w_i^t + \sum_{i=1}^{N_a} a_i^t + s_+^t(\mathbf{Y}) - s_-^t(\mathbf{Y}) = \sum_{j=1}^{N_d} d_j^t,\ \forall t,\ \forall \mathbf{Y}\in\mathcal{U} \tag{2.2}$$

$$-RD_i^t \le p_i^t(\mathbf{Y}) - p_i^{t-1}(\mathbf{Y}) \le RU_i^t,\ \forall i,\ t,\ \forall \mathbf{Y}\in\mathcal{U} \tag{2.3}$$

$$-f_l^{\max} \le \mathbf{a}_l^{'}(\mathbf{B}_p\mathbf{p}^t(\mathbf{Y}) + \mathbf{B}_w\mathbf{w}^t + \mathbf{B}_a\mathbf{a}^t - \mathbf{B}_d\mathbf{d}^t) \le f_l^{\max},\ \forall t,\ l, \forall \mathbf{Y}\in\mathcal{U} \tag{2.4}$$

$$-f_{l,i}^{\max} \le \mathbf{a}_{l,i}^{'}(\mathbf{B}_p\mathbf{p}^t(\mathbf{Y}) + \mathbf{B}_w\mathbf{w}^t + \mathbf{B}_a\mathbf{a}^t - \mathbf{B}_d\mathbf{d}^t) \le f_{l,i}^{\max},\ \forall t,\ \forall l\in CT,\ \forall \mathbf{Y}\in\mathcal{U} \tag{2.5}$$

$$p_i^{\min} \le p_i^t(\mathbf{Y}) + q_i^t(\mathbf{Y}) \le p_i^{\max},\ \forall i,\ t,\ \forall \mathbf{Y}\in\mathcal{U} \tag{2.6}$$

$$p_i^{\min} x_i^t \le p_i^t(\mathbf{Y}) \le p_i^{\max} x_i^t,\ \forall i,\ t,\ \forall \mathbf{Y}\in\mathcal{U} \tag{2.7}$$

$$q_i^t(\mathbf{Y}) \le \overline{q}_i^t,\ \forall i,\ t,\ \forall \mathbf{Y}\in\mathcal{U} \tag{2.8}$$

$$\sum_{i=1}^{N_g} q_i^t(\mathbf{Y}) \le \overline{q}^t,\ \forall t,\ \forall \mathbf{Y}\in\mathcal{U} \tag{2.9}$$

The objective function (2.1) has two parts. The first part is the commitment cost. The second part is the worst case dispatch and penalty costs. In the robust model, the commitment decision takes into account all possible future load, wind and interchange characterized in the uncertainty set. Such a UC solution remains feasible, thus robust, for any realization of uncertainty. The uncertainty set $\mathcal{U}$ is equal to $\mathcal{U}^{NU}$ under the R-NU approach, while it is $\mathcal{U}^U$ under the R-U approach.

Under the Det approach, the system reserve requirement $\overline{q}^t$ is increased by $\Delta\overline{q}^t$, which is the incremental reserve. The incremental reserve $\Delta\overline{q}^t$ is used to accommodate potential changes in renewable generation. The resulting UC formulation under the Det approach is very similar the base UC model (1), except that the reserve requirement in the last constraint (1.23) is replaced by the sum of the reserve requirement and the incremental reserve. More specifically, the UC model under the Det approach is the following:

$$\min_{\mathbf{x,u,v,p,q}} \sum_{t=1}^{T}\sum_{i=1}^{N_g} x_i^t F_i^t + u_i^t S_i^t + v_i^t G_i^t + C_i^t(p_i^t) + M \times (s_+^t + s_-^t) \tag{3.1}$$

s.t. $(\mathbf{x,u,v,p,q})$ satisifies (1.2) – (1.12)

$$\sum_{i=1}^{N_g} q_i^t \le \overline{q}^t + \Delta\overline{q}^t \ , \ \forall t \tag{3.2}$$

