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Stability of optical bullets in 2-dimensional fiber arrays

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ABSTRACT

Long-lived light-bullets fully localized in both space and time can be generated in continuous-discrete optical media such as multi-core optical fiber or waveguide arrays. In this paper we present detailed theoretical analysis on the existence and stability of the discrete-continuous light bullets using a most general model that occurs in a number of applications.

Key words: optical solitons, fiber arrays, optical bullets, stability

1. INTRODUCTION

Light-matter interaction continues to be an active and fascinating area of research in nonlinear science. Over the past 30 years, advances, in particular in fiber optics technology, has had an impact not only in applications, but it has provided a laboratory to display nonlinear phenomena such as modulation instability, solitons formation and interactions, supercontinuum generation, parametric amplification, optical wave turbulence and many others. Even earlier, when the first experimental evidence of nonlinear properties of crystals came about, the possibility of balancing diffraction with self-focusing so that localized wave-packets could propagate undistorted emerged as an important topic, one with broad applicability. At that time, the theory of soliton formation and its instability in the 1+2 dimensional nonlinear Schrödinger equation (NLSE), $i\partial_z u(x,y;z) + \Delta_T u + |u|^{2\sigma} u = 0$, proved that for the Kerr nonlinearity (σ=1), a 2-d spatial soliton is unstable in that it either collapses (for powers above critical) or diffracts. In this equation, *u* represents the envelope of the electric field; *(x,y)* are the spatial variables (T)ransverse to the direction of propagation *z.* At this so called critical (σd=2, d=transverse dimension), critical collapse is easily arrested by many additional terms in the model even if small. Most common examples are losses, nonlinear saturation. In particular if instead of a continuum field we have a discrete model with the corresponding discrete Laplacian, our earlier seminal work [1,2] demonstrated in a particular rectangular geometry of waveguide arrays that stable bullets propagate in such model. As in has been the case in other instances, it was years later that this theoretical discovery was demonstrated experimentally. At the time, we witnessed technological advances in photonic crystals and multi-core fibers. While these were driven by major challenges in optical communication in providing methods and techniques capable to offer transmission capacity above the limitations of the single mode-fiber communication channels [3,4]. A multi-core fiber (MCF) allows one to implement spatial division multiplexing, enabling a scale-up in transmission capacity per-fiber and more recently, they were used for the experimental demonstration of Light Bullets (LB) in a multi-core optical array [5,6]. Furthermore, as the technology of multicore fibers continues to advance so are the possible applications of such arrays. Spatial de-multiplexing is also important in emerging applications of multi-core fiber is in the field of high-power fiber lasers [7,8]. The reason this technology has been instrumental in novel design of fiber lasers is the limitation in generating high power outputs in a single laser. The power transmitted through the single mode fiber is limited by the different

non-linear effects. To overcome this threshold and generate high powers, a promising approach is to coherently combine multiple amplifier outputs. Here, the use of multi-core fiber allows one to split the total high power into channels with power below any undesirable nonlinear effects. In other words, laser beams in each core may be transported safely being below the threshold of the detrimental nonlinear effects while the total coherently combined power can be high. In recent work [8] multi-core fibers in a hexagonal configuration was used to efficiently combine the output of multiple fiber amplifiers so that one can produce coherent output power that scales to the number of cores (equal to the number of amplifiers). We should emphasize that a critical component of the coherent beam combining scheme, is the ability to design of multicore structures with a geometrical configuration that has a more general coupling scheme that the simplest nearest neighbor. In this respect, our work presented here represents an important contribution to this application in that it studies for the first time the stability properties of LB under more general coupling schemes. Finally, this new multi-core fiber technology opens up new perspectives for the fascinating research on light bullets (see e.g. [9-15] and references therein) and can be a natural laboratory to study fundamental phenomena such as nonlinear Anderson localization, light filaments and vortices and optical rogue wave formation to name some. The MCF is a specific realization of fiber arrays with flexible mutual arrangement of cores. It is important to understand how the mutual arrangement of fibers will affect the existence the LB and their stability, which is the subject of present paper

Given the importance of LB in nonlinear science and applications mentioned above, the main objective of this work is to analyze the optical bullet features in continuous-discrete optical media including their stability under the most general coupling schemes. Our focus is on the generic models that may be applied in various applications. This theoretical work we believe will provide a framework in the design of multicore elements aiming at optimizing desirable and specific applications such as routing, switching and coherent beam combining.