In particular, $\Delta\overline{q}^t$ is chosen in the following way:

$$\Delta\overline{q}^t = \beta^t \left(\hat{d}_j^t + \hat{w}_i^t + \hat{a}_i^t \right),$$

where the parameter β^t varies between 0 and 1 to control the conservatism of the deterministic UC solution. When $\beta^t = 0$, model (3) is equivalent to the original deterministic model (1). As β^t increases, the system procures more incremental reserve. The Det approach assumes that the worst-case scenario occur when $\beta^t = 1$, i.e. load reaches its upper bound while wind and interchange reach their lower bounds in their intervals. Such a worst-case scenario does not take into account the transmission constraints, so the resulting UC solution is not necessarily robust.

For a given unit commitment solution $\mathbf{x}^*$, an economic dispatch problem is solve for realized load, wind and interchange $(\tilde{\mathbf{d}}, \tilde{\mathbf{w}}, \tilde{\mathbf{a}})$ in the simulation. The dispatch problem is formulated as follows:

$$\min_{\mathbf{x,u,v,p,q}} \sum_{t=1}^{T}\sum_{i=1}^{N_g} C_i^t(p_i^t) + M \times LS^t \tag{4.1}$$

$$\sum_{i=1}^{N_g} p_i^t + \sum_{i=1}^{N_w} (\tilde{w}_i^t - ws_i^t) + \sum_{i=1}^{N_a} \tilde{a}_i^t + LS^t = \sum_{j=1}^{N_d} \tilde{d}_j^t \ , \forall t \tag{4.2}$$

$$-RD_i^t \le p_i^t - p_i^{t-1} \le RU_i^t \ , \ \forall i \ , \ t \tag{4.3}$$

$$-f_l^{\max} \le \mathbf{a}_l^{'}(\mathbf{B}_p\mathbf{p}^t + \mathbf{B}_w(\tilde{\mathbf{w}}^t - \mathbf{ws}^t) + \mathbf{B}_a\tilde{\mathbf{a}}^t - \mathbf{B}_d\tilde{\mathbf{d}}^t) \le f_l^{\max} \ , \ \forall t \ , \ l \tag{4.4}$$

$$-f_{l,i}^{\max} \le \mathbf{a}_{l,i}^{'}(\mathbf{B}_p\mathbf{p}^t + \mathbf{B}_w(\tilde{\mathbf{w}}^t - \mathbf{ws}^t) + \mathbf{B}_a\tilde{\mathbf{a}}^t - \mathbf{B}_d\tilde{\mathbf{d}}^t) \le f_{l,i}^{\max} \ , \quad \forall t \ , \ \forall l \in CT \ , \tag{4.5}$$

$$p_i^{\min} \le p_i^t + q_i^t \le p_i^{\max} \ , \ \forall i \ , \ t \tag{4.6}$$

$$p_i^{\min} x_i^{*,t} \le p_i^t \le p_i^{\max} x_i^{*,t} \ , \ \forall i \ , \ t \tag{4.7}$$

$$q_i^t \le \overline{q}_i^t \ , \ \forall i \ , \ t \tag{4.8}$$

$$\sum_{i=1}^{N_g} q_i^t \le \overline{q}^t \ , \ \forall t \tag{4.9}$$

$$\mathbf{ws}^t \le \tilde{\mathbf{w}}^t \ , \ \forall t \tag{4.10}$$

In the dispatch problem (4), the incremental reserve $\Delta\overline{q}^t$ is not included in the reserve requirement constraint (4.9). This is because the incremental reserve is used to hedge against load, wind and interchange uncertainty in problem (3) under the Det approach, while the original reserve requirement $\overline{q}^t$ is designed for loss of largest generation contingency. In the simulation, we only sample the uncertain load, wind and interchange to test the robustness of the UC solutions, so the incremental reserve is not needed in problem (4).

Additionally, we also model wind spillage in the dispatch problem. Because the cost wind power is very small, when there is a supply surplus in the system, the dispatch model also to curtail wind at no cost to maintain energy balanced. In the balance constraint (4.2), we include the variable ws_i^t to allow wind spillage. In addition, the constraint (4.10) restricts that the amount spillage should not exceed the realized wind.