2. MATHEMATICAL DESCRIPTION OF THE SYSTEM

For arrays of waveguides where light propagates mainly in the central (core) region of the individual elements and for which transverse exchange of energy is due to tails of the field overlapping neighbor waveguides, the field is well approximated by a superposition $E(x,y,z,t)=\sum_{nm} U_{nm}(z,t)F(x-x_n,y-y_m)e^{i(kz-\omega t)}+cc$ where we assume each waveguide is identical and supports a single mode *F*. The center of each waveguide is at location (x_n,y_m).

2.1 Basic equations

The equations, in dimensionless form, describing the propagation of the envelopes $U_{nm}(z,t)$ propagating in the fiber at site *(n,m)* read:

$$i\partial_z U_{nm}+(\underline{\underline{C}}\underline{U})_{nm}+\partial_{t^2}^2 U_{nm}+2\,|U_{nm}|^2\,U_{nm}=0 \qquad (1)$$

where $(\underline{\underline{C}}\underline{U})_{nm}$ represents the linear coupling profile at site (*n,m*). Two cases of interest are the uniform square and hexagonal geometries, for which the respective coupling operators are

$$(\underline{\underline{C}}\underline{U})_{nm}^{square}=c(U_{n-1,m}+U_{n+1,m}+U_{n,m-1}+U_{n,m+1}) \qquad (2a)$$

$$(\underline{\underline{C}}\underline{U})_{nm}^{hexagon}=c(U_{n-1,m-1}+U_{n-1,m+1}+U_{n,m-2}+U_{n,m+2}+U_{n+1,m-1}+U_{n+1,m+1}) \qquad (2b)$$

This system has two known conserved quantities; the Hamiltonian $H = \sum_{nm} \int (N(\underline{U};\underline{U})_{nm} + |\partial_t U_{nm}|^2 - |U_{nm}|^4) dt$ and the total power $P = \sum_{nm} \int |U_{nm}|^2 \, dt$.

The form of the discrete-continuous light bullets (that are extrema of the Hamiltonian under fixed total power P) $U_{nm}(z,t) = A_{nm}(t,\lambda) e^{i\lambda z}$ is given by the equation:

$$-\frac{\delta}{\delta U^*_{nm}}(H + \lambda P) = -\lambda U_{nm} + (\underline{\underline{C}}\underline{U})_{nm} + \partial^2_{t^2} U_{nm} + 2|U_{nm}|^2 U_{nm} = 0 \qquad (3)$$

2.2 Localized bullet solutions

Highly localized bullet solutions in some limits can be derived using an asymptotic approach. The consideration is that spatially most of the energy is concentrated in one site, (0,0) and small satellite pulses of decreasing amplitude propagate in subsequent layers. Mathematically, this means we seek solutions of the form $U_{nm}(z,t) = A_{nm}(t) e^{i\lambda z}$ with $\lambda >> 1$, where each envelope has an expansion of the form $A_{nm}(t) = A^{(0)}_{nm}(t;\lambda) + A^{(1)}_{nm}(t;\lambda) + A^{(2)}_{nm}(t;\lambda) + \ldots$.

Solution for the central core and first layer: Here the indices reflect a square array. The functional form is the same for a hexagonal array, but the indexing varies and the first layer consists of 6 cores instead of 4.

$$\begin{aligned} A_{00}(t) &= a_0(t) + O(\frac{1}{\lambda^{3/2}}) = \frac{\sqrt{\lambda}}{\cosh(\sqrt{\lambda}\, t)} + \frac{1}{\sqrt{\lambda}} \frac{\tanh(\sqrt{\lambda}\, t)}{\cosh(\sqrt{\lambda}\, t)} + O(\frac{1}{\lambda^{3/2}}) \\ A_{\pm 1,0}(t) &= a_1(t) + O(\frac{1}{\lambda^{3/2}}) = \frac{c}{2\sqrt{\lambda}} [e^{\sqrt{\lambda}\, t} \ln(1 + e^{-2\sqrt{\lambda}\, t}) + e^{-\sqrt{\lambda}\, t} \ln(1 + e^{2\sqrt{\lambda}\, t})] + O(\frac{1}{\lambda^{3/2}}) \end{aligned} \qquad (4)$$

These solutions fail to be uniformly valid beyond $|t| \geq O(\lambda)$. In fact at the pulse tails, all waveguides have solutions of the same order $A_{nm} = O(\lambda e^{-\lambda |t|}), t \geq \lambda$.

What follows are numerical simulations corresponding to these highly localized solutions as well as more general nonlinear states.

2.3 Numerical simulations

We calculate now numerically exact solutions describing continuous discrete light bullets in the equation (1) where we varied the number of elements in the array, which is an important consideration for practical systems where the number of cores is finite. The asymptotic functions $a_0(t), a_1(t)$ coincide with numerical solutions $A_{00}(t)$ and $A_{\pm 1,0}(t) = A_{0,\pm 1}(t)$ of the system (3) up to the order $O(\frac{1}{\sqrt{\lambda}^3})$. To fully assess the validity of the asymptotic analysis, we compared them with full numerical solutions for the case of a coupling constant c=1 and for the rectangular structure with NxN crossed. As figures 1and 2 show, there is good correspondence between the theory and numerical modeling for large values of λ. Figure 1 depicts dependence of the amplitude of the solution in the central and neighboring cores on the parameter lambda. Figure 2 shows comparison of the time domain structure of the light bullets for three different values of λ. As expected correspondence is good for values of λ equal to 10 and 15 and fails at λ= 5, thus the limiting value of λ for which the asymptotic state is

valid is around $\lambda_c \approx 4$. Figure 3 summarizes the spatio-temporal amplitude and phase features of the light bullet. Altogether, key features such as peak amplitude and pulse width dependence on λ are in complete agreement with those presented in figures 2b,2e and 2f in [4].

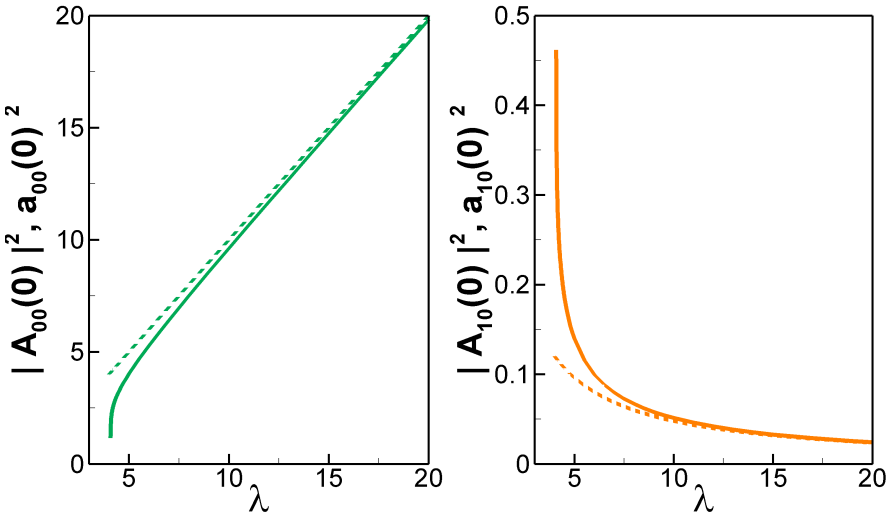

Figure 1 Dependence numerical (solid) and theory (dashed) peak powers vs the parameter λ in central (left green) and closer neighbor waveguides (right) for waveguides structure 17x17