Appendix B
Features Used in Clustering Forecast Data

Using the historical forecast data described in the Methods section, we identified three key wind features used to cluster days into groups: 1) the day's total level of wind across all hours and locations, 2) the trend over the day, defined as the difference between the late day (hours 21-24 ET) and the early morning (hours 3-6 ET) averages (across all locations), and 3) the mid-day deviation from the trend (average of 10-16 ET wind average across all locations minus the average of the late day and early morning averages, as defined earlier). Figure 1 below shows a schematic of the different levels of two of these features.

These three features were extracted for each of the 1096 day-long wind trajectories and used to cluster the days with the k-means clustering algorithm (Hartigan and Wong 1979). The algorithm requires specifying the number of clusters. The degree of cluster separation (as measured by the ratio of between-cluster sum of squares to total sum of squares) and practical considerations (using too many clusters can result in too few observations in any given cluster to estimate the parameters of the error distribution in the cluster) suggested using 4 clusters.

The load forecast data, on the other hand, suggested the use of only one feature, the day's total load, and using 3 clusters in the k-means algorithm. Note that this clustering was performed independently of the wind feature clustering as the examination of the two quantities indicated a lack of a significant correlation.

Thus, a given day can be assigned to one of 4 wind groups and one of 3 load groups. Since there are 4x3 = 12 possible combinations of these groups, to cover an entire range of wind and load patterns, one needs to run the robust UC algorithm and economic dispatch for each of these 12 combinations.

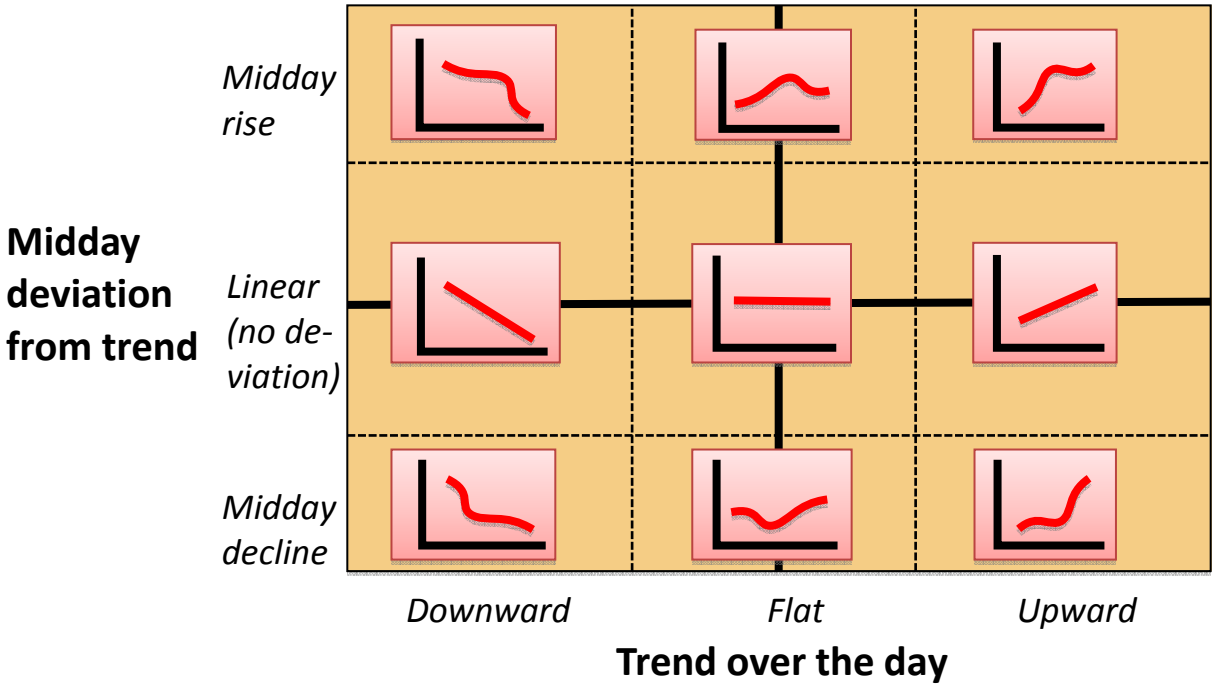

Figure 1. Schematic of wind patterns corresponding to combinations of various levels of the trend and midday deviation from the trend features.