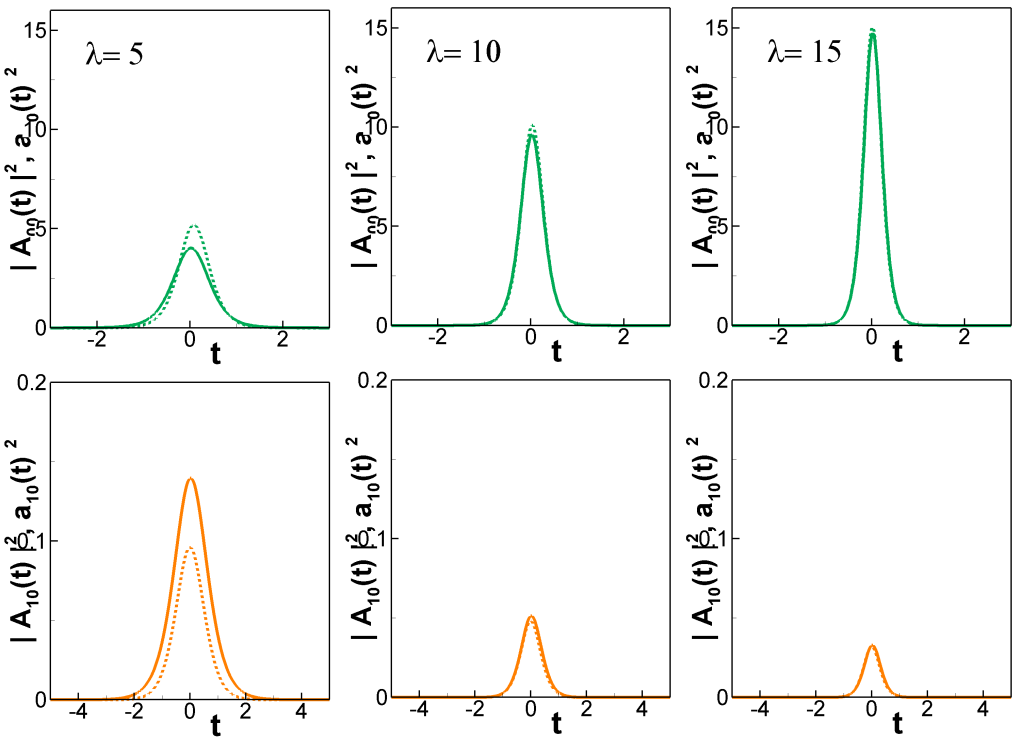

Figure 2 Comparison of numerical solutions (solid) with their analytical approximations (dashed) for different λ in the central waveguide (top green) and closer neighbor waveguide (bottom)

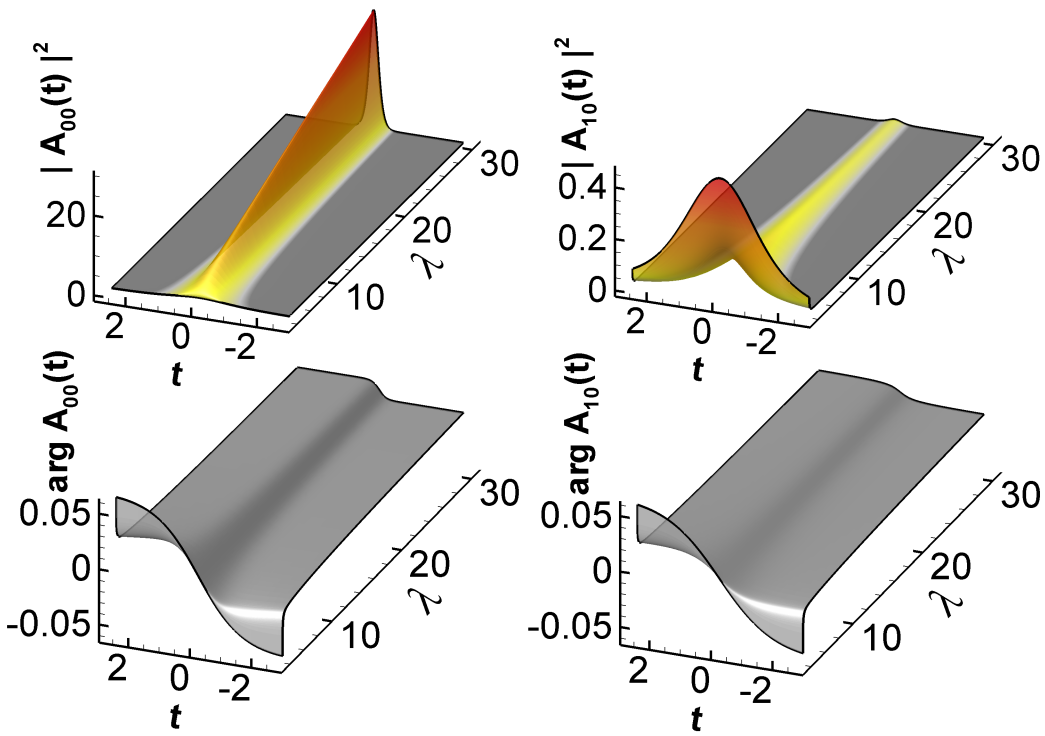

Figure 3 3D power and phase time distributions vs parameter λ for central (left) and closer neighbor waveguide (right)

Finally, global quantities of the LB such as power and the Hamiltonian are shown in figure 4. Here we observed universal behavior for different number of fiber elements. Specific to the power dependence on λ, below we discuss how it relates to the stability properties of the LB. This behavior coincides with that of figure 2a in [4].