Appendix C

Model for the Observed Wind and Load and Parameter Estimation

The examination of the forecast error data described in the Methods section motivated the use of the following relative error model for the observed 24-hour trajectory of each of these quantities:

$$\boldsymbol{Y} = (\boldsymbol{F} + b) \times \exp(\boldsymbol{\varepsilon}) - a, \qquad (1)$$

where

$\boldsymbol{Y}$ is an observed vector of wind generation or load for a given day, by time and location combination

$\boldsymbol{F}$ is the day's day-ahead forecast wind generation or load, by time and location combination,

$\boldsymbol{\varepsilon}$ is the vector of forecast errors associated with wind generation or load, by time and location combination, and is modeled as a multivariate normal random variable with

mean vector $\boldsymbol{\mu_\varepsilon}$ and covariance matrix $\boldsymbol{\Sigma_\varepsilon}$, and *a* and *b* are scalar constants, determined from the data, as described below. As alluded to earlier, the parameters of the forecast error distribution, $\boldsymbol{\mu_\varepsilon}$ and $\boldsymbol{\Sigma_\varepsilon}$, are cluster-specific and are determined from the observed data, as described below.

The values of the constants *a* and *b* were determined empirically by examining the data and finding the values that would yield an approximately multivariate normal distribution for the error vector $\boldsymbol{\varepsilon}$. In the case of wind, the best values for this purpose proved to be the mean value of the observed and forecast values, respectively. In the case of load, 0 was found to be the best value for both constants.

The values of the error mean vector $\boldsymbol{\mu_\varepsilon}$ and covariance matrix $\boldsymbol{\Sigma_\varepsilon}$ in each cluster were determined by matching the cluster's mean and covariance of the observed values $\boldsymbol{Y}$, so as to ensure that the generated 24-hour trajectories (scenarios) of wind or load have the statistical characteristics of the wind and load trajectories observed in the data.

Specifically, it is straightforward to show that if the vectors $\boldsymbol{Y}$ in a particular cluster have a mean vector $\boldsymbol{\mu_Y}$ and covariance matrix $\boldsymbol{\Sigma_Y}$, then if model in (1) is assumed, the error mean vector $\boldsymbol{\mu_\varepsilon}$ and the *(i,j)* entry of the error covariance matrix $\boldsymbol{\Sigma_\varepsilon}$ are given by

$$\boldsymbol{\mu_\varepsilon} = \log((\boldsymbol{\mu_Y} + a)/(\boldsymbol{F} + b)) - 0.5*\log(\boldsymbol{\sigma_Y} / (\boldsymbol{\mu_Y} + a)^2 + 1) \qquad (2)$$

$$\sigma_\varepsilon^{ij} = \log(\sigma_Y^{ij} / ((\mu_Y^i + a)*(\mu_Y^j + a)) + 1) \qquad (3)$$

where

$\boldsymbol{\sigma_Y}$ is the variance vector of $\boldsymbol{Y}$ (diagonal of the covariance matrix $\boldsymbol{\Sigma_Y}$),

σ_Y^{ij} is the *(i,j)* entry of the covariance matrix $\boldsymbol{\Sigma_Y}$, and

μ_Y^i is the *i*-th entry of the mean vector $\boldsymbol{\mu_Y}$

Note that the model in (1) can produce values of $\boldsymbol{Y}$ that are outside the physically feasible range for either wind (0 to capacity) and load (a range of positive values, with minimum and maximum known from historical data). This happened for a very small fraction of the values (3% for wind and 1% for load), and in these cases the values outside the range were truncated to either the lower or the upper bound, whichever was closer.