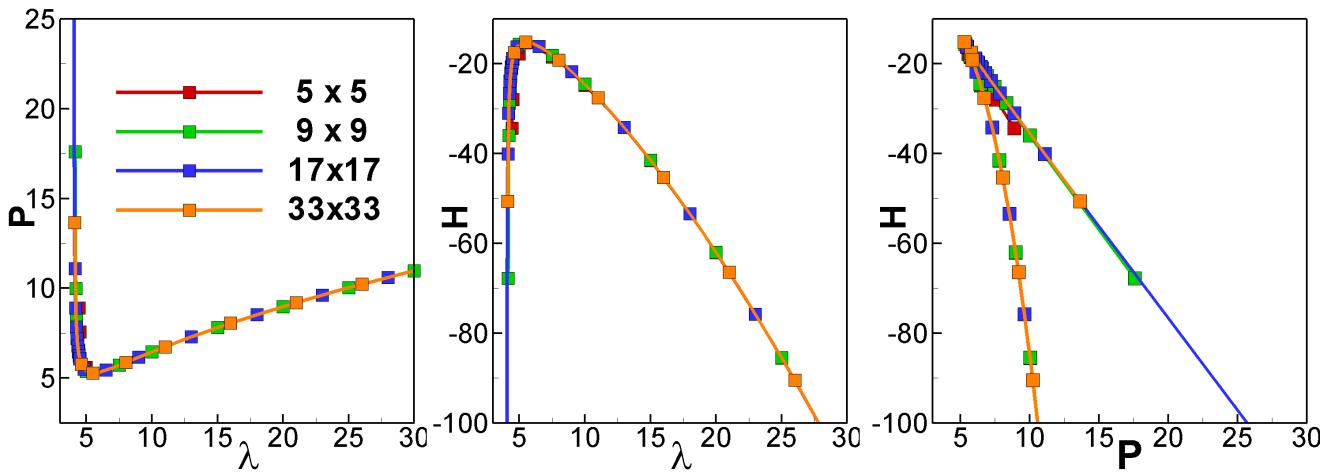

Figure 4 From left to right: dependence of the total power on the parameter λ; dependence of the Hamiltonian on the parameter λ; the Hamiltonian vs the total power Figure 4 From left to right: dependence of the total power on the parameter λ; dependence of the Hamiltonian on the parameter λ; the Hamiltonian vs the total power

2.4 Stability analysis

We now investigate the stability of solutions of the form $U_{nm}(z,t) = A_{nm}(t)e^{i\lambda z}$ by linearization $U_{nm}(z,t) = (A_{nm}(t) + f_{nm} + ig_{nm})e^{i\lambda z}$, leading to the system of linear equations

$$-\partial_z g_{nm} = -(\underline{\underline{C}}\underline{f})_{nm} - \partial^2_{t^2} f_{nm} + \lambda f_{nm} - 6\,|A_{nm}|^2 f_{nm} = (H_- f)_{nm}$$

$$\partial_z f_{nm} = -(\underline{\underline{C}}\underline{g})_{nm} - \partial^2_{t^2} g_{nm} + \lambda g_{nm} - 2\,|A_{nm}|^2 g_{nm} = (H_+ g)_{nm}$$

or

$$-\partial^2_{z^2} f_{nm} = \left[H_+(H_- f)\right]_{nm}$$

For square arrays

$$\langle \underline{f}, H_+\underline{f}\rangle \equiv \sum_{nm}\int f_{nm}(H_+ f)_{nm}\,dt = \sum_{nm}\int\left[A^2_{nm}\left(\partial_t\left(\frac{f_{nm}}{A_{nm}}\right)\right)^2 dt\right] + \sum_{nm}\int\left[\left(\sqrt{\frac{A_{n,m-1}}{A_{nm}}}f_{nm} - \sqrt{\frac{A_{n,m}}{A_{n,m-1}}}f_{n,m-1}\right)^2 dt\right]$$

$$+\sum_{nm}\int\left[\left(\sqrt{\frac{A_{n-1,m}}{A_{nm}}}f_{nm} - \sqrt{\frac{A_{n,m}}{A_{n-1,m}}}f_{n-1,m}\right)^2 dt\right]$$

and for hexagonal arrays

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$$\langle \underline{f}, H_+\underline{f}\rangle \equiv \sum_{nm}\int f_{nm}(H_+ f)_{nm}\,dt = \sum_{nm}\int\left[A^2_{nm}\left(\partial_t\left(\frac{f_{nm}}{A_{nm}}\right)\right)^2 dt\right] + \sum_{nm}\int\left[\left(\sqrt{\frac{A_{n,m-1}}{A_{nm}}}f_{nm} - \sqrt{\frac{A_{n,m}}{A_{n,m-1}}}f_{n,m-1}\right)^2 dt\right]$$

$$+\sum_{nm}\int\left[\left(\sqrt{\frac{A_{n-1,m}}{A_{nm}}}f_{nm} - \sqrt{\frac{A_{n,m}}{A_{n-1,m}}}f_{n-1,m}\right)^2 dt\right] + \sum_{nm}\int\left[\left(\sqrt{\frac{A_{n+1,m+1}}{A_{nm}}}f_{nm} - \sqrt{\frac{A_{n,m}}{A_{n+1,m+1}}}f_{n+1,m+1}\right)^2 dt\right]$$

$$+\sum_{nm}\int\left[\left(\sqrt{\frac{A_{n+1,m-1}}{A_{nm}}}f_{nm} - \sqrt{\frac{A_{n,m}}{A_{n+1,m-1}}}f_{n+1,m-1}\right)^2 dt\right]$$

In both cases and similar to the 1+1+1 case [16,17,18] the following properties of the linear operators hold:

(i) $\langle \underline{f}, H_+ \underline{f} \rangle \geq 0$ and it is equal to 0 if $f_{nm} = 0$ or $f_{nm} = A_{nm}$.

(ii) There exist some $\underline{F}$ for which $\langle \underline{F}, H_- \underline{F} \rangle < 0, \langle \underline{F}, \underline{A} \rangle = 0$

While we present two specific geometries, (i) and (ii) will be generally true for a large class of coupling schemes.

Properties (i) and (ii) allows us to conclude that the existence of negative eigenvalues of the operator H_- is a sufficient condition for instability of the nonlinear state $\underline{A}$. Furthermore one can show this condition is equivalent to the Vakhitov-Kolokolov condition on the sign of $\frac{d}{d\lambda} P = \frac{d}{d\lambda} \langle \underline{A}, \underline{A} \rangle$.

For the highly localized solutions of the previous section, one finds that the power $P(\lambda) = 2\lambda + \frac{K}{\lambda^3}$, where the constant K depends on the coupling coefficient and the geometry. Observe that a minimum is achieved at $\lambda_c = \left(\frac{3K}{2} \right)^{1/4}$, so that stability of the localized (λ>1) bullet is assured for coupling strengths below some critical value $K_c = 2/3$. This stability criterion is also in agreement with the known stability of one dimensional solitons which in this model corresponds to the limit $K \to 0$. As it was the case above, this result is consistent to that shown in figure 2c in [4].

3. CONCLUSIONS

In this work we have studied a spatio-temporal continuous-discrete model of an optical medium consisting of a multi-core fiber or a general array of waveguides which as this and other recent work indicates, can be used in a variety of applications. We have put forward a solid theoretical framework based on asymptotics and stability conditions that not only validates the existence of stable light bullets, representing light localization in space and time, but it shows these are not only achieved for a particular array configuration and instead are generic and stable for a multitude of geometries. This can represent an opportunity to design array configurations whose topology is suitable for a particular application including preparing a LB for propagation in bulk Kerr-media [19]. It can also be a starting point in the study other phenomena in nonlinear arrays when the model can be easily extended by incorporating additional effects. Two relevant examples that come to mind, are first the inclusion of higher order temporal effects which can be important for the most intense (and temporarily short) bullets. If as figure7 in [4] suggests, there is an additional time shift that is λ dependent (for λ large), then one should be able to verify it theoretically. Clearly such dependence would be quite useful for time delay lines and for efficient coherent pulse combining [20]. The second extension that comes to mind is coupling active fibers, where at first approximation we can add to the model linear gain and saturation. Finally we limited our work to the study of light bullets in the anomalous regime. A natural extension is to perform similar studies to discrete spatio-temporal vortices and other nonlinear modes. With respect to the normal regime, recent experimental results [21] demonstrate the existence of X-waves in 1d-semiconductor waveguide arrays. Theoretically, X-waves carry infinite energy, so once a proper renormalization of Power and the Hamiltoninan is done to the work presented here, an improved existence and stability analysis to 1d and 2d arrays n the normal dispersion regime could be done.

